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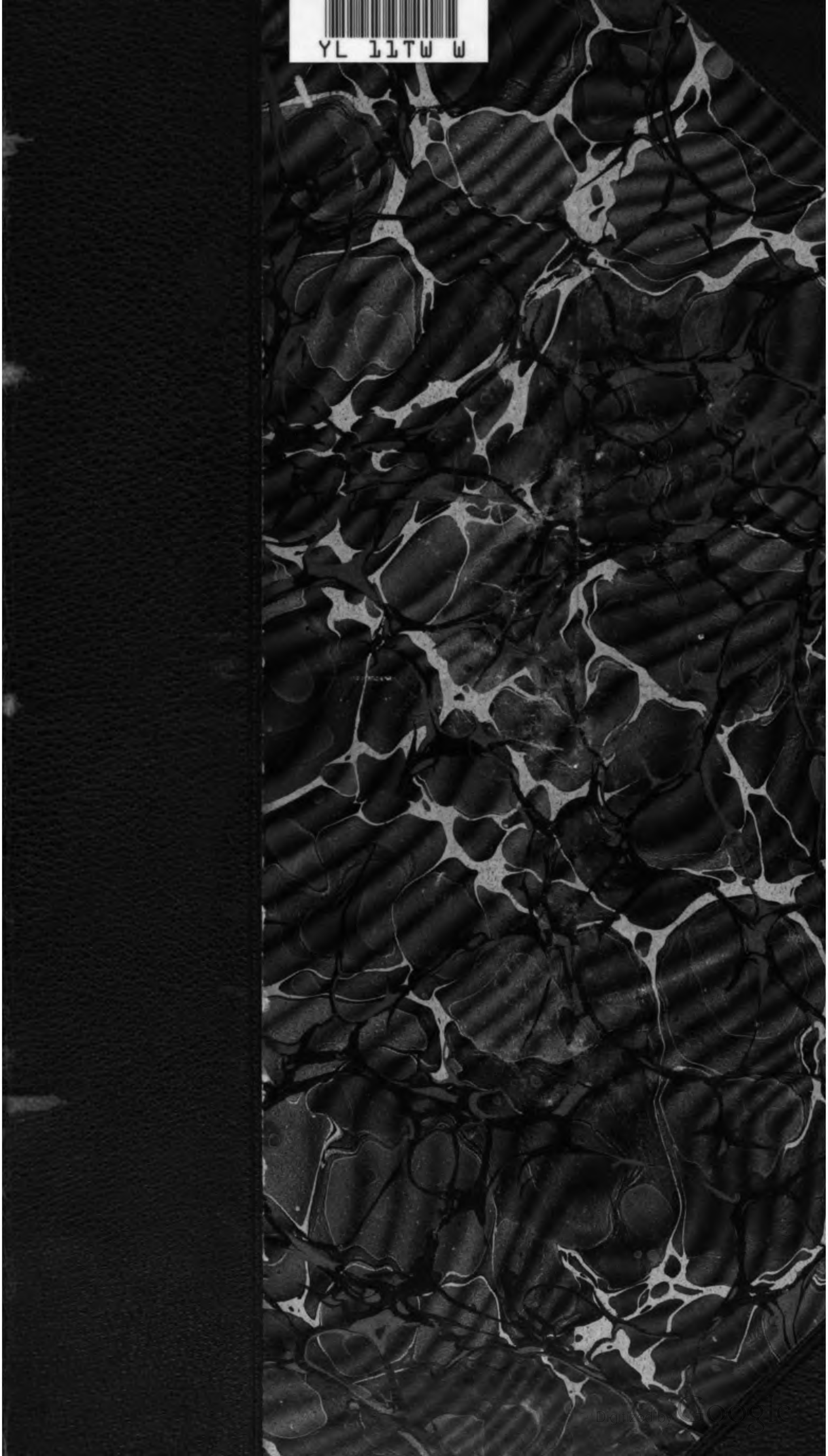

One of the Books of
Hark! the world so loud, and these, the movers of the world, so still.
George W. Pierce

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THE

PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS

CONDUCTED WITH THE CO-OPERATION
OF THE
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BY
EDWARD L. NICHOLS, ERNEST MERRITT,
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THE

PHYSICAL REVIEW.

SOME NEW RECTIFYING EFFECTS IN CONDUCTING GASES.

By P. G. Nutting.

THE passage of electricity from a metal to a gas or from a gas to a metal occurs in a manner not yet well understood. The present investigation was undertaken to determine what conditions of the metal or gas favor the transfer of electricity from one to the other and if possible to throw some light on the mechanism of surface condition. While the results here presented are crude, they bear directly on the theory of metal-gas conduction and the method, though very simple, is believed to be new. The work suggested itself during some spectroscopic work with Plücker tubes.

If we use a direct current in studying the electrode drop in potential, we are confronted by the great difficulty in keeping the condition of the surface, temperature, pressure, etc., constant and in reproducing these conditions in successive tests. But if we use an alternating current of a maximum voltage just a little above the maximum total drop in potential between electrodes, conditions cannot appreciably change between successive alternations and the excess of current in one direction may be taken as a rough measure of the relative surface conductivity at the two electrodes. It is hardly conceivable that there can be any rectifying effect due to the gas itself away from the electrodes, for this would require not only molecular or atomic dissymmetry but a fixed orientation of these particles. Hence we must regard rectification as due to conditions existing at the metal-gas surface and in the immediately adjacent metal and gas. The general form assumed by a single phase alternating current wave when a conducting gas is included in the circuit is discussed later on in relation to the dissymmetry of the wave caused by partial rectification.

The greater part of the work here described was done with the current from a transformer of 100/2,000 volts and 600 watts capacity. Occasionally some of the work was repeated on smaller 100/1,000 volt transformers of 100 or 300 watts capacity. To control the current in the secondary, a variable non-inductive resistance of about 100 ohms was placed in the primary of the transformer. The current in the secondary varied from ten to forty milliamperes according to the density of the conducting gas. Most of the work was done with ordinary Plücker tubes slighly attered according to the conditions desired. They were of the short stout form now preferred and had either rod or disk aluminum electrodes. The tubes were connected with rubber pressure tubing to a Geryk pump and hydrogen generator by a three way cock so that they could be quickly exhausted and filled or washed with hydrogen. A direct current Weston milliammeter in series with the tube gave the amount of current rectified while an alternate current voltmeter also in series gave the total current. In this way rectification as small as half of one per cent. could be easily measured and a tenth as much detected. It is difficult to prepare a tube that will not show as much as one per cent. rectification. The effects here recorded are as large as five to forty per cent. and decidedly reversible.

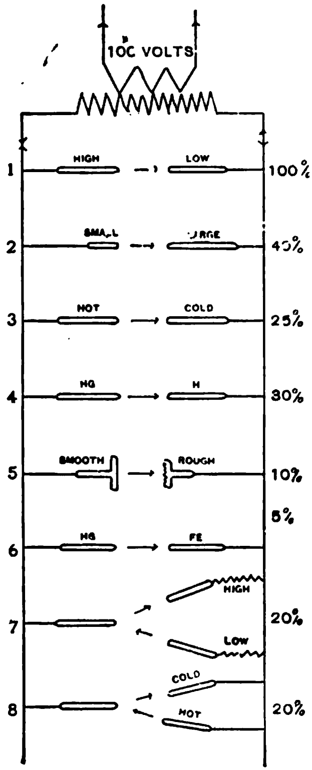

Fig. 1.

The principal rectifying effects obtained are indicated diagrammatically in the accompanying figures. The number at the left corresponds with the number of the descriptive paragraph in the context. The titles *pressure*, *temperature*, etc., indicate the differences which cause rectification, the percentages at the right indicate the maximum percentage of the current rectified, while the intermediate arrow indicates its direction. Except the first, the effects showed but little variation in percentage of rectification with the density of the conducting gas as the pressure on this was varied from 6 to 0.1 mm.

1. The difference in pressure of the gas about the two electrodes was produced by constricting the tube to a fine capillary between the electrodes and rapidly exhausting. If the exhaustion is sufficiently rapid, complete rectification occurs, *i. e.*, half of the current wave fails to pass at all. Judging by the width of the cathode dark space the corresponding pressures in the two bulbs are about 4 mm. and 0.3 mm. Sudden admission of gas into an exhausted tube reverses the rectification.

We can hardly attribute this effect to any differences within the electrodes or to the ease of escape from metal to gas. But if we consider the current carried by the negative electrons alone, then the current will be able to start much more easily from the electrode surrounded by the more rarefied gas, for here the electrons will have a greater free path and will be more easily able to attain ionizing velocity. Hence the (negative) current will travel most easily from low to high pressure, as observed. A cruder and less tenable explanation is possible if we consider the current carried by both positive and negative electrons. The strong current of gas through the capillary from high to low pressure, would tend to sweep the larger positive electrons with it rather than the much smaller negative electrons. Hence the (positive) current would flow from high to lower pressure as observed.

2. The relative size of the electrodes is an important factor in producing rectification. The excess of current flows from the small to the large electrode. When the large electrode was three times the area of the smaller a rectification of 12 per cent. was observed; when it was ten times the area, 35 per cent. was observed. The tests were made with aluminum electrodes in Plücker tubes. When the pressure is so great and the current so small that the

glow covers only a portion of the larger electrode, the effect is null or reversed. After the current has been passing for some time, the heating effect next described (3) may mask the effect of relative size. There appears to be little if any difference in size of the cathode dark space at the large and small electrodes.

The current density being less at the larger electrode, any competition between the electrons carrying the current will be less at this electrode. At the larger electrode, negative electrons starting out would be less interfered with by the pressure of positive electrons. The effect can hardly be traced to the difference in capacity of the large and small electrodes, for connecting one electrode to a large capacity or even to earth produces but a slight rectifying effect.

3. A difference in temperature of the electrodes gives a partly rectified current from the hot to the cold electrode. When one bulb of a Plücker tube was heated to about 250° and the other bulb kept cold by a wet cloth and protected by an asbestos screen, a rectification as large as 25 per cent. was observed on starting the current. This decreased rapidly as the current heated the cold electrode. Using tubes with disk electrodes the cathode dark space was at least 50 per cent. larger at the hot electrode than at the cold. At the same time the canal ray glow within the dark space was scarely perceptible while at the cold electrode it nearly filled the cathode dark space.

A continuous current heats the cathode much more energetically than the anode. Hence when an alternating current passes between a hot and a cold electrode we should expect the rectified portion to pass in such a direction as to heat the cold electrode more than the hot and so produce equilibrium. This is what actually occurs. But the electron theory would lead up to expect a current from cold to hot. At the hot electrode negative electrons escape from the metal most easily, they have a longer free path to get started and produce ionization and this movement is less effect from the mercury vapor toward the hydrogen. In spectroscopic work I have frequently observed complete rectification under similar conditions.

In some previous work[1] on the spectra of mixed gases it was shown that in a mixture, other things being equal, gases or vapors of great atomic weight appear to give always greater spectral energy than those of lower. If we take this to mean that they carry current more readily or are more readily ionized, we have a simple explanation for the above rectifying effect. The current is more easily started in the bulb containing heavier vapor so that half the alternating current wave is reduced in area according to the diagram discussed later.

5. The condition of the surface of the electrodes appears to determine a slight but decided rectifying effect. Aluminum disk electrodes were used, one polished and the other cut in ridges like a file with a sharp knife. An excess of current passed from the smooth to the rough electrode. Whether this effect was anything more than that due to a greater exposed surface could not be determined.

6. With one electrode mercury and the other iron of the same size only a slight rectifying effect from mercury to iron was observed on starting the current. After the current had been running some time, mercury vapor over the mercury electrode tended by (4) to increase the rectification, while the much hotter iron electrode by (3) tended to decrease it. It would appear that the action of the Cooper Hewitt rectifier on a three phase circuit is largely due to the much greater surface exposed by the mercury electrode and the greater density of mercury vapor in its immediate neighborhood. If cathode drop of potential played an important rôle we should have obtained greater rectification with the tube just described and a single phase current.

Tests 7 and 8 were made with tubes provided with three mercury electrodes. One of these was connected to one 2,000-volt terminal ance electrode becoming more heated by the greater current. When connections were made with equal resistances and one electrode heated, the current passed from hot to cold in accordance with (3) and (4).

We may get a better idea of what actually occurs in the process of partial rectification from the accompanying figure (Fig. 2). Let the curve $\epsilon(i)$ be the characteristic gas conduction curve for the tube used, *i. e.*, the fall of potential between electrodes as a function of the current. Impress the alternating voltage $E(t)$ on the tube having such an external (ohmic) resistance in circuit that the line $E–iR$ will have the slope indicated, then $i(t)$ will represent the form of the corresponding curve of current through the tube. Any one who has ever plotted these current curves knows how entirely their form changes with even a slight change in the form of the gas curve $\epsilon(i)$, and these $\epsilon(i)$ curves change in form greatly as conditions in the tube are varied. For instance, at very low pressures the curves are elevated and flattened. At high pressures it may rise continually, particularly if the electrodes be small or pointed.

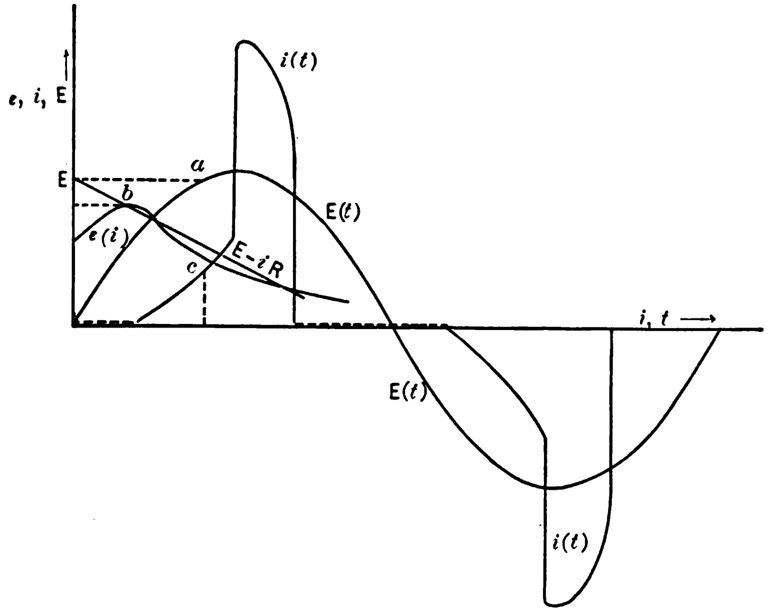

Fig. 2.

If the tube be illuminated by X-rays the maximum of the $\epsilon(i)$ curve will be sharpened, lowered and displaced towards the left. If then the electrodes are under different conditions, the $\epsilon(i)$ curve will vary in form with the direction of the current through the tube, and therefore the current curves $i(t)$ will no longer be symmetrical about the horizontal axis. The reversed (lower) part of the curve will be more flattened or rounded say than the upper and the effect will be shown as partial rectification.

A tube having one internal and one exterdal electrode showed no trace of rectification. This might have been expected since in this case the $\epsilon(i)$ curve rises steeply and continuously and the current curve $i(t)$ is a low broad sine curve which would show little if any dissymmetry.

National Bureau of Standards,
April, 1904.

THE ABSORPTION OF ENERGY IN CONDENSERS.

By R. F. EARHART.

LORD RAYLEIGH has described a method for making certain measurements in alternate current phenomena which is particularly suitable for measuring energy losses in circuits having low power-factors.[1] This instrument has been frequently referred to as Rayleigh's phasemeter. Its application to measurements of losses due to magnetic hysteresis has been successfully employed by Johonnott[2] and others.

It occurred to the writer that this method might be employed in the measurement of losses due to the absorption of energy in dielectrics. There is no doubt that it affords an accurate and simple method for determinations of this general class.

In the experiments to be described some results were obtained which may appear satisfactory. In other cases, while consistent results were obtained, the exact quantitative results are of somewhat doubtful value.

In a condenser of comparatively small capacity, say one microfarad or less, the amount of energy that will be transmitted when a perodic E.M.F. of 100 volts is applied will be small; this will apply particularly when resonance conditions are lacking. If the condenser is a good one, the amount of energy absorbed in such transmission will be a small per cent. of the energy entering the system. It will appear that the difficulty of measuring such a quantity exactly is inherent in the problem itself. It is hoped that the results obtained will show, that in certain cases at least, this method may be successfully applied. Under other conditions the possibilities for error are sufficiently great to cast serious doubts on the value of the quantitative results.

The problem will be discussed under three general heads: First, an outline of the general theory of the phasemeter and its application to the general problem of dielectric hysteresis. Second, an expression for the determination of the efficiency of a condenser subject to a simple perodic E.M.F. Third, experimental results and conclusions.

---

[1] Phil. Mag., May, 1897.

[2] PHYSICAL REVIEW, Nov., 1902.

GENERAL THEORY OF THE INSTRUMENT.

In measurements of power in alternate current circuits, it is necessary to know three things, *viz.*, the current, E.M.F., and the angle of advance or lag of the current with respect to the E.M.F. Lord Rayleigh has suggested a simple method for determining these quantities.

A piece of soft iron wire is suspended between two coils having a common axis and lying with the center of the needle in this axis. (See Fig. 1.)

If an alternate current be passed through one of these coils the needle will be subject to a couple varying with the square of the current and with 2θ, where θ is the angle the axis of the needle makes with the common axis of the coils. This will apply for any fixed position of the coils with respect to the needle. The couple will be a maximum when $\theta = 45°$. If we consider such a needle suspended in the manner described the couple on the needle due to the magnetic field created by the current will vary simply as the square of the current. A suspended needle will give deflections proportional to the couple acting and it may be shown that for small angular deflections, the deflections will be proportional to the square of the current passing through the coil. The current passing through the coil may therefore be measured by the deflection produced and expressed in scale divisions. By calibrating the instrument for the coil in a given position these readings may be reduced to any desired units.

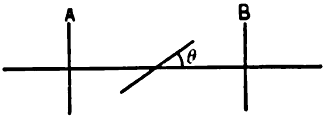

Fig. 1.

Similarly if an alternate current be passed through the second coil the current therein could be measured in a similar manner.

Again if currents be taken from points having a common P.D. and passed simultaneously through both coils, the deflections would depend on the value of the currents passing through (*A*) and (*B*) and upon their phase relation.

Consider currents of the sine type passing through (A) and (B). Let $a(\cos wt - \varphi)$ represent the instantaneous current in (A) and $b \cos wt$ the instantaneous current in (B) then

$$\frac{1}{\tau}\int_0 [Ha \cos(\omega t - \varphi)]^2 dt = \frac{H^2a^2}{2} \qquad \text{(I.)}$$

and

$$\frac{1}{\tau}\int_0^{\tau} [Kb \cos \omega t]^2 = \frac{K^2b^2}{2} \qquad \text{(II.)}$$

The constants H and K being such that when multiplied by the value of the instantaneous current will give the instantaneous values of the intensity of the field produced at the needles by the respective coils.

The constant $(H^2a^2)/2$ we may put $= A^2$ and will be proportional to the mean square value of the current and may be represented by the deflection of the needle.

Similarly put $(K^2b^2)/2 = B^2$.

When the currents act simultaneously through both coils giving couples we may have two cases.

First acting in the same direction

$$\frac{1}{\tau}\int_0^{\tau} Ha[\cos(\omega t - \varphi) + Kb \cos \omega t]^2 dt = C_1^2 \qquad \text{(III.)}$$

Second, in opposite directions

$$\frac{1}{\tau}\int_0^{\tau} Ha[\cos(\omega t - \varphi) - Kb \cos \omega t]^2 dt = C_2^2 \qquad \text{(IV.)}$$

Also

$$\left.\begin{aligned} C_1^2 &= A^2 + B^2 + 2AB \cos \varphi. \\ C_2^2 &= A^2 + B^2 - 2AB \cos \varphi. \end{aligned}\right\} \qquad \text{(V.)}$$

Solving for $\cos \varphi$ we have

$$\cos \varphi = \frac{C_1^2 - C_2^2}{4AB.} \qquad \text{(VI.)}$$

By using one coil to obtain readings giving the P.D. between two points, the second coil to obtain the current flowing through any system from these same points and then using both coils simultaneously, all the data necessary for calculating energy loss in the system will be afforded.

In the application of this method to energy losses in condensers, the condenser was placed in a circuit containing one such coil; the other serving to measure the P.D. The arrangement is shown in Fig. 2. S_1 and S_2 are switches, the one is the circuit containing the condenser being a reversing switch.

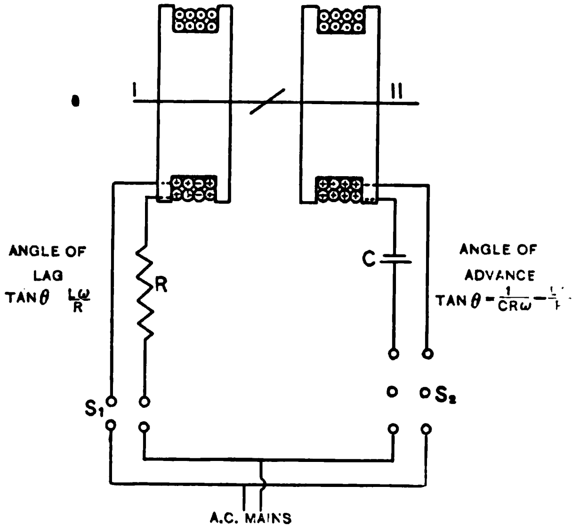

Fig. 2.

The angle of advance of the current over the E.M.F. in the condenser circuit may be calculated from the well-known formula

$$\tan\theta = \frac{1}{CR\omega} - \frac{L\omega}{R} \qquad \text{(VII.)}$$

The coil contained in the condenser circuit had an external diameter of 8.5 cm. and consisted of 24 turns of heavy copper wire. The resistance of this entire branch, obtained by short-circuiting the condenser was .67 ohms. For a condenser of 1 microfarad capacity and on a circuit having a frequency of 60 cycles we can calculate the angle of advance assuming no absorption occurs

$$1 \text{ microfarad} = 1 \times 10^{-15} \text{ C.G.S. units.}$$

$$.67 \text{ ohms} \quad .67 \times 10^{9}$$

$$\omega = 376$$

Whence from VII. $\tan\varphi = 3968$.

The second term of the right hand member (Equation VII.) can be entirely neglected since this additive term will be very small and the first term large. The angle corresponding to $\tan\theta$ is 89° 59′ 20″ or for a perfect condenser in such a circuit the current leads the impressed E.M.F. almost 90°. The departure from this angle of lead is less than one minute.

In the E.M.F. circuit a coil similar to the current coil is used. In this circuit is placed noninductive resistance amounting to about 4,000 ohms, so that in this circuit we have the condition for the current to be in phase with the E.M.F. In such a system the condenser current leads the E.M.F. by 90° provided no absorption of energy occurs. It remains then only to measure the actual advance by means of the four readings suggested previously and the angle of advance computed from formula VI. The difference between these two angles will give the angle of hysteretic lag. It is to be remembered that the phasemeter gives the cosine of the difference in phase which exists between the current and E.M.F. and for a good condenser this will not differ greatly from 90° and here the cosine changes most rapidly, hence the advantage of the method. It is desirable that the resistance of the current coil be kept as low as possible. The relation $\tan\theta = 1/CR\omega - L\omega/R$ will show that if the resistance of this coil and connections be several ohms we could not assume that the current would lead the E.M.F. by 90°. It would then be quite possible to compute what the advance would be for a perfect condenser and to obtain the actual advance, obtaining the angle of hysteretic lag in this way. This is objectionable, however, owing to the fact that the cosine of the angle changes essl rapidly and a sacrifice in sensibility would result. There is no difficulty in obtaining the first condition referred to. Using a delicate suspension with a concave mirror for determining deflections, and with a scale distance of 1 meter, deflections of something like 90 divisions were obtained for a 1 microfarad condenser and on a 110-volt circuit. This gives sufficient sensibility in this case.

DETERMINATION OF THE EFFICIENCY OF A CONDENSER SUBJECT TO A SIMPLE PERODIC ELECTROMOTIVE-FORCE.

In deducing the expression for cos φ (Equation VI.) which measures the actual advance of the current over the E.M.F., Lord Rayleigh has shown that this relation holds good for E.M.F.'s departing from the sine form. In calculating the efficiency of a condenser when the angle of advance is known, we may proceed from the following considerations, assuming the current to be of the sine type. In Fig. 3 let the heavy curves represent the E.M.F. and current. The power curve will be represented by the dotted line. If we define efficiency as the ratio of the work done by the condenser in discharge, to the work done in charging it, then we can obtain the efficiency.

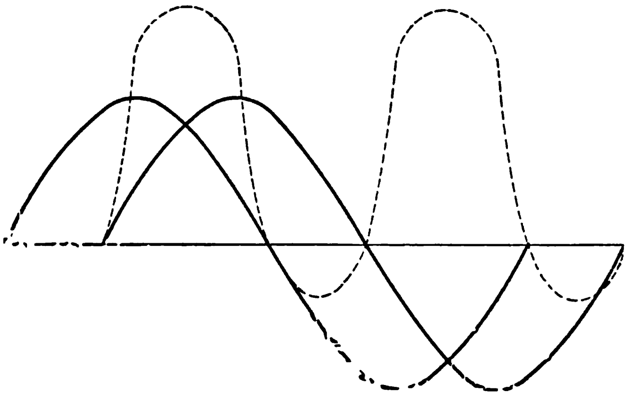

Fig. 3.

$$\text{Efficiency} = \frac{\text{Area of negative loop}}{\text{Area of positive loop}}\,.$$

It will be the ratio of the area of the negative loop to the area of the positive loop when a given phase relation exists between E.M.F. and current.

The area of the positive loop will be

$$\frac{E_1 I_1}{\omega}\cos\varphi\left[\frac{\omega t}{2}-\frac{\sin 2\omega t}{4}\right]_0^{\pi-\phi}-\frac{E_1 I_1}{4\omega}\sin\varphi\left[\cos\omega t\right]_0^{\pi-\phi}.$$

For the negative loop we have the same expression between the limits $\pi - \varphi$ and π or

$$\frac{E_1 I_1}{\omega}\cos\varphi\left[\frac{\omega t}{2}-\frac{\sin 2\omega t}{4}\right]_{\pi-\phi}^{\pi}-\frac{E_1 I_1}{4\omega}\sin\varphi\left[\cos\omega t\right]_{\pi-\phi}^{\pi}.$$

Where E_1 is the maximum value of the impressed E.M.F.

I_1 is the maximum value of the instantaneous current.

$\omega = 2\pi n$ where n is the frequency.

For the analytical work see Rosa on the Dissipation of Energy in Condensers, Physical Review, Jan., 1899.

It is therefore possible to obtain the phase relation from the phasemeter readings and apply them in this case to the determination of efficiencies.

Results.

Referring again to Fig. 2, it will be seen that circuits I. and II. proceed from points having a common difference of potential. The low resistance coils forming the phasemeter portion of the circuit were wound on wooden spools and all large masses of metal avoided in the construction, it being desirable to avoid any unknown effect due to eddy currents caused by the varying field. The soft iron needle was about .8 cm. long and was suspended by a phosphor-bronze strip. The suspended system consisted of the needle, a concave mirror and a mica vane. The resistance included in the E.M.F. circuit consisted of a series of incandescent lamps. Such resistances have practically negligable inductance and capacity. The current passing through them will be small and while the resistances will vary with temperature, after a current has been maintained through them for a short time, the resistance will not change appreciably. The time required for a complete set of readings is short and it is only necessary that the resistance should not change during that time. It is unessential just what that resistance shall be, so long as the inductance of the circuit is low.

Several condensers were used. The following measurements were made on a Leeds 10 microfarad condenser, subdivided into sections of 1 microfarad each. These condensers are guaranteed to withstand a P.D. of 1,000 volts. The dielectric consists of waxed paper concerning which the manufacturer does not speak.

The alternate current source was supplied by the mains extending to the university buildings. The fluctuations in E.M.F. in such a circuit cannot be eliminated and are the source of much trouble. In this case a second phasemeter was placed across the mains and by the indications of this "pilot" phasemeter any disturbances on the mains could be detected. It is necessary to take sets of readings under standard conditions and readings were taken when the pilot instrument indicated no disturbance during a set of readings. The readings given apply to a frequency of 60 cycles.

Condenser Section.	A^2	B^2	C_1^2	C_2^2	$\cos\phi = \frac{C_1^2 - C^2}{4AB}$	ϕ Angle Advance.	Efficiency.	Equivalent. Rest in ohms.
1	83	109	190.0±.2	192.8±.2	.0074	89° 35′	97.8%	18.5
2	91	108	197.0	200.2	.0080	89 32	97.5	21.3
3	117	109	224.2	227.8	.0079	89 33	97.6	19.1
4	94	108	200.4	203.4	.0075	89 35	97.8	16.4
5	92	108	199.0	202.0	.0075	89 35	97.8	16.6
6	87	108	194.2	198.0	.0115	89 21	96.5	30.3
7	88	108	194.0	197.0	.0077	89 34	97.7	20.2
8	88	107	194.0	197.4	.0087	89 30	97.3	21.0
9	102	109	209.2	212.6	.0080	89 32	97.5	19.9
10	77	109	185.2	188.0	.0076	89 34	97.7	22.4

In the table of readings the headings of the columns are self-explanatory with the exception of the last one. Rowland has shown that a condenser possessing absorption behaves like a perfect condenser with a resistance in series. The last column gives this effective resistance. The readings in column (A^2) indicate that the capacities of various sections are not equal. In computing the effective resistances for the different sections, the true capacity of the section was taken. The readings given represent the mean of several readings all of which were consistent and lie within the limits of error indicated.

By using coils having different resistances and at varying distances from the needle, results were obtained which show good agreement with the recorded observations. It was necessary in this case to compute the theoretical angle of advance and determine the true angle of advance. From this data the results were computed.

A long series of measurements were undertaken which leave much to be desired. An attempt was made to study the change in absorption in a condenser when changing its capacity, without changing the amount of the dielectric possessing absorption. With this end in view, several condensers were laid up, consisting of tinfoil and paper. These condensers were placed in a letter press; so that by varying the pressure the capacity could be varied, and an attempt made to determine whether the absorption changed under these conditions. The condensers after being prepared as described were heated to a temperature of approximately 120° C. and maintained at that temperature for several hours in order to rid the paper of water vapor, so far as possible. The whole arrangement was then placed in an air-tight box and the press operated through a stuffing box, the object being to prevent any change in the dielectric. Paper to a certain extent is hygroscopic and the absorption of water from the air will alter its character. Before adopting this precaution, results of a rather erratic character were obtained. This precaution does not, of course, eliminate water but merely makes possible a constant hygrometric state. In the case of certain condensers variations in capacity of from .5 to 1.2 microfarads were obtained, but the results will hardly admit of quantitative discussion. Such condensers as I have described are practically air condensers and possess so small a coefficient of absorption that the results are of little value. The efficiency in most cases is above 99 per cent., corresponding to a power factor of .003. Unsteadiness in the line potential was sufficient to mask quantitative results. I have, however, persisted in attempting to accomplish this notwithstanding.

Ten condensers were laid up at different times and the results based on several hundred sets of readings.

In eight of the ten cases the condensers showed an increase in absorption with increase in capacity. In the other cases no change could be detected. It seems possible to explain the increase in absorption in this way.

A condenser of the type described is made up of two dielectrics: first air, possessing no absorption; second of a medium possessing some absorption. In increasing the capacity we are squeezing out air or the nonabsorbing medium, while the absorbing medium forms a larger proportion of the combination dielectric. If my results are of any qualitative value they are certainly too small upon which to base any quantative statement.

The preliminary portion of this experiment was carried on at the Rose Polytechnic Institute of Terre Haute, Ind., the latter portion at Columbus, Ohio.

Physical Laboratory, Ohio State University,
Columbus, Ohio, March, 1904.

STUDIES IN LUMINESCENCE.

By Edward L. Nichols and Ernest Merritt.

III. On Fluorescence Spectra.

IN a recent paper[1] the authors have described spectrophotometric measurements of fluorescent solutions of fluorescein, eosin and naphthalin-roth, the results of which fully confirm the contention of Lommel[2] that in the case of these substances Stokes' law is not fulfilled. In the opinion of Lommel all other fluorescent substances, with the probable exception of chlorophyll, conform to Stokes' law, the shortest wave-length of the fluorescence spectrum being always of less refrangibility than the longest wave-length capable of exciting fluorescence. We have since extended our measurements to various other fluorescent substances, determining in each case by means of the spectrophotometer the location and character of the absorption band with which fluorescence is associated and the form of the curve of relative intensities in the fluorescence spectrum.

Rhodamin and Resorcin-blau.

Of the substances examined rhodamin and resorcin-blau belong to the same group as those previously examined, being organic dye stuffs which show fluorescence in solution. The fluorescence of these solutions was of sufficient intensity to permit of the use of the method described in our previous paper which, briefly, was as follows: the light of a Nernst filament was dispersed by means of a large spectrometer with a flint prism, and that portion of the spectrum which was to be used for excitation was isolated by means of a slit mounted in place of the eyepiece in the observing telescope. Two nearly monochromatic regions of the spectrum were selected in each case, one lying well within the absorption band of the solution and the other as far toward the red as it was possible to go without reducing fluorescence to an extent which rendered measurements inexact. The fluorescence spectrum was observed by means of a Lummer-Brodhun spectrophotometer from a direction at right angles to that of the exciting ray. The solution was placed in a glass cell with plane faces, the exciting light being introduced through a slit and passing through the layer of liquid lying next to the wall of the cell through which the fluorescent light passed.

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[1] Nichols and Merritt, Physical Review, Vol. XVIII., p. 403.

[2] Lommel, Poggendorff's Annalen, vols. 143, 159 and 160.

Figures 11 and 12 give in graphical form the results of these measurements. The curves of the fluorescence spectra are of the same type as those of fluorescein, eosin and naphthalin-roth given in the paper already cited. They rise from within the region of the absorption band to a well-defined maximum, which is located, generally speaking, near the infra edge of the band; beyond the maximum they fall away rapidly with increasing wave-length. The comparison spectrum in these, as in the previous cases, was that obtained from the light diffusely reflected by a block of magnesium carbonate illuminated by an acetylene flame.

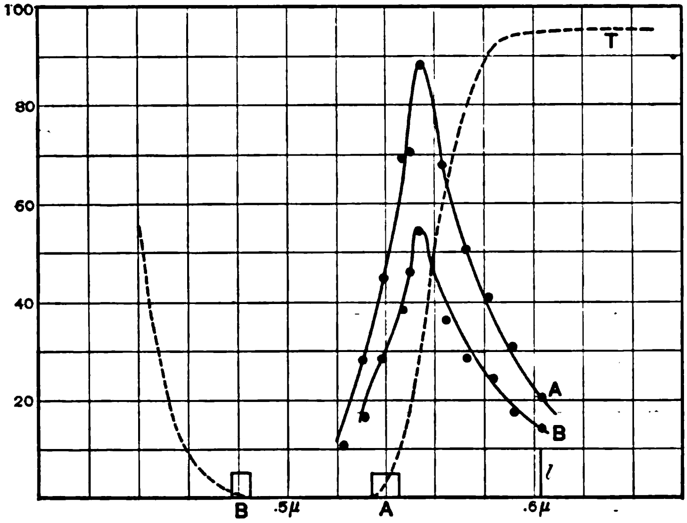

Fig. 11.

Rhodamin. Curve *A*. Fluorescence spectrum when excited by green light in the region marked *A*.

Curve *B*. Fluorescence spectrum when excited by blue light (*B*).

Curve *T*. Transmission spectrum. Layer 1.1 cm. thick.

Neither of these solutions conforms even approximately with the law of Stokes. Curve A of Fig. 12 (resorcin-blau) was obtained by means of light none of which was of shorter wave-length than .642 μ whereas the fluorescence is of measureable intensity to 600 μ. The rhodamin solution affords a striking example of the non-validity of the law; for although the *maximum* of the fluorescent spectrum lies at .554 μ an observable fluorescence was produced by means of a very nearly monochromatic light, obtained by the method of double dispersion, using two spectrometers in series as described in our previous paper, the shortest wave-length of which was .602 μ. This limiting wave-length is indicated by the line marked *l* in the figures. This substance, like eosin, which has its maximum of fluorescence at .580 μ will respond to the excitation of light of the *D* lines of sodium or to even longer waves if of sufficient intensity.

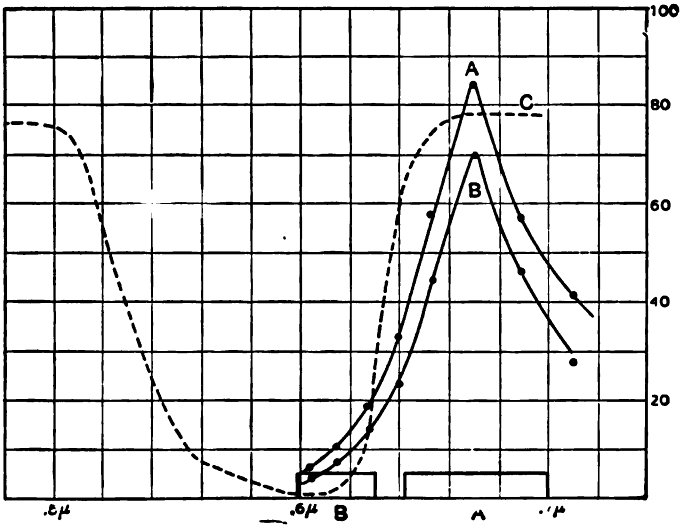

Fig. 12.

Resorcin blau in methyl alcohol. Curve *A*. Fluorescence spectrum when excited by red light lying in the region marked *A*.

Curve *B*. Fluorescence spectrum when excited by orange light (*B*).

Curve *C*. Transmission spectrum of layer 1.1 cm. thick.

The other substances to be considered in this paper are more difficult subjects for quantitative work because, being less strongly fluorescent, it is necessary to have recourse to powerful illumination, and because in several instances the fluorescence band lies near the limits of the visible spectrum — either in the violet or in the extreme red.

Quinine Sulphate.

The quinine sulphate subjected to measurement was prepared by making a saturated solution in water containing a trace of sulphuric acid. The solution was exposed to diffuse daylight in a cubical cell of glass, the faces of which measured 8 cm., and the fluorescence was observed through a slit in one of the faces at right angles to that exposed to the light. This slit was placed as near the corner of the cell as possible, so as to bring the layer of liquid lying next the glass on the exposed side into the field of view. The reference spectrum, as usual, was obtained from the light diffusely reflected from a block of magnesium carbonate, but in this case the surface of the block was illuminated by daylight so that the fluctuations of intensity might affect both spectra equally and in the same sense.

The transmission of the solution was determined by placing a second block of magnesium carbonate behind the cell and determining the intensity of the light transmitted by the liquid in the various regions as compared with that reflected from the first block. The instrument used in this and in many of the subsequent measurements to be described was a spectroscope, the collimator of which was furnished with a double Vierordt slit. Light for the comparison spectrum was introduced by means of a right-angled prism and the adjustment of intensities was made in the usual manner by means of micrometer screws.

A second set of measurements with this substance was made with the Lummer-Brodhun spectrophotometer. A mercury arc lamp was used in place of the Nernst filament of our previous experiments and the ultra-violet line .365 μ, isolated by dispersion with the large spectrometer, was employed for excitation. The comparison spectrum in this case was the magnesium block illuminated by means of the acetylene flame.

It is obvious that owing to the different distribution of intensities in the spectra of daylight and of the acetylene flame these two sets of observations would not be strictly comparable. In order, however, to make them at least approximately so, the following correction was applied to the observations by daylight. From certain data obtained by Vogel[1] and published by him many years ago, a curve was plotted giving the distribution of intensities in the spectrum of diffuse daylight with clouded sky, the spectrum of a petroleum gas flame being taken as the standard. Vogel's measurements agree as well as could be expected, considering the fluctuating character of daylight, with those subsequently made by W. H. Pickering[1] and by Nichols and Franklin.[2] As the sky was overcast at the time of making the observations on quinine sulphate just described it was assumed that the quality of daylight would on that occasion be represented with sufficient accuracy by means of this curve. By means of a similar curve giving the distribution of intensities in the spectrum of the acetylene flame as compared with the petroleum flame it is possible to determine the relation between daylight and the acetylene flame.

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[1] Vogel, Berliner Monatsberichte, 1880, p. 801.

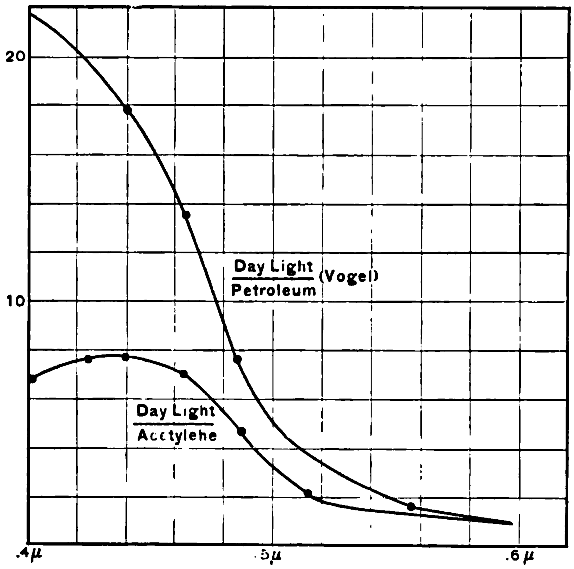

Fig. 13.

The two sources being so adjusted as to have the same intensity at the *D* line, the curve shows the ratio of intensities for other wave-lengths.

nesium carbonate is the standard. This curve (*B*, Fig. 14) corresponds precisely as regards the location of its maximum with curve *A* in the same figure, which shows the results of the measurement of the fluorescence of this substance when excited by means of the ultra-violet of the mercury lamp. Since these two sets were made with different types of spectrophotometer their agreement affords a desirable check upon the performance of the two instruments.

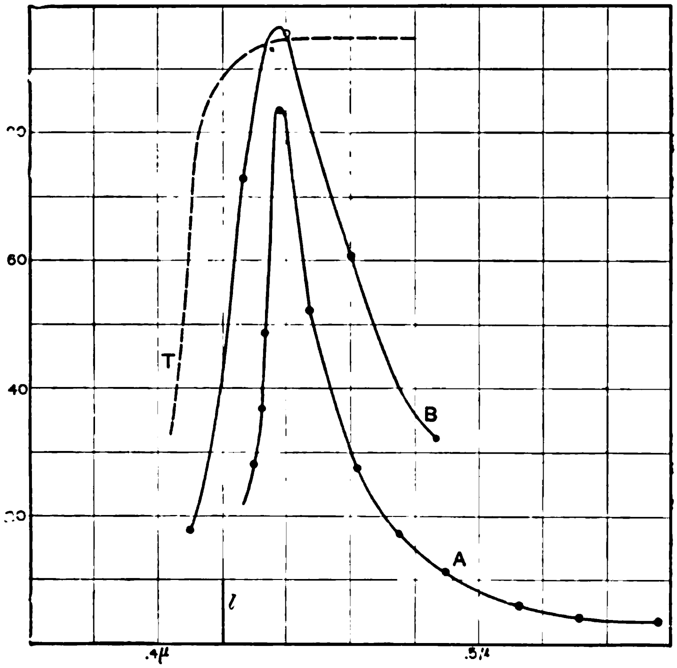

Fig. 14.

Quinine Sulphate in water. Curve *A*. Fluorescence spectrum when excited by Hg line, .3650 μ.

Curve *B*. Fluorescence spectrum when excited by daylight.

Curve *T*. Transmission spectrum. Layer 8 cm. thick.

The curve *T* gives the transmission of the solution as measured by the method already described. It was not found possible, owing to the fact that this solution remains transparent almost to the limits of the visible spectrum, to go very far into the absorption band; but the observations suffice to locate the infra edge of the band very closely. To gain further information concerning the nature of the absorption band a series of photographs of the transmission spectrum were taken by means of a Rowland grating with sunlight as the source of illumination. The infra edge of the band as shown by the disappearance of the Fraunhofer lines was found to lie in the region indicated by the spectrometric measurements shown in Fig. 14. Absorption became almost complete in the neighborhood of the *H* lines and continued throughout the ultra-violet, at least up to the point where glass becomes opaque.

In order to determine if possible the position of the ultra edge of the band the quinine solution was placed in a cell with quartz walls and photographs were taken, using an arc light into which zinc had been introduced as a source. Lines due to this metal were distinguishable in the comparison spectrum as far as wave-length $.2558\mu$ to which point the opacity of the solution of quinine sulphate was found to be complete.

It will be seen from the curves *A* and *B* (Fig. 14) that the fluorescence spectrum of quinine sulphate is of the same type as that of the various fluorescent dye stuffs. It consists of a single band with a sharply defined maximum at $.437\mu$. Since the curves obtained with daylight and with monochromatic light from the mercury arc are identical, as regards the position of the maximum and in general form, and since they are of the same type as the various other fluorescence spectra already described, it is evident that quinine sulphate belongs to the same class as fluorescein, eosin, etc. It is true that in the case of this substance it is possible to trace the fluorescent light throughout nearly the entire spectrum, a point which is to be considered further in a subsequent paragraph, but beyond $.5\mu$ intensities are very small.

Contrary to the view expressed by Lommel the fluorescence of quinine sulphate appears to be independent of the wave-length of the exciting light. Since, however, this is one of the substances which has been cited in support of Stokes' law, it was deemed important to determine as carefully as possible the longest wave-length of monochromatic light capable of exciting fluorescence. This is a matter of considerable difficulty on account of the weakness of the effect. By means of the method of double dispersion already described, the electric arc being used as a source, monochromatic light thoroughly free from all stray radiation was obtained. When this was used for excitation the last trace of observable fluorescence was found to disappear at wave-length .420μ.

In no other case have we found this limiting wave-length to lie so close to the ultra edge of the fluorescence band. It will be noted, however, that in making measurements by daylight readings were obtained at wave-length .41 μ and the fluorescence under strong excitation is traceable still further into the violet, certainly almost, if not quite, to .40 μ. While quinine sulphate then approaches more nearly to conformity with Stokes' law than the other substances that we have studied, the evidence is distinctly in favor of the view that it, like other fluorescent materials, is capable of being excited by all wave-lengths of light lying in or on the infra edge of its absorption band; and that the longest wave-length capable of producing an observable fluorescence is appreciably less refrangible than the shorter wave-lengths of its fluorescence spectrum.

Chlorophyll.

The fluorescence spectrum of chlorophyll was studied by Stokes[1] who found, in addition to the usual red band, a fainter excitation in the green of the spectrum. Hagenbach[2] subsequently made an exhaustive study of this substance. It was his opinion that it did not conform to the law of Stokes. Lommel[3] placed chlorophyll in his first class, to which belong all substances the fluorescence of which is independent of the wave-length of the exciting light.

The solution of chlorophyll used in our measurements was made by digesting green leaves in absolute alcohol and filtering. The transmission curve (Fig. 15) shows four well defined bands, of which the one to which fluorescence is due is more intense and broader than the others. Measurements made in the extreme red appeared to indicate the presence of still another region of diminished transparency which it was not possible to map with the spectrophotometer. In the hope of determining more definitely the character of this band and of ascertaining whether the series of striking absorption bands in the visible spectrum extends into the infra-red, we requested Mr. W. W. Coblentz, who is engaged in the study of the infra-red by means of a radiometer and a mirror spectrometer with rock salt prism, to explore the spectrum of this solution. The results of his measurements are plotted in Fig. 16. His observations extend from .61 μ to 1.45 μ at which wave-length the alcohol in which the chlorophyll was dissolved becomes so nearly opaque as to prevent further readings. The fluorescence band at .675 μ is clearly shown; also the band at the extreme edge of the visible red .745 μ. The latter however is of little intensity. In Mr. Coblentz's measurements comparison was made between a cell filled with the chlorophyll solution and the same cell when filled with alcohol in which no chlorophyll had been dissolved. The curve therefore indicates the effect of the chlorophyll and other dissolved matters upon the transparency of the alcohol. It will be noted that between .8 μ and .9 μ the material in solution has no absorbing power. At the latter wave-length absorption again begins to show itself and increases steadily to 1.45 μ where the transmitting power of the solution is only 60 per cent. as great as that of the alcohol itself.

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[1] Stokes, Philos. Trans., 1853.

[2] Hagenbach, Poggendorff's Ann., 141, page 245.

[3] Lommel, l. c.

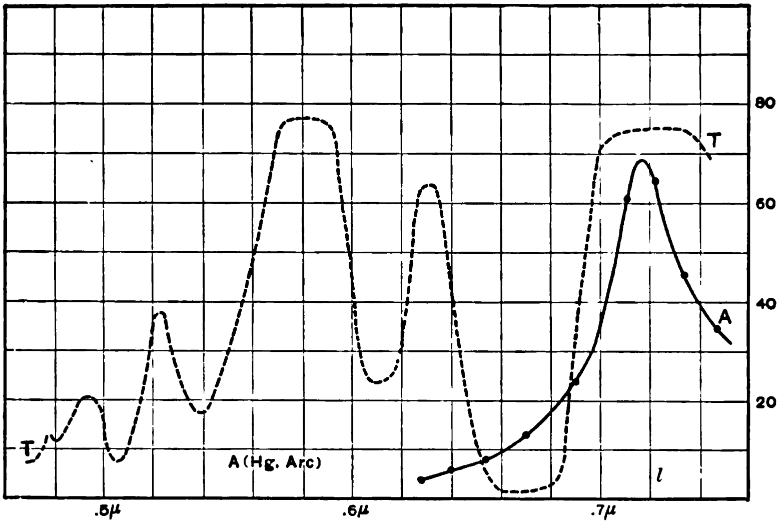

Fig. 15.

Chlorophyll (fresh alcoholic solution from green leaves). Curve *A*. Fluorescence spectrum when excited by Hg arc.

Curve *T*. Transmission spectrum of a layer 1.1 cm. thick.

In determining the fluorescence curve *A* (Fig. 15) which was obtained by using the green line of the mercury arc (.546 μ) for excitation, we were able to trace the fluorescence to wave-length .624 μ, which lies well beyond the ultra side of the absorption band. This had not been found possible in the case of the fluorescence spectra previously described, partly on account of the greater width of the bands, and partly for the following reason. The fluorescence spectrum of chlorophyll is doubtless traceable to an unusual distance towards the violet because the maximum lies in the extreme red, in a region of very low luminosity. Fluorescence in this region, to be appreciable, must be of great intensity, and on the side towards the violet the rapidly falling intensity is largely counterbalanced by increase in luminosity. The same is true of the infra side of the fluorescence band of substances like quinine sulphate; so that, although the curves are of the same type as those described in our previous paper — fluorescein, etc. — the fluorescence can be followed nearly through the spectrum. The curves of resorcin-blau, fluorite, and æsculin afford other examples of this phenomenon. Where, on the other hand, the maximum is in the middle of the spectrum, as in the case of rhodamin, eosin, fluorescein, etc., the fluorescence soon be-

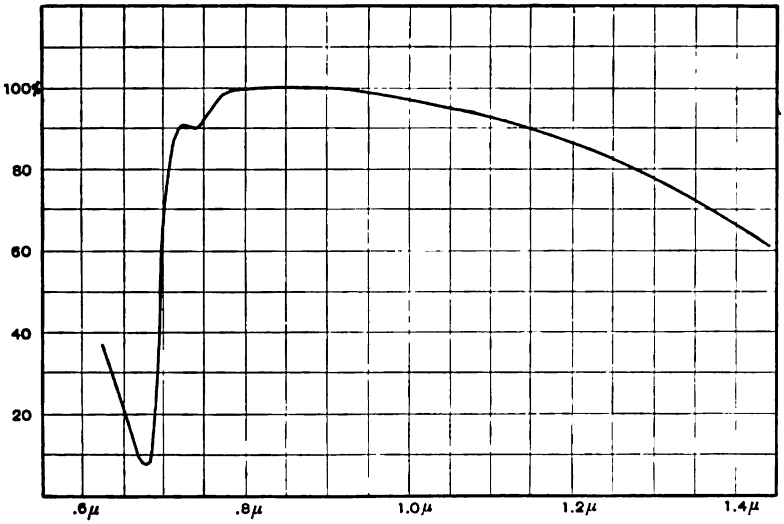

Fig. 16.

Transmission of chlorophyll in the infra-red. (Measurements by W. W. Coblentz.)

comes too small for measurement on both sides of the maximum because of the diminution of luminosity toward the ends of the visible spectrum.

The green fluorescence described by Stokes was faintly discernible under strong illumination, but its intensity would not permit of measurements. This band is probably due to the presence of some slightly fluorescent substance other than chlorophyll.

The question of the applicability of Stokes' law to this solution was tested by observing with monochromatic light, by the method already described, the longest wave-length capable of producing appreciable fluorescence. The limiting wave-length (.720 μ), which gave faint but unmistakable fluorescence (see line l, Fig. 15), lies on the infra side of the maximum of the curve A.

Canary Glass.

In our experiments upon canary glass one corner of a rectangular slab 1 cm. in thickness and 7 cm. wide was used. This was mounted in front of the slit of the Lummer-Brodhun spectrophotometer in place of and in a position corresponding to the cell employed in the study of the fluorescent solutions. Fluorescence was excited in the manner already described by means of light entering the glass at right angles to the axis of the collimator tube. Measurements were made using the Hg line .407 μ (see curve A, Fig. 17) and green light from the spectrum of the Nernst filament (curve B).

To determine the location of the absorption band, light transmitted through the entire width of the glass, giving a layer 7 cm. thick, was measured. The transmission curve C shows a minimum, but by no means complete opacity, at .472 μ. The absorption spectrum of this specimen did not exhibit the complex character of that studied and described by Stokes in his memoir already cited. Our quantitative exploration however did not extend beyond the determination of the infra edge of the band exciting fluorescence. The fluorescence spectrum, contrary to expectation, appears to belong to the same simple type as that of the fluorescent substances previously examined. The glass was optically very imperfect and the fluorescence measurements were seriously interfered with by the presence of stray light. This shows itself in the case of curve B, which is distorted in the regions corresponding in wave-length to the exciting light. The observations upon the ultra side of the curve are, as will be seen from the figure, considerably raised above the probable normal trend of the curve, which is shown by means of the dotted line. Curves *A* and *B* however have their maxima at the same wave-length, and, apart from the discrepancy just mentioned, their type corresponds precisely with that of the numerous other fluorescent spectra examined.

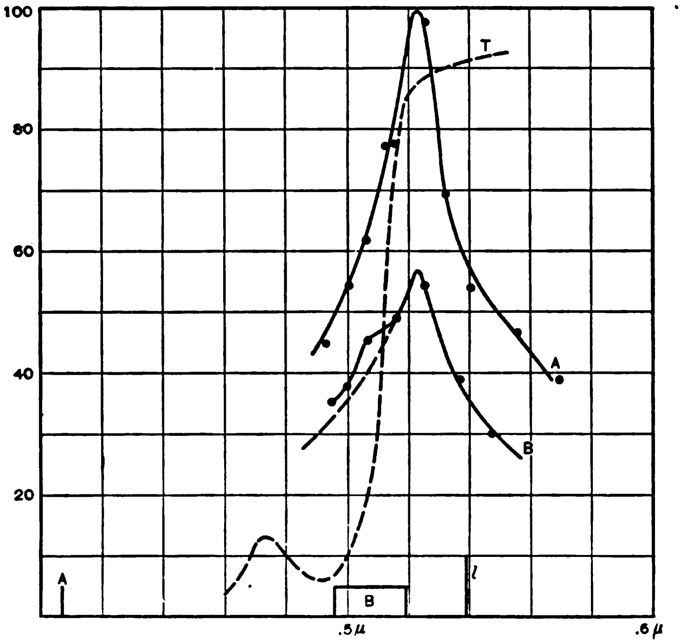

Fig. 17.

Canary Glass. Curve *A*. Fluorescence spectrum when excited by violet light, $\lambda = 0.407\ u$.

Curve *B*. Fluorescence spectrum when the exciting light lay in the green, in the region marked *B*.

Curve *T*. Transmission spectrum for a layer 7 cm. thick.

In the determination of the maximum wave-length capable of exciting fluorescence it was especially necessary to distinguish between fluorescent light and ordinary diffusion due to the imperfect optical properties of the glass. When one allows the spectrum to pass through this glass the path of the rays remains visible throughout the entire spectrum instead of disappearing when fluorescence ceases, as is the case in an optically perfect medium. By means of a Nicol prism held before the eye it was however easy to distinguish sharply between fluorescence and ordinary diffusion. The vertical line *l* marks the limiting wave-length (.539 μ) at which fluorescence becomes inappreciable. It lies far to the infra side of the maximum, showing that in this, as in all previous cases, there was not even approximate conformity to the law of Stokes.

It is probable that the glass studied by Stokes, and by means of which he observed such extraordinary fluorescence phenomena, was altogether different from our specimen. It contained perhaps other fluorescent materials in solution which were absent in the glass now under consideration. The glass which we have examined appears to belong in the same class with all the other fluorescent substances upon which measurements were made: its fluorescence spectrum

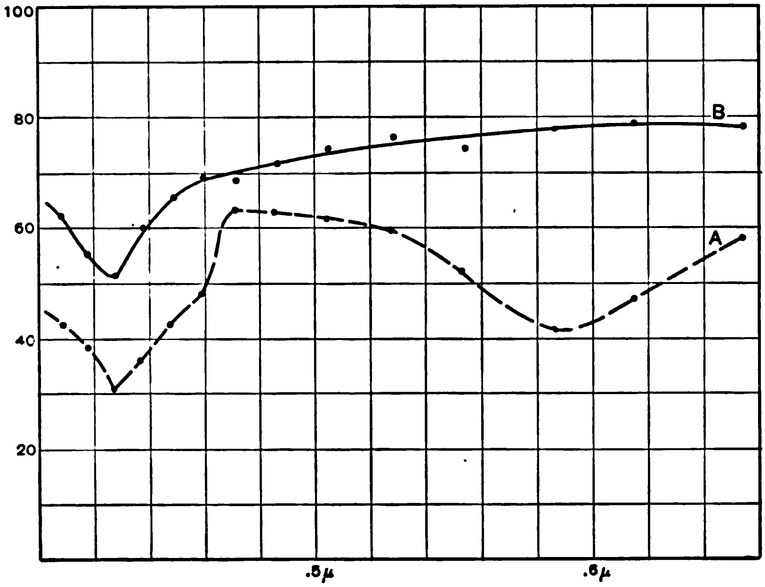

Fig. 18.

Transmission spectra of green fluorspar (*A*) and white fluorspar (*B*).

Fluorite.

From a collection of small crystals of fluorspar from Derbyshire, England, placed at our disposal by Professor Gill of the Department of Mineralogy, two specimens were selected for study. One of these was green by transmitted light, the other colorless. Both showed the well known blue-violet color due to fluorescence. The transmission curves of the spectrum of these crystals are brought together for comparison in Fig. 18. Curve *A* of the green spar (thickness 0.8 cm.) contains a broad, well-marked band in the yellow and a second rather narrow band, to which the fluorescence is due. The maximum of absorption in the latter band is at .424 μ. Curve *B*, which relates to the white crystal, contains only the second band. This crystal, although thicker (1.72 cm.), is more transparent, the absorption in the middle of the band being less than 50 per cent.

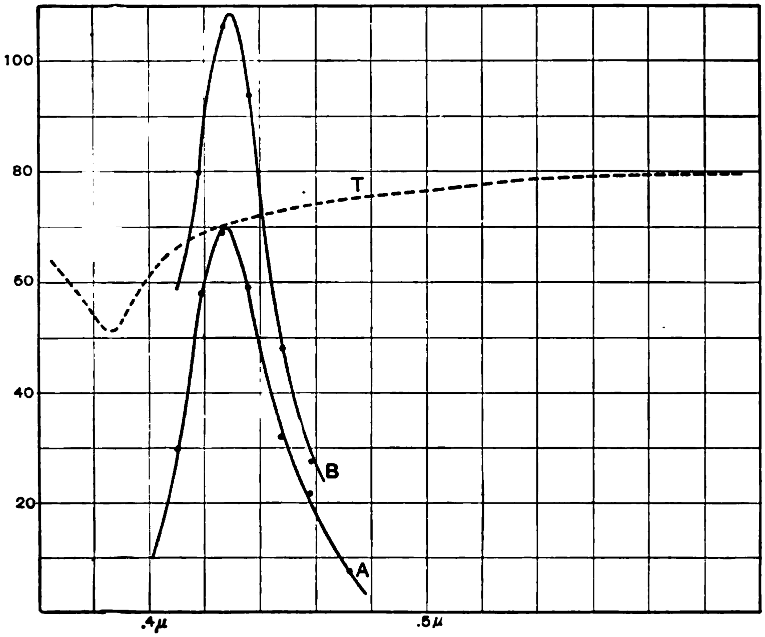

Fig. 19.

Green Fluorite. Curve *A*. Fluorescence spectrum when excited by the carbon arc. Curve *B*. Fluorescence spectrum when excited by Hg arc.
Curve *T*. Transmission spectrum of a layer 0.8 cm. thick.

It is interesting to note the presence of easily measurable fluorescence accompanying such feeble absorption. The fluorescence of the green crystal was determined by means of the undispersed light of the ordinary arc (curve *A*, Fig. 19) and of the mercury arc direct (curve *B*). The effective light from the latter source comes chiefly from the lines .407 μ and .365 μ. The fluorescence conforms strictly to that of all substances previously examined by us. The spectrum consists of a single band, the maximum of which at .433 μ is independent of the character of the exciting light.

In the measurement of the fluorescence of the white crystal diffuse daylight was employed, and the observations were made with the spectroscope with a single collimator tube and Vierordt slit, already described. The two sets of readings obtained are plotted in Fig. 20. The curves correspond very closely in character with those obtained with the green fluorite, although the maximum appears to be shifted very slightly towards the violet. This shift may be due

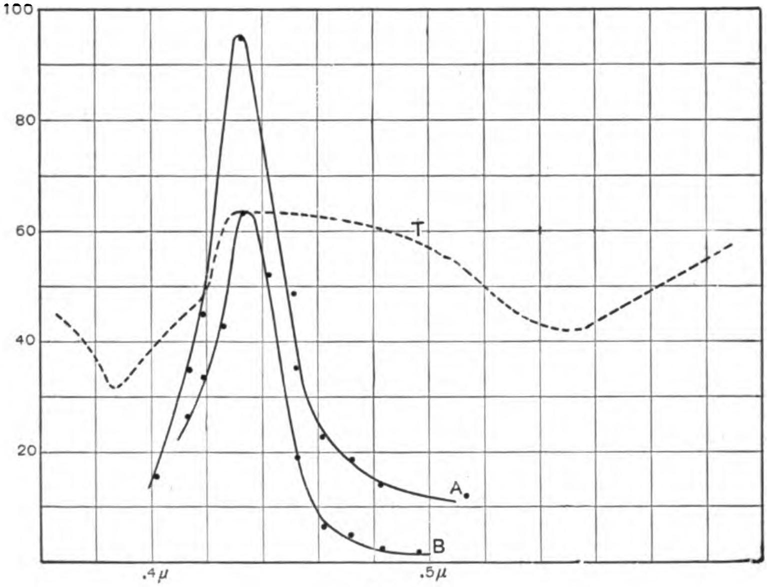

Fig. 20.

White Fluorite. Curve *A*. Fluorescence spectrum when excited by diffuse daylight.
Curve *B*. The same repeated on another day.
Curve *T*. Transmission spectrum of a layer 1.72 cm. thick.

to a combination of errors of observation and of instrumental errors, the instruments employed being different throughout, or it may result from some slight difference in the character of the absorption band. It will be noticed by reference to Fig. 18 that the infra edge of the absorption band of the green crystal does not rise so sharply as is the case of the white fluorite. The broadening of the band upon this side may result in a slight shifting of the fluorescence. It has already been shown by Stenger[1] and by O. Knoblauch[2] that fluorescent substances, such as eosin and naphthalin-roth, have the position of the fluorescent spectrum shifted by dissolving them in different liquids; and that the shift corresponds with that of the absorption band previously described by Kundt.[3]

Æsculin.

This substance was cited by Lommel[4] as typical of his second class, the characteristics of which are a fluorescence spectrum independent of the wave-length of the exciting source and an absorption band beginning at that point in the spectrum at which the first trace of fluorescence shows itself and extending towards the violet, so that there is no overlapping of the band of the exciting light with the fluorescence spectrum.

Our measurements of the fluorescence of æsculin were made upon a solution consisting of water in which freshly broken twigs of the horse-chestnut tree had been immersed. When freshly prepared this shows the well-known fluorescence characteristic of æsculin, but the solution loses its activity upon standing. We were unable to procure pure æsculin, but the results so far as the form and composition of the fluorescence curve is concerned would probably be the same had measurements been made upon the solution of the chemically prepared substance. The fluorescence curve, Fig. 21, is of the usual form and its position with reference to the transmission curve *T* is similar to that of the substances previously examined. On the infra side, as has been pointed out by Lommel, it is possible to trace the fluorescence throughout the entire spectrum; but this, as in the instances already considered, is due to the greater luminosity as we move toward the middle of the spectrum rather than to any unusual maintenance of the intensity of the fluorescence in that direction. The band falls off quite sharply on that side, differing not at all in this respect from the bands of other substances such as quinine sulphate and fluorite. The measurement of the transmission showed a very dilute solution, in which the maximum of absorption is scarcely more marked than in the case of the bands of the specimens of fluorite already described.

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[1] Stenger, Wied. Ann., 28, p. 201.

[2] O. Knoblauch, Wied. Ann., 54, p. 193.

[3] Kundt, Wied. Ann., 4, p. 34, 1878.

[4] Lommel, Poggendorff's Annalen, 143, p. 38.

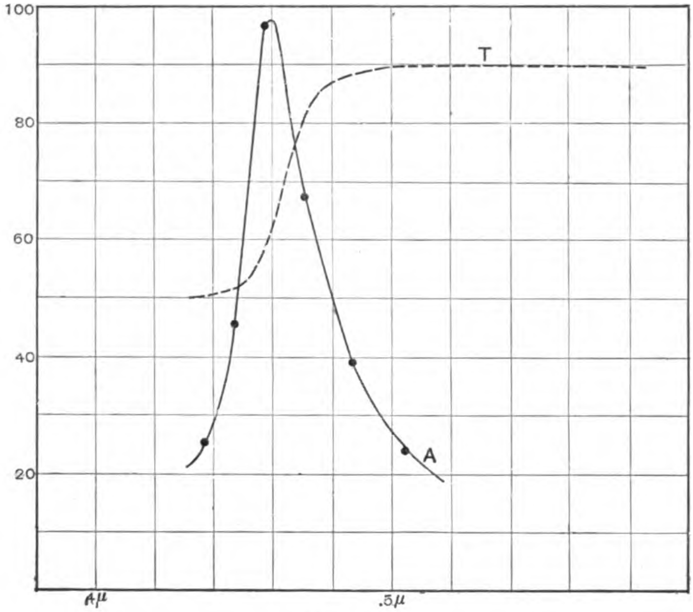

Fig. 21.

Æsculin (from horse chestnut). Curve *A*. Fluorescence spectrum when excited by arc light.

Curve *T*. Transmission spectrum of a layer 8 cm. thick.

From the location of the absorption band with reference to the maximum of fluorescence it would be safe to predict for this substance, on the basis of previous experience, a wide divergence from the law of Stokes. Determination of the limiting value of the exciting light is somewhat difficult to make. It was found, using the spectrum of the arc lamp as a source, that the fluorescence due to light of wave-length .45μ could still be discerned.

It is not possible, with the spectrophotometer, to make a complete exploration of the absorption band of æsculin, since the band extends into the ultra-violet. Photographs of the absorption spectrum of this solution, made by Miss Eleanor Burns at our suggestion show this band to be comparatively narrow, its ultra-edge lying at about .34 μ, beyond which there is no absorption capable of detection by photographic means.

There is nothing in these measurements which would lead one to place æsculin in a class different from that of the foregoing substances. It is distinguished, as regards the character of its fluorescence spectrum, from quinine sulphate and fluorite, the other two substances on our list the fluorescence of which lies in the blue, only in that the maximum lies at a somewhat greater wave-length. The curve *A*, Fig. 21, was obtained by the use of the direct light of the arc, undispersed. Since the curve corresponded in every form to the general type it did not seem necessary to repeat the observations with exciting light of other composition.

SUMMARY.

From the measurements described in this and in our previous paper the following conclusions may be drawn:

1. Eosin, naphthalin-roth, fluorescein, rhodamin, resorcin blau, quinine sulphate, chlorophyll, canary glass, green fluorspar, white fluorspar, and æsculin all exhibit fluorescence of the same type.

2. The fluorescence spectrum in every case consists of a single band situated near the infra edge of the absorption band with which fluorescence is associated.

3. The position of the maximum of the fluorescence band is in all cases independent of the wave-length or composition of the exciting light.

4. The distribution of intensities in the fluorescence spectrum is independent of the wave-length of the exciting light.

5. Fluorescence occuring at the red end of the spectrum is traceable to a greater distance towards the violet, and fluorescence at the violet to a greater distance towards the red, than is the case with fluorescence bands in the middle of the spectrum because of the increase of luminosity in those directions.

6. Stokes' law holds for none of the numerous fluorescent substances thus far examined by us.

7. Fluorescent substances having absorption bands of shorter wave-length than that with which the fluorescence is associated are capable of excitation by the absorbed light in each band, whatever its wave-length.

8. Fluorescent substances may have absorption bands of greater wave-length than that with which fluorescence is associated, as in the cases of green fluorspar and chlorophyll. The light absorbed in these bands does not however produce fluorescence.

9. Where more than one fluorescence band exists, as in the case of chlorophyll, it is probable that each band is due to a distinct fluorescent material contained in the solution. No case is known of double or multiple fluorescence where only one substance capable of fluorescence is present.

We propose in a forthcoming paper to consider the influence of fluorescence upon the absorption and electrical conductivity of fluorescent solutions.

THE VISCOSITY COEFFICIENT OF AIR, WITH AN INQUIRY INTO THE EFFECT OF RÖNTGEN RAYS THEREON. II.

By F. G. Reynolds.

Description of Apparatus.

A VERTICAL glass tube 92.71 cm. long and 6.85 cm. in diameter has its lower end fitted air-tight into a brass collar 7 cm. high which screws into a brass circular bed-plate, 28 cm. in diameter. The lower surface of the plate is ground and polished so that the ground edge of a bell jar when placed against it may form an air-tight connection. The upper end of the glass tube is capped with a brass cylinder in the form of an inverted cup whose upper surface is 3.49 cm. thick and is pierced by a conical shaped opening into which fits the cone of a torsion head, which being carefully ground is capable of easy motion but at the same time is air-tight. The brass collar at the base is fitted with a plane glass window which is adjusted air-tight over a part of the glass tube which has been cut away. In this way the observer may see clearly within that part of the cylinder, since the rays of light are not interfered with by the cylindrical surface of the tube. The bed-plate rests upon a tripod fitted with adjusting screws in its legs so that the glass cylinder may be made vertical. By an arrangement of clamps acting as a wedge, a bell-jar may be attached to the tripod and held tightly against the under surface of the bed-plate.

In the first and second sets of experiments a hollow brass spherical shell was used and careful measurements of its diameter were made for the purpose of determining any variations from a truly spherical form with the result that the diameter used as an axis was found to be very slightly smaller than the diameters perpendicular to it but the difference was so small as to cause no appreciable error in considering the surface spherical. The dimensions were therefore taken as follows: External diameter 12.68 cm. Mass 235.7

grammes and a moment of inertia (calculated) 6316.06. This sphere hung within the bell jar and was attached to the suspending wire by means of a brass rod 28.8 cm. long whose lower end screwed into the sphere and whose upper end was fitted with a cross pin in the shape of a T which fitted into a hook-shaped saddle attached to the wire. The wire was of German silver .0254 cm. in diameter and 79.3 cm. long and was used throughout the entire set of experiments. A small plane mirror was attached to the brass connecting rod, and this was capable of a vertical adjustment so that the mirror could be brought directly opposite the opening in the base of the apparatus. The whole suspended apparatus was thus protected from currents of air, and a very small turn to the torsion head would suffice to set the system into motion, which could be observed through the agency of the mirror. The readings were made by means of a tangent scale and telescope directly in front of the opening opposite the mirror and from 95 cm. to 100 cm. distant therefrom. The temperature of the gas was observed from a thermometer placed within the bell jar and easily read from without. The length of time taken for a swing from one position of rest to the next was obtained by means of a break-second chronometer with an electrical attachment to a recording apparatus from which one one-hundredth second could be estimated. The barometer from which the readings were taken hung near the apparatus.

A preliminary set of experiments was made and the results were calculated for the purpose of determining:

1. The best working limits of the apparatus as used.
2. Whether either the logarithmic decrement or the period of one swing depends upon the amplitude of the same.
3. Whether within the range of future experiments there is any change corresponding to the varying barometric pressure.
4. Any peculiarities liable to occur and therefore to be avoided.

Preliminary Set.

The method of proceeding in this instance was as follows: The sphere was first attached to the brass rod and the latter was then placed with its end in the saddle; the bell jar was then brought up and with as little jar as possible was clamped into position. Then by means of the torsion head, which was moved first in one direction and then in the reverse, an oscillating movement would be given to the sphere. In many cases a slight pendulum motion would also be given, but with a little practice it was found that this could be entirely stopped by a slight pressure of the hand on the outside of the apparatus. The oscillations were allowed to continue sometimes at the start for an hour, whereby over two hundred vibrations would have occurred, the object of this being to accustom the suspending wire to its load and to the torsional vibrations, so that when the readings proper were taken the damping effect of the internal friction of the wire itself would be reduced to as small and as constant a quantity as possible. The necessity of this precaution was shown by the work of Tomlinson[1] where he found that rest and change of temperature as well as any sudden shock would temporarily raise the internal friction of the wire. After this the sphere would be given a fresh swing and when the amplitude of the swing was that desired, the temperature, barometric pressure and time of start would be noted as well as the starting points on the scale. The readings on the scale were always taken in groups of ten successive swings and at the end of every two or three groups a pressure on the electrical key would record the time. This would continue till two hundred readings had been taken. The method of getting from this the logarithmic decrement is as follows: Suppose $a_1, a_2, a_3, a_4, a_5, a_6$ and b_1, b_2, b_3, b_4, b_5 are eleven consecutive readings from the left and right of the scale. The ten corresponding arcs from rest to rest are therefore $a_1 + b_1, b_1 + a_2, a_2 + b_2, b_2 + a_3, a_3 + b_3, b_3 + a_4, a_4 + b_4, b_4 + a_5, a_5 + b_5, b_5 + a_6$. The mean of the first and last, second and ninth, third and eighth, etc., would then be taken and if these did not vary much, as was generally the case, the mean of these five would be taken and the logarithm of the corresponding number of seconds noted. This would also be done with the second and succeeding sets of ten readings and the mean of the ten differences between the first and eleventh, second and twelfth, etc., logarithms would when divided by 100 be taken as the logarithmic decrement. These differences would vary slightly but no particular law of variation was noticeable, the variation being sometimes in one direction and sometimes in the other. The accompanying set shows more particularly the method:

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[1] The Influence of Stress and Strain on the Physical Properties of Matter. Phil. Trans., 1886, (2), CLXXVII, p. 801.

s	A	$\text{Log}_{10} A$	$n\lambda$
110.33	22.666	4.35537	
103.65	21.303	4.32844	
97.40	20.027	4.30161	
91.60	18.841	4.27510	
86.19	17.693	4.24780	
81.12	16.695	4.22258	
76.33	15.714	4.19628	
71.85	14.795	4.17011	
67.62	13.926	4.14382	
63.73	13.127	4.11816	
59.96	12.353	4.09177	.26360
56.41	11.623	4.06531	.26312
53.06	10.935	4.03881	.26279
50.05	10.314	4.01342	.26167
47.06	9.700	3.98677	.26103
44.28	9.127	3.96032	.26225
41.75	8.607	3.93485	.26143
39.12	8.065	3.90660	.26351
36.81	7.589	3.88018	.26364
34.67	7.148	3.85418	.26398

in which s represents the respective means of ten consecutive amplitudes as noted on the scale.

A the amplitudes reduced to seconds.

$n\lambda$ a multiple of the logarithmic decrement found by subtracting 11 from 1, 12 from 2, etc.

The mean of the ten differences $n\lambda$ gives .26270 and therefore the average logarithmic decrement for one swing is taken as .002627. The results of such observations taken at different times under varied conditions of barometric pressure and temperature are shown by the table:

Temp.	Barom.	Period.	Amplitude.	Log. Dec.
12.65	30.21	15.98	121.1- 36.9	.0025749
15.25	30.2	15.91	129.5-199.5	.0026061
16.40	29.95	16.	113.7- 33.7	.0026270
16.67	30.15	15.97	168.7- 50.2	.0026283
17.85	30.08	15.98	164.4- 84.2	.0026385
18.15	30.06	15.99	74.1- 38.1	.0026417
18.22	29.81	15.97	200.5-199.5	.0026419
18.30	3.018	15.94	205.1- 60	.0026543

From which it appears:

1. The small difference of barometric pressure has no effect on the logarithmic decrement.

2. The period of oscillation is unaffected by the amplitude of swing or by the temperature.

3. The logarithmic decrement is unaffected by the amplitude of swing.

4. The temperature is the only element which affects the logarithmic decrement. This was further emphasized in two cases where during the observation the temperature changed gradually to the extent of one degree. In one case where the temperature increased, the value of the logarithmic decrement increased steadily instead of varying in one way and the other around the mean value; and in the other case where the temperature gradually fell, there was a steady falling off in the value of the logarithmic decrement.

This sensitiveness to the changes in temperature rendered it advisable in the future observations to confine them to a shorter period of time so that there would be less liability that a change in temperature would occur, and therefore in the following sets the observations were confined to 100 complete vibrations.

Second Set.

With the swinging apparatus as described it was of course impossible to know how large a part of the logarithmic decrement was due to the internal friction of the wire itself and to the action of the air on the mirror. A change therefore was necessary so that this effect could either be eliminated or accounted for. This was accomplished by an addition to the apparatus as before described. A hollow brass tube 14.818 cm. long was pierced by a hole in the plane of its mid-section so as to permit the hanging brass rod to pass through it and to which it was fastened so as to hang 8.6 cm. from the lower end. Great care was necessary in the arrangement of this so that when finished it should hang in a perfectly horizontal position and at right angles to the suspending rod in order not to deflect the mirror. Two solid brass cylindrical weights were now constructed which fitted snugly into the ends of the hollow cylinder and each weight was carefully cut down till it was half the weight of the spherical shell (*i. e.*, 177.85) when the length of each was found to be 4.842 cm. The moment of inertia of the spherical shell had been calculated to be 6316.06 and it was desired to place each cylinder at such a distance from the axis as to have its moment of inertia equal to 3158.03. The moment of inertia of each around an axis through its center of inertia is 229.93 and therefore 2928.09 is the moment about the original axis of the whole mass considered as placed at the center of inertia. If the distance of the center of inertia from the axis be denoted by x, then $Mx^2 = 2928.09$ from which $x = 4.988$ cm. and since it was also necessary to have the ends of the weights flush with the ends of the enveloping cylinder this determined the proper length of the cylinder 14.818 cm. The aim in this was to reproduce the weight and moment of inertia of the spherical shell in the weight and moment of inertia of the two cylindrical weights.

The method of procedure in this set was as follows:

The cylinders would be removed and the sphere attached to the brass rod and the apparatus then left for several hours, usually over night. At the end of this time, by means of the torsion head the sphere would be oscillated for some time to accustom the wire to the oscillations, and then the vibrations would be started again for the readings, care always being taken to avoid any pendulum motion. In this case 100 complete swings from rest to rest were taken and the time thermometer and barometer noted as before. The sphere would then be detached, care being taken not to jar the wire and the cylinders would then be inserted into place, the preparatory oscillations given and then 100 more readings from the scale with temperature and pressure would be noted. The two swinging systems being the same in every respect save that of the extra surface exposed by the spherical shell, the difference between the decrements when the sphere was swinging and when the counter weights were inserted was that due to the action of the spherical surface alone. The method was varied by taking alternately the sphere readings first and then the readings with the weights first, and the whole extended over a range of eight degrees in temperature during a series of twenty-seven observations. A slight discrepancy in the period of swing was noticed when the sphere was attached and when the cylinders were in place which led to a further investigation to reconcile the results. The cylinders were removed and circular paper caps were attached to the horizontal hollow cylinder and the time and decrement noted, and compared with the time and decrement when the caps were removed exposing the interior of the cylinder. With the use of the caps the times were found to agree and a slight correction was found necessary to the logarithmic decrement of the sphere as found originally. The corrected results are inserted in the following table:

Time.	Temp.	$\lambda_1-\lambda_2$	Time.	Temp.	$\lambda_1-\lambda_2$
18.965	16.58	.002672	18.97	21	.002819
18.99	16.91	.002679	18.972	21.78	.002851
19.02	17.56	.002691	18.977	22.36	.002873
18.98	18.27	.002717	18.99	23.29	.002904
19.005	19.63	.002760	19	24.61	.002924
18.955	20.72	.002808			

Kirchoff in his "Mechanik"[1] derives a formula involving the coefficient of viscosity, the logarithmic decrement, the time of swing and certain other known elements as follows:

$$\lambda = \frac{2\pi R^4 \sqrt{n\mu}}{3K} \sqrt{2\pi T_0}.$$

In which $n =$ coefficient of viscosity.

$\mu =$ density of the medium.

$R =$ radius of the swinging sphere.

$\lambda =$ the logarithmic decrement.

$T =$ the swinging time in the fluid.

$T_0 =$ the swinging time free from influence of friction.

$K =$ moment of inertia of the system, which in the case of a spherical shell $= \frac{2}{3} Ma^2$.

The relation between the observed T and T_0 is given by:

$$T = T_0\left(1 + e\sqrt{\frac{2T_0}{\pi}}\right) = T_0\left(1 + \frac{\lambda}{\sqrt{\pi}}\right)$$

Using for the density of the air the value .001293 in the above formula the value of the coefficient of viscosity at the temperature 20.72 was calculated to be .00018697.

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[1] Vierte Aufl. 26 Vorlesung, p. 383.

Third Series.

The object of this series was to see if, with an entirely different swinging system, results could be obtained which would approximate closely the results obtained with the spherical shell. Such agreement if obtained would justify the mathematical theories giving rise to the formulæ used.

For this purpose two hollow brass cylinders were procured and after careful tests were found to depart but the slightest from exact uniformity of measurement. The dimensions were as follows:

Cylinder.	Length.	Weight.	Inside Diameter.	Outside Diameter.
A	30.4 cm.	327.8 gr.	5.00	5.08
B	30.4	316.3	4.70	4.78

A thin circular brass cap was soldered to one end of the smaller cylinder in the center of which was fastened a perpendicular brass rod 16 cm. long so that it coincided exactly with the axis of the cylinder. In a manner similar to the one used with the sphere, this rod was fitted at the top extremity with a cross-pin to rest in the hanging hook on the wire. The wire was the same one as used before, its length being this time 80.1 cm. By means of two small felt collars, one on the lower end of the smaller cylinder and the other on the upper end of the larger one, the larger when placed over the smaller was held snugly in place and the whole, when placed on the suspending wire, hung in a perpendicular position with the axis of the two cylinders coinciding with the wire. This telescopic arrangement of the cylinders permitted the outer one to be drawn out over the inner one or pushed back so as to cover it and in this way was it possible to eliminate the effect of the friction of the wire and the action of the air on the mirror. Two marks 25.4 cm. apart were made on the inner cylinder and the extension was limited to this amount. In the same manner as before the readings were made from the scale and the temperature, time, and pressure were noted. The observations were always made in pairs; one set with the cylinders collapsed and the other with the cylinders extended and alternately, one set and then the other being taken first. The results of 34 sets with temperature varying about six degrees were of a very satisfactory character. The time of swing averaged 13.09 seconds, any one differing from this by less than .05 seconds.

Temp.	Cylinders Extended.	Temp.	Cylinders Collapsed.
21.4	.001504	21.25	.000953
22.11	.001507	22.73	.000954
22.75	.001509	23.35	.000956
23.6	.001518	24.3	.000960
24.35	.001522	25.52	.000961
25.21	.001526	25.61	.000962
25.95	.001529	25.7	.000964
26.38	.001531	26.73	.000966
27.1	.001533	27.4	.000969

The differences between the two corresponding logarithmic decrements vary from .000551 to .000564 and these differences were taken for the logarithmic decrement corresponding to a cylinder of the internal and external diameters of the smaller cylinder and 25.4 cm. in length.

From the mathematical deductions of Stokes the logarithmic decrement due to the action on the walls of the cylinder will be given by:

$$\lambda = \frac{M\mu T}{I\rho} \log_{10} e(\sqrt{2} \times f + \sqrt{2} \times 0.375 f^{-1} - \sqrt{2} \times 0.4922 f^{-3} + \text{etc.})$$

where

$$f = \sqrt{\frac{\pi\rho}{\mu T}} \times a.$$

In which λ = the logarithmic decrement.

M = mass of air confined in a cylinder of same length and mean radius of the one considered.

μ = the coefficient of viscosity.

T = the observed time.

I = the moment of inertia of the cylinder considered.

ρ = the density of the air considered as about half saturated with moisture.

a = the mean radius of cylinder.

In applying the formula a first approximation was made for the value of μ and this used in a second approximation. A third was found unnecessary and the value of μ resulting from an observed time 13.09 seconds, a logarithmic decrement .000551 and at a temperature 21.33 was calculated to be .00018711.

This value agrees very closely with the value found by the preceding method, and both agree well with the later results of other investigators, as may be seen by the table.

FOURTH SERIES.

Attention has been called by Thomson[1] to the facility with which a gas can be changed from a conductor to a non-conductor by the application and removal of Röntgen rays; and in particular he caused the gas so exposed to be swept past a wire charged with electricity and in this way part of the charge would be carried to an electrometer. When the ray was not acting on the gas no charge was carried over by the gas. The gas it was found loses this conductivity when a current of electricity passes through it and also when forced through a plug of glass wool, which latter fact would indicate that the structure in virtue of which the gas conducts is of a coarse character since it does not survive the passage through the fine pores of the glass wool. The study of any other property of a gas when in the state into which it is thrown by the Röntgen ray might therefore lead to further interesting and valuable results. The object of this phase of the investigation is to ascertain if there is any difference in the coefficient of viscosity of air when it is affected by the influence of the Röntgen rays.

In this set the brass spherical shell used before was hung in position and the bulb from an X-ray apparatus was brought within 15 cm. of the surface of the sphere. It was found necessary to have the coil in an adjoining room in order to keep the floor and air free from the vibrations of the apparatus. Furthermore in order to shield the air surrounding the sphere from the electrostatic influences of the bulb, the latter was surrounded by a metal gauze screen which was connected to the earth. The observations were taken in pairs, with the current alternately on and off, and attention was first directed to the period of swing. The following table, where the average time of ten consecutive swings is given, at different temperatures and barometric pressures, gives no evidence of any uniform law of change:

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[1] Discharge of Electricity through Gases, Phil. Mag., 1896 (5), 208, p. 392.

Temp.	Bar.	Without Rays.	With Rays.	Amplitude.
15	29.6	15.985	16.	80 -70
"	"	15.97	16.	19 -13.5
"	"	15.95	16.94	
16.6	29.77	15.98	15.98	35 -30
17.75	"	15.94	15.96	70 -64
17.75	"	15.965	15.94	65 -56
		15.97	15.96	84.7-71.5

At a later time further sets were taken with a slight change in the swinging apparatus and at higher temperatures and the logarithmic decrement as well as the time were noted for comparison. The method of procedure consisted in taking the readings of thirty complete oscillations with the current off, then thirty more with the current on, and a final thirty with the current off again. The interval between each two sets of thirty being the same, usually one complete oscillation. The average of the first and third sets was taken and compared with the second set with the following results :

Time Off.	Time On.	Current Off.	Current On.	Amplitude.
15.68	15.7	.0025498	.0025428	112 - 94.9
15.7	15.73	.0025681	.0025647	56 - 47.6
15.7	15.7	.0026147	.0026042	38.4- 32.4
		.0026893	.0026223	84.7- 71.5
15.7	15.73	.0025472	.0025471	120.6-102.3
		.0026653	.0026273	151.3-126
15.7	15.7	.0025697	.0025648	100.8- 85.4
15.7	15.7	.0025791	.0025729	70.6- 59.8

In this set the periods agree closely. It might be noted that the variation favors usually a slight increase in the period of swing when the air is affected by the Röntgen ray, but the difference is so small that this might easily be caused by errors in observation. In the case of the logarithmic decrement the difference is also very slight but points to a decrease when the gas is under the influence of the Röntgen ray. The effect furthermore seems to be an increasing one within the limits of the time given, since in striking the average decrement for the gas when under the influence of the rays, the final terms were less and the first terms greater than this average as given. During each set the current was kept practically uniform, but it was varied in the different sets wherein the more pronounced effect accompanied the stronger current.

College of the City of New York.

THE APPLICATION OF ALLOTROPIC SILVER FOR THE PREPARATION OF CONDUCTING FIBERS.

By N. A. Dubois.

A NUMBER of articles discussing allotropic modifications of silver have appeared in the various journals since 1889. These articles treat rather of the methods of preparation and physical condition of the different allotropic modifications, than of any application of them. A complete list of references is given by Barus and Schneider.[1]

Three forms of allotropic silver are described, all of which have certain properties common to themselves and which distinguish them from ordinary silver.

When a suspension or emulsion of any of these forms of allotropic silver is spread over paper and dried, the particles are left in optical contact and thus form an apparently continuous film.

Treatment with halogens produces brilliant interference color effects. All the forms are very readily changed to an impalpable powder. Treatment with acids instantly changes all forms to ordinary silver, with apparently no chemical action. No gas is evolved.

It is the purpose of this article to give a simple and efficient means for the preparation of conducting fibers; whether they be used for suspension, as in the quadrant electrometer, or otherwise; and whether the fiber material be quartz or any other suitable substance.

The allotropic silver used was prepared according to the process described by Carey Lea.[2]

A mixture of 200 c.c. of a 30 per cent. solution of ferrous sulphate and 280 c.c. of a 40 per cent. solution of sodium citrate was treated with a 10 per cent. solution of sodium hydroxide, just till no permanent precipitate formed. This was then mixed with 200 c.c. of a 10 per cent. silver nitrate solution stirred thoroughly, allowed to settle and the liquid decanted. The precipitated allotropic silver was treated with about 200 c.c. of distilled water, which dissolved a part of it, and was again precipitated by treatment with a saturated solution of ammonium nitrate. This process was repeated three times, and the final precipitate filtered with suction and washed with 95 per cent. alcohol. The product was transferred, in the form of a brown mud, to a bottle and kept covered with alcohol.

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[1] Wied. Ann., XLVIII., p. 327, 1893.

[2] Am. Jour. Sci., XXXVII., p. 476, 1889.

The allotropic silver was used for the experiments as an emulsion in alcohol, alcohol being used rather than water as it dries more quickly and thus the dry film of allotropic silver can be obtained in a very few minutes after spreading the emulsion.

This emulsion spread on any surface and dried gave a copper-colored film. When dried on a glass plate and viewed through the glass, it gave a good mirror, but of a dark color.

When these films were treated with gaseous hydrochloric acid they were immediately changed to ordinary silver and presented a dull silvery appearance. The films are changed to a bright silver surface by slight friction. It was the action of the gaseous hydrochloric acid, which suggested the investigation.

Glass plates were painted with the emulsion and dried. The conductivity of the films in this condition was found to be very poor. It was tested by placing the terminals of an ordinary dry battery upon the films with an ammeter in circuit. The electromotive force of the battery was about two volts and the terminals were placed eight or ten centimeters apart.

The films were then treated with gaseous hydrochloric acid by blowing through an arrangement like a reversed wash-bottle, the bottle containing a little strong hydrochloric acid. By this means a stream of gaseous hydrochloric acid was very readily directed upon the allotropic silver films, thus changing them to sheets of ordinary silver. These films were then tried with a battery as before and found to be excellent conductors.

The use of the silver was then tried on the quartz film of a quadrant electrometer as follows: The quartz fiber, suspending the whole was enclosed in a copper case to prevent disturbance by air currents and a beam of light reflected through a glass window in the copper case, from a mirror fixed on the needle, served as a pointer on a transparent scale arranged at the proper distance.

The instrument having been adjusted, experiments were tried to see how quickly the needle responded on being charged. It was found to respond instantaneously.

Various tests on the electrometer with the fiber prepared as above stated were perfectly satisfactory.

Conclusions.

The dried films of allotropic silver, although poor conductors, become very good conductors on treatment with gaseous hydrochloric acid.

The emulsion can be used admirably for the preparation of conducting fibers; and can be applied to the most delicate apparatus in a very few minutes, without danger of injury.

The above work was done in the physics laboratory of Brown University. In closing, the writer wishes to express his thanks to Professor Carl Barus, for his valuable instruction and friendly aid.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

APPARATUS FOR PLATINUM RESISTANCE THERMOMETRY.[1]

BY C. W. WAIDNER AND H. C. DICKINSON.

THIS paper contained:

1. An outline of the development of resistance thermometry.

2. A description of some special forms of resistance thermometers, which are taken to establish the scale of temperature used by the National Bureau of Standards in the interval from — 200° C., or lower, to upwards of 600° C.

3. The description of a special form of resistance bridge that has been designed for use with resistance thermometers of the 'compensated lead' type, which combines high accuracy with rapidity in working. This bridge may be briefly described as a mercury contact bridge, with a series of coils embodying the essential features of standard resistance coils, ranging from 0.01 ohm to 50 ohms; the finer settings being made on three 10-step shunt dials, which take the place of a bridge wire and which read in steps of 0.001, 0.0001, and 0.00001 ohm respectively. The adjustment of these coils in the shunt dials need only be approximate, without introducing an appreciable error. This bridge also contains all the necessary reversing battery and galvanometer keys and an Ayrton-Mather shunt, and four sets of ratio coils that can be conveniently reversed for eliminating inequalities in the ratio arms.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

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[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 23, 1904.

THE INTERCOMPARISON OF PRIMARY STANDARD MERCURIAL THERMOMETERS.[1]

BY C. W. WAIDNER AND H. C. DICKINSON.

ONE of the first problems to present itself in connection with the work of the Thermometry section of the Bureau of Standards was that of a temperature scale in the interval 0° to 100° C. together with a suitable means for accurate and convenient intercomparison of primary standard and other accurate mercury thermometers in this range.

The scale of temperature in almost world wide use is the International Hydrogen Scale established by the experiments of Chappuis. Hydrogen was chosen because of its near approach to a perfect gas throughout a great range of temperature and its consequent close agreement with the absolute thermodynamic scale. This scale is made available by the work of the International Bureau of Weights and Measures who have made a complete study of the hydrogen thermometer and have compared with it various mercury in glass thermometers constructed of different kinds of glass. It is thus possible to reduce the indications of a mercury thermometer to the hydrogen scale.

The advantage resulting from the adoption of a standard temperature scale cannot be too strongly emphasized. Before this work was carried out by the International Bureau each experimenter was obliged to set up his own air thermometer and establish his own temperature scale. This often required far more time than the experiment itself, *e. g.*, in the classical determinations of the mechanical equivalent of heat by Rowland. This had the further disadvantage that the results of different experimenters could not be compared to the greatest advantage. As an illustration of the advantage of the world wide use of a single scale we need only refer to the explanation of the discrepancy that had been found in the mechanical equivalent of heat when measured by mechanical and electrical methods. When the apparently discordant results were reduced to the same temperature scale they were found to be in most satisfactory agreement in the variation of the specific heat of water, and further strongly indicated an error in the International electrical units, which was almost simultaneously confirmed by the redeterminations of the E.M.F. of the Clark cell by Kahle and by Patterson and Guthe.

The Bureau of Standards possesses a number of primary standards made from French hard glass by Baudin and by Tonnelot. These thermometers are of the very finest construction, graduated in tenths of a degree with lines (on those by Baudin) less than 20 microns in width. Each thermometer has been calibrated at the International Bureau and some of them have been compared directly with their standards. The temperature scale of the Bureau of Standards in the interval 0° to 100° C. is established by the mean of a large number of intercomparisons of these primary mercurial standards in a specially constructed comparator which is designed to satisfy the requirements for the very highest accuracy and at the same time permits great rapidity and convenience of manipulation so that it can be used readily for even coarse comparisons of ordinary thermometers.

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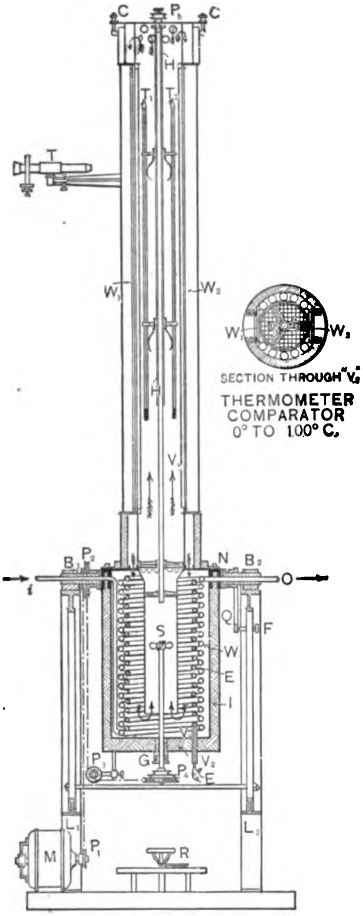

THERMOMETER COMPARATOR 0° TO 100° C.

This comparator (see accompanying cut) is essentially a triple walled water bath which can be used in either horizontal or vertical position, with water coil for rapid heating and cooling, electric coil for heating, and rapid circulation of water produced by an electrically driven propeller. The upper part of the tank is made of three concentric brass tubes and contains within the inner one a holder with places for fourteen thermometers. Between the outer two of these tubes is a packing of hair felt and between the inner two the downward circulation of water which comes upward around the thermometers in the inner space. Plate glass windows (W_1) (W_2) 60 cm. long and about 8 cm. wide are let in on opposite sides for the purpose of reading. These windows being opposite each other afford excellent direct illumination.

The reading is done by means of the micrometer microscope (T) sliding on a vertical support beside the window. The holder (H) may be conveniently rotated to bring the thermometers successively into the field of the microscope, by means of a belt running from the pulley (P_6) in the water tight cover, over idlers to a wheel (not shown) at a convenient height on the back of the tank. The lower portion of the comparator consists of an inner brass chamber covered with a heavy layer of felt and a removable outer brass casing. In this inner chamber are contained all the means for securing temperature regulation and circulation of water. First there is a coil of three-eighths inch copper tubing (W) making sixteen turns with inlet and outlet (i and o) through the axle on which the comparator is supported. Through this coil either hot or cold water may be circulated at will. For temperatures below that of the room, regulation is secured by using water at 0°C. from an ice tank, but at higher than room temperature hot water is used only for rapid heating, and for exact regulation the electric heating coil (E) furnishes the energy. This electric heating gives better regulation than hot water. The coil (E) is of a type made by the Simplex Electric Heater Company for the general trade and was found to fulfil the requirements of this apparatus very well indeed. It consists of three-eighths brass tube in which are inserted two coils, combinations of which give three rates of heating for any E.M.F. and in conjunction with the rheostat (R) permit regulation from a fraction of a watt to 800 watts.

Stirring is effected by means of a propeller (S) driven by the one-tenth horse power motor (M) which is belted to a wheel on the axle of the comparator, from which another belt runs over idlers to the pulley on the propeller shaft. This shaft passes through the stuffing box (G).

The reason for this arrangement was to permit one of the most convenient features of the comparator, *i. e.*, the adaptation to horizontal as well as vertical comparison. By simply drawing the spring key (F) tipping the comparator to a horizontal position and releasing the key, the comparison of a set of thermometers may be continued in horizontal position without any change whatever in the working of any part of the comparator other than possibly a slight change in speed of circulation due to difference in the fluid friction.

With this apparatus and with reasonable constancy in the temperature of the room any temperature within the range of 10° C. to 90° C., may be quickly attained at a rate of two to four degrees per minute and may be held constant to within the limit of the best eye estimations for many minutes. In fact the rise of temperature may be made slower than is desirable for a good set of observations.

It has seemed in most of our work with practice in estimation that a large number of comparisons by eye estimation are better than a limited number of micrometer measurements. These estimations have been found to differ very seldom by as much as three thousandths of a degree from measurements with a micrometer, and the time required for such an eye observation is only a small fraction of that required for the use of the micrometer. The accompanying table gives the observations and results for a set of five double series of eye estimated readings on four primary standards. There were six thermometers in the tank at the time and the interval between successive readings was fifteen seconds by the chronometer. Thus between the two readings of the last thermometer three minutes had elapsed and allowing for time between series the whole set required not less than twenty minutes, probably thirty; thus it will be seen that during this time the rise of temperature was approximately .02

Intercomparison of Primary Standard Thermometers.

	Baudin 15282	**Tonnelot** 4334	**Tonnelot** 4335	**Tonnelot** 4624
	30.013	30.206	30.000	30.128
	30.014	30.205	30.000	30.127
Corrected T. =	29.987	29.992	29.992	29.986
	30.013	30.207	30.002	30.133
	30.020	30.208	30.004	30.133
Corrected T. =	29.990	29.994	29.995	29.993
	30.023	30.210	30.006	30.140
	30.026	30.213	30.008	30.142
Corrected T. =	29.998	29.997	29.997	30.000
	30.031	30.218	30.008	30.145
	30.030	30.218	30.009	30.150
Corrected T. =	30.005	20.004	30.000	30.006
	30.035	30.222	30.010	30.150
	30.035	30.222	30.012	30.153
Corrected T. =	30.009	30.008	30.003	30.010
Mean =	29.998	29.997	29.997	29.999

Sample
Computation
Baudin
15282

Observed.	30.013
Calibration.	+.055
Ext. Pressure.	−.007
Int. Pressure	+.042
Fundamental.	+.032
Zero Corr.	−.046
To Hydrogen.	−.102
Temperature.	29.987

degree. The observations here copied are an entirely fair sample being taken directly from one of the record volumes and in many cases the temperatures have varied much less than in this example.

The final results of the complete series of intercomparisons of the primary standard mercurial thermometers will be given in a subsequent communication.

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ABSOLUTE MEASUREMENT OF CAPACITY.[1]

BY E. B. ROSA AND F. W. GROVER.

MEASUREMENTS of the capacity of air and mica condensers were made by Maxwell's bridge method, using a rotating commutator in the fourth arm of the bridge, for charging and discharging the condenser. This commutator was of new design and was built in the instrument shop of the Bureau of Standards. Its special feature consisted in having the two contact brushes, through which the charge and discharge took place, so arranged as to afford air insulation, when the brushes were not in contact with the metal segments, and so prevent any appreciable leakage across the bridge arm.

The commutator was driven by a large electric motor, which received current from a storage battery, and the speed was maintained constant by means of a carbon rheostat in the armature circuit of the motor. The criterion of uniform speed was the balance of the bridge, which was indicated by zero deflection of the galvanometer. It was found possible to maintain speeds more nearly constant in this manner than when using a stroboscopic method with a tuning fork. The exact average value of the speed was determined by registering every fiftieth revolution of the commutator on a chronograph cylinder by means of an electric contact.

Numerous of determinations of capacity were made; the following determinations illustrate the order of accuracy obtained.

Two determinations of the capacities of each condenser were made, and then they were joined, first in parallel, and secondly in series, and two determinations were made in each combination. The means of the two in each case are as follows:

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No. 1 = 0.029292 microfarad
" 2 = 0.029481

Sum = 0.058773
Measured sum = 0.058770

Difference = 0.000003
¼ sum = 0.014693
Measured in series = 0.014699

Difference = 0.000006

Two determinations of the capacity of two mica condensers, separately and then in series and parallel were made, the means being the following:

No. 1 = 0.200882
" 2 = 0.200145

Sum = 0.401027
Measured sum = 0.401004

Difference = 0.000023
¼ sum = 0.100257
Measured in series = 0.100215

Difference = 0.000042

This method is used in determining the capacities of condensers submitted to the Bureau for test.

Absolute Measurement of Inductance.[1]

By E. B. Rosa and F. W. Grover.

THE method used was originally proposed by Joubert, but, so far as the authors know, it has never before been employed for an accurate determination of inductance.

The method consists in joining a non-inductive resistance in series with the inductive resistance to be measured, and passing an alternating current through them. An electrometer is then connected so that it may measure successively the difference of potential at the terminals of the inductive resistance and the non-inductive resistance, and the latter is varied until the two readings of the electrometer are equal. The current and voltage being the same for each, it follows that the impedance of the inductive resistance is equal to the resistance of the non-inductive resistance. The inductance of the former can then be calculated when the frequency of the current and the resistance of the inductive coil are known.

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The frequency is determined in the same manner as when measuring capacity absolutely, namely, by recording every fiftieth revolution of the generator on a chronograph. To keep the speed constant, a condenser is joined up to a Wheatstone bridge, and the galvanometer deflection is kept zero by hand regulation of the motor current. In other words, the condenser experiment is here used as an auxiliary to the inductance measurement.

If the wave form is not a true sine curve (as it never is), it is necessary to apply a correction to the result, computed on the assumption of a sine wave, and the value of this correction depends on what harmonics are present, and on their relative magnitudes. If the harmonics are very small, the correction is also small. The authors used a generator, constructed especially for the Bureau of Standards, so as to give as nearly a sine curve as possible and the only harmonic present to an appreciable extent was the third. The analysis showed this to be 1.6 per cent. of the fundamental, no other being more than one or two tenths of one per cent. The correction calculated for this harmonic amounted to about twelve parts in ten thousand.

The work is to be repeated and an effort made to ascertain more exactly the values of the small higher harmonics, if they are present at all.

The following determinations of the inductance of two coils were given by the authors.

$$L = \frac{F}{p}\sqrt{R^2 - r^2}.$$

	Coil No. 1.		Coil No. 2.	
$R =$	962.55	681.85	949.40	672.30
$r =$	93.5	93.5	93.5	93.5
$n =$	150.106	105.808	150.104	105.809
$F =$	0.9988	0.9988	0.9988	0.9988
$L =$	1.01455	1.01472	1.00055	1.00023
	Mean = 1.0146		Mean = 1.0004	
	Ratio = 1.0142			

R = non-inductive resistance to balance.
r = resistance of the coil.
n = frequency of the alternating current.
F = correction factor for presence of harmonics

Testing of Mica Condensers.[1]

By E. B. Rosa and F. W. Grover.

In estimating the value of a condenser, it is necessary to know:

1. Its change of capacity with the time of charge, *i. e.*, its capacity curve.
2. The magnitude of its absorbed charges.
3. Its leakage resistance.
4. Its temperature coefficient.

The author made determinations with about thirty mica and paper condensers, belonging to the Bureau of Standards, and made by some of the best manufacturers of England, France, Germany and America, with a view to ascertaining the performance of good condensers.

I. To study the variation of capacity with the time of charge, for charging periods of one half second and upward a ballistic galvanometer was used. This had a period of 17 seconds for a double vibration and a remarkably steady zero, so that deflections as great as one meter could be read to two or three parts in ten thousand, and the mean of several determinations would give the deflection to about one part in ten thousand. The deflections being very nearly equal, the relative capacities as the time of charge is varied may be obtained directly from the ratio of the deflections.

For shorter charging periods such as occur with alternating currents, comparisons with a standard may be made with an accuracy of one part in ten thousand, using the alternating current bridge method, with an oscillating galvanometer, tuned to the frequency of the current, for a detector.

To combine these two methods so as to obtain a curve showing the change in capacity from a charging period of a few thousandths of a second up to half a minute we used an air condenser. Since the capacity of an air condenser is independent of the charging period within the prescribed limits the condenser first compared with the air condenser by the alternating current bridge and then a ballistic comparison of the two condensers was made, the voltages being chosen so that they gave very nearly equal deflections. The charging voltage was measured and maintained constant by a potentiometer. The air condensers recently constructed in the instrument shop of the Bureau of Standards were employed. Their combined capacity is nearly 0.06 m.f., and their leakage is negligible.

The mica condensers tested showed curves very much alike as will be seen from the following table for seven condensers. The capacities at the different charging periods are all referred to the capacity at 10 seconds charge as unity.

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Time of Charge.	Condenser. 1	2	3	6	7
$\frac{1}{360}$ sec.	0.9914	0.9902	0.9906	——	——
$\frac{1}{2}$ "	0.9964	0.9952	0.9962	0.9952	0.9971
1 "	0.9977	0.9975	0.9977	0.9968	0.9980
2 "	0.9984	0.9985	0.9984	0.9978	0.9989
5 "	0.9996	0.9996	0.9996	0.9994	0.9995
10 "	1.0000	1.0000	1.0000	1.0000	1.0000
20 "	1.0004	1.0005	1.0005	1.0006	1.0005

II. Residual charges were measured by a very sensitive ballistic galvanometer, the following schedule being used: (1) Charge to seconds. (2) Insulate 1 sec. (3) Short-circuit 1 sec. (4) Insulate 30 sec. and then discharge, a moment observing the quantity. This was called the first residual charge. (5) Insulate 30 sec. and then observe the second residual charge and so on. Two values of t were used, $t = 1$ sec. and $t = 10$ sec.

The following table shows the values of the first five residuals for three mica condensers for two different periods:

$t = 1$ sec.			$t = 10$ sec.		
		Condenser.			
2	6	7	2	6	7
0.403	0.328	0.237	0.644	0.832	0.511
0.028	0.028	0.020	0.054	0.114	0.059
0.004	0.010	0.012	0.018	0.036	0.024
——	0.008	0.005	0.014	0.021	0.013
——	0.006	——	0.007	0.016	0.009

These residual charges are expressed in per cent. of the original or CV charge.

Some other mica condensers have shown still larger residuals.

III. The leakage resistance was measured by charging the condenser, insulating it and filling it up again through a ballistic galvanometer at stated equal intervals, the process being repeated until the approaching constancy of the deflection showed that the quantity disappearing by absorption was approaching zero. The values below illustrate the order of magnitude found for the best condensers. Much smaller values have been found for some mica condensers.

Capacity.	Leakage Resistance.
0.1 mf.	2,120,000 megohms
0.2 "	1,500,000 "

IV. The authors have not as yet investigated the temperature coefficients of the condensers, but are about to take up the subject at length. Preliminary observations show, however, that the coefficient for mica condensers may be either positive or negative, but is small, not being very different from 0.0001 per degree.

OPTICAL PYROMETERS AND SOME PROBLEMS IN OPTICAL PYROMETRY.[1]

BY C. W. WAIDNER AND G. K. BURGESS.

THIS paper, which will shortly be published in full, contains a study of the sources of error, attainable accuracy, necessary precautions in the use of, and the calibration and range of certain optical pyrometers: The LeChatelier, Wanner, the Holborn and Kurlbaum, and the Féry. The advantages and limitations of optical methods in pyrometry are discussed and the results of investigations are given with experimental data on the influences that may affect the indications of the various instruments, such as polarized light emitted by incandescent surfaces, diffuse and reflected light from neighboring luminous bodies, and selective radiation from the sources studied including platinum, copper and iron oxides, lime, flames and other incandescents.

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NEW BOOKS.

Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light. By Lord Kelvin. Cambridge University Press; Baltimore, Johns Hopkins Press, 1904. Pp. xxii + 694. Price $4.50 net.

Before attempting a review of this volume it may be well to say a few words concerning its personal history. In October, 1884, Lord Kelvin, on the invitation of President Gilman of the Johns Hopkins University, Baltimore, delivered a course of lectures in the University before an audience of students and teachers who were assembled from all parts of America. As Lord Kelvin says in the preface to the present volume, he chose "as subjects the Wave Theory of Light with the intention of accentuating its failures." There were twenty lectures, the first on October 1, the last on October 17. The permanent audience numbered twenty-one, and by a happy suggestion of Professor George Forbes, these members of the audience were called the "twenty-one coefficients," reference being made, of course, to the twenty-one coefficients required in Green's dynamical treatment of an elastic solid. In giving these lectures Lord Kelvin spoke with perfect freedom, almost extemporaneously, preparation being made to-day for the lecture of to-morrow, so that the listeners had the inestimable benefit of observing how the master mind attacked and developed problems of difficulty. The lectures were reported stenographically by Mr. A. S. Hathaway, who had been a Fellow in Mathematics in the Johns Hopkins University, and this report was published in the fall of 1884, and has served as a basis of this present volume. In preparing these lectures for the Press, Lord Kelvin found so many new problems and new points of interest that he believed it unwise to print the lectures without a thorough revision. This has occupied the past twenty years, and the volume before us contains the substance of fifteen of the original lectures with numerous additions, comments, etc., together with Lord Kelvin's present views concerning the subjects of the last five lectures, and twelve appendixes on allied subjects.

There are several difficulties in reviewing this volume. One of these elastic solid; dynamics of a row of connected particles; fluorescence and phosphorescence; metallic reflection; dynamics of absorption; energy of waves; dynamics of dispersion; mechanical value of sunlight and possible density of the ether; molecular dimensions; the color of the sky; opacity of metals; Freshnel and Green's theories; kinetic theory of gases; clustering of gravitational matter; the equilibrium of several "electrions" in an atom; the viscosity of ice; waves and ripples; the molecular dynamics of a crystal; electric and magnetic screening; electric waves and vibrations in a submarine cable; etc. Another difficulty in attempting a review comes from the fact that within the past ten years physics has developed along lines essentially different from those which are treated in this volume, and therefore one lacks that interest which would have been aroused if it had appeared twenty or even fifteen years ago.

The object of the lectures is to attempt to explain the properties of the ether, both in interstellar space and in matter, the treatment being based definitely on the elastic solid theory. At the close of the course of lectures in October, 1884, there were still many points for which no such explanation had been given, but in the preface to the present volume Lord Kelvin says: "It is in some measure satisfactory to me, and I hope it will be satisfactory to all my Baltimore coefficients still alive in our world of science, when this volume reaches their hands; to find in it dynamical explanations of every one of the difficulties with which we were concerned from the first to the last of our twenty lectures of 1884."

There are many problems in light the solution of which does not require any explicit statement as to the assumptions concerning the nature of the ether and of its connection with matter, but by far the most interesting and important phenomena are those which do involve some conception as to these properties. In all these cases Lord Kelvin assumes that the ether has only those properties which ordinary matter is known to have; and this material conception of the ether forms the basis of the whole volume. (There is no mention made of the theories of Lorentz or of Larmor.) By a series of most ingenious hypotheses Lord Kelvin shows how an elastic solid theory of the ether can be devised, starting from the theory of Green, so as to explain the ordinary phenomena of reflection and refraction. He further expresses in the form of an equation the properties of an atom which will explain dispersion. But it is difficult to imagine chapters to be more difficult to read than these: the subject matter is so condensed and the discussions are so mathematical. If it were not for the frequent brilliant illustrations and analogies one might say that certain chapters could not be read. One notes with regret the absence of reference to many recent investigations which have important bearing on the subjects discussed, and it is difficut to understand why metallic reflection should be treated as independent of anomalous dispersion.

There has rarely, if ever, appeared a book which contains discussions of so many physical problems as this. In fact for many years to come the student of physics will find it to his advantage to refer to these lectures for help in almost any difficulty that may occur to him. He will surely find in it what he needs. It is impossible to express in words the enthusiasm that was felt by Lord Kelvin's audience as they listened to his daily lectures, and it is equally impossible to give any proper idea of the importance of the present volume, in as much as it contains the matured thoughts of the greatest master of physical science now living. One may regret that the line of activity is that chosen, but nothing can detract from the importance of the subject and its treatment.

The volume is a handsome book of 694 large octavo pages and is a credit to its publishers, the Cambridge Press. There is an elaborate table of contents but no index, an inexcusable neglect in any book, but specially in this case.

J. S. AMES.

Volume XIX. *August, 1904.* *Number 2*

THE

PHYSICAL REVIEW.

THE CONDUCTION OF ELECTRICITY IN MERCURY VAPOR.

By A. P. Wills.

When a current of electricity is made to pass through a tube containing mercury vapor, it may, or may not, encounter a large resistance at or near the junction of the vapor with the negative electrode. If an induction coil or a static machine be employed as a source of current supply, this large resistance invariably exists. But under certain conditions it disappears almost completely; in fact, it may so far disappear that instead of a resistance of thousands of ohms one of less than one ohm may obtain. Under such conditions, the current carrying capacity of a tube may become enormous. When the high negative electrode resistance is present, the current is conveniently measured in milli-amperes; but when it is not, the ampere is none too large for a convenient unit. In some cases, a current of over one hundred amperes has been passed through a tube containing mercury vapor.

The phenomena which accompany the passage of large currents through mercury vapor are exceedingly numerous and diverse, offering opportunities for investigation along many different lines giving promise of interesting and important results. In particular, any information which can be derived that will enable us to specify the quantities upon which the conductivity of the vapor depends, and the manner in which the variation of these quantities affects the conductivity, is likely to prove of considerable importance in connection with the electron theory of conduction in gases. The discussion in the present paper is chiefly concerned with this phase of the subject, that is, with the experimental investigation of the conducting properties of the so-called "positive column." Before going into a detailed discussion of this matter, some general phenomena to be observed in a tube carrying large currents will be noticed.

General Phenomena.

Fig. 1 represents a tube exhausted save for mercury vapor. The positive electrode, *a*, is of iron, although by slightly varying the construction of the tube a positive electrode of liquid mercury might be used. In fact, any metal may be used as a positive electrode with good results, provided it does not undergo physical or chemical change under the conditions of use. The negative electrode, *b*, always consists of liquid mercury, because this material although easily vaporized soon condenses and falls to the bottom of the tube thus becoming again part of the electrode; and this action has no deleterious effect upon the walls of the tube. Furthermore, it has been shown that electricity passes from a gas to a metal much more readily when the latter is easily capable of disintegration of some sort. When the tube is operating normally, there is about the anode a region extending to *c*, which is practically non-luminous. At *c* the luminous column begins and ends at *d*. At *d* there is a narrow dark space, then a narrow luminous section, and following this a dark space, extending to the negative electrode but penetrated by a conical flame extending from a brightly luminous spot on the surface of the electrode to the bright layer just mentioned.

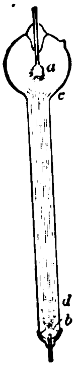

Fig. 1.

The anode, under normal conditions, presents nothing of particular interest in a CASUAL observation of its action except a slight luminous protuberance which projects out into the non-luminous region about it; and the electrode itself does not appear to be heated to any extent. But if the tube is made to carry an abnormal amount of current, the luminous column extends up to the electrode and joins with it, sometimes in one, sometimes in numerous bright spots, and at these spots the electrode itself is sometimes heated to a red or even a white heat and the tube is said to "arc." Under ordinary conditions, the current appears to be leaving the electrode over a considerable extent of surface. Again, under certain conditions, a blue velvety glow will appear over the entire surface of the electrode. When this is present, and likewise when arcing takes place, there is a large increase in the potential drop over the anode, and also in the temperature measured by a thermo-couple in the neighborhood of the anode.

The cathode presents, even in a casual examination, numerous phenomena of much interest. The bright spot at which the current enters the electrode indicates that the current must encounter conderable resistance just there. This bright spot does not remain stationary but dances around over the surface in very lively fashion. This dancing effect can be made to disappear, as Dr. Hewitt has shown, by causing a platinum wire to protrude slightly above the surface of the mercury. When this is done, the current seems to prefer to enter the electrode where the wire emerges from its surface. At the point where the current enters the electrode, there is a marked depression in the surface of the mercury. The depth of this depression is sometimes as much as three or four millimeters.

The length of the first negative dark space seems to depend very much upon the current and the diameter of the tube, also upon the vapor density. An attempt was made to find how the current and diameter of the tube affect the length of the dark space. The results are to be found in the following table.

The numbers in the table indicate the length of the dark space in centimeters.

TABLE I.

Current in Amperes.	Diameter of Tube in Centimeters.			
	2.54	5.1	8.5	9.0
1	2.5	4.0	6.0	6.0
2	3.0	5.0	6.5	6.5
3	3.0	5.5	6.5	6.7
4	2.0	6.0	6.8	6.9
5	Ill Defined.	6.5	7.0	7.3

The effect of increasing the density is to decrease the length of cause. This view is supported by the fact that when the tube is on the pump the dark space is longer with a given current than it is when operated off the pump with the same current.

The temperature in the dark space except near its junction with the luminous column, and possibly in the negative flame, is low in comparison with that in the luminous column ; for, in this region, the vapor condenses readily upon the walls of the tube.

The behavior of the " negative flame " proceeding from the bright spot on the negative electrode under the action of magnetic force has been investigated by Dr. Hewitt, and the results will probably appear in another place.

The Positive Column, viewed from the side, usually appears uniformly luminous except in the neighborhood of its extremities *c* and *d*; viewed from the end, it appears brightest at the axis of the tube; and at low vapor densities the luminosity falls off gradually towards the walls of the tube; but at high vapor densities the luminosity is confined to a small circular cylinder concentric with the axis of the tube. The cross-section of this cylinder increases with increase of current, and decreases with increase of vapor density.

Sometimes, in the early stages of exhaustion, a tube will show beautifully defined striations which may be made to dissolve into a uniformly luminous column by varying the current slightly in either direction from a certain value favorable to the striated condition. The most beautiful striations are obtained with tubes of large diameters.

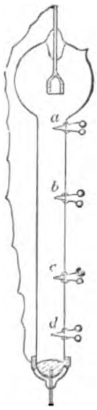

Fig. 2.

The Electromotive Intensity and the Temperature Along the Positive Column. — In order to find out how the temperature and the electromotive intensity vary along the length of the column, the tube shown in Fig. 2 was constructed. At *a*, *b*, *c* and *d* are thermocouples serving also as terminals for an electrostatic voltmeter. The tube was 1.9 cms. in diameter and the distance between successive electrodes 38 centimeters. The results contained in Table II. were found, the electromotive intensity being the average between the points indicated.

TABLE II.

Current in Amperes.	Electromotive Intensity in Volts per cm. between			Temperature in Degrees Centigrade at			
	a — b	*b — c*	*c — d*	*a*	*b*	*c*	*d*
1.25	.645	.660	.641	123	83	105	134
1.75	.612	.612	.590	143	88	143	160
2.25	.580	.585	.557	172	125	175	200
2.75	.552	.559	.537	200	123(?)	205	241
3.25	.535	.548	.561	243	167	242	297

The numbers in this table are for the case of the tube operating in air at the room temperature of about 20° C. The results given are typical of those which could be obtained from a tube of any diameter.

The Drop in Potential over the Anode. — By the phrase "drop in potential over the anode" is meant the difference in potential between the anode itself and a small wire (thermo-couple) sealed into the tube with its end about one centimeter in front of the anode. The following table gives the results obtained from a tube 1.9 centimeters in diameter; the temperatures given are those furnished by the thermo-couple which at the same time served as one of the terminals for the electrometer.

TABLE III.

Current in Amperes.	Anode Drop in Volts.	Temperature in C°.
1.25	6.5	123
1.75	5.8	143
2.25	5.4	172
2.75	5.1	200
3.25	4.6	243

These figures were obtained while the tube was behaving normally at the anode; that is, there was no arcing and no bluish haze over the electrode. The effect of these is to greatly increase the drop in potential over the anode, and also to cause a large increase of the temperature. The anode drop may rise to fifteen volts when the anode is behaving abnormally. Much larger currents than the last given in the table could not be used without producing arcing. As the main object of the investigation was to study the behavior of the vapor column, no exhaustive study of phenomena at the electrodes was made but, in all probability, the results given above are typical of those which would be obtained from all tubes. It is noteworthy that the larger the current the less the anode drop. If the anode consist of liquid mercury instead of iron, the anode drop is considerably greater under similar circumstances. The current probably does extra work in vaporizing the mercury.

The Drop in Potential over the Cathode. — From experiments upon a number of different tubes it appears that the drop in potential over the cathode is very nearly independent of the current, and about five volts in magnitude. Dr. von Rechlinghausen has shown by using very short tubes of large diameter, so that the resistance of the vapor could be neglected, that the sum of the cathode and anode drops decreases slowly as the current increases. He employed in one instance a current as large as 100 amperes. The lowest value for the sum of the two drops which he obtained was 7.5 volts. From what has been said above, it is probable that this low value is due to the favorable action of large currents in decreasing the resistance encountered by the current at the anode.

Methods of Starting Tubes. — The methods employed in starting tubes have already been described by Dr. Hewitt and will be but briefly referred to here.

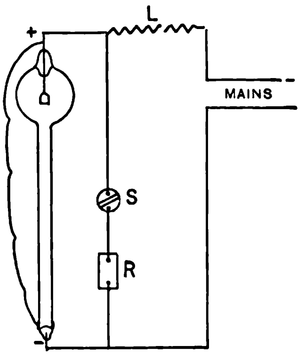

Fig. 3.

The method of starting by high tension discharge is the one which was employed in the experiments described in the present paper. A convenient arrangement in the application of this method is shown in Fig. 3. In multiple with the tube there is placed an oil switch in series with a resistance. When the switch is closed a current passes from the positive to the negative main through the inductance coil L shown in the figure. Now if the switch be suddenly opened a high tension discharge passes from the positive to the negative main by way of the tube, thus breaking down the high internal resistance of the tube and allowing the ordinary difference of potential of the mains to start and maintain a continuous current, provided the external resistance in series with the tube is such as to permit the passage of a current which exceeds a certain minimum value. A variation of this method is obtained by removing the oil switch and making the inductance coil in series with the tube the secondary of an induction coil, the current in the primary of this coil being broken by a Wehnelt interrupter, for instance, when it is desired to start the tube. After the tube starts, the current through the primary is, of course, discontinued.

The method of starting by contact consists simply in tilting the tube until the mercury from the negative electrode runs along the tube and makes contact with the positive electrode, then, upon tilting the tube back, a break is made, and the tube starts. On the whole, the high tension discharge method is the most satisfactory in practice.

The Starting Band. — It was early discovered by Dr. Hewitt that, if a band of metal be wrapped around the outside of the tube in the neighborhood of the surface of the liquid mercury constituting the cathode and connected by a wire to the positive terminal of the tube, then the tube will start under a less high potential difference when the method of starting by high tension discharge is employed. The exact nature of the action of this starting band is somewhat obscure.

The Action of Inductance in Series with the Tube. — A tube is almost always operated with a certain amount of inductance and resistance in series with it. The action of both is to help towards steadiness of operation. But inductance and resistance are not necessary when the tube is small enough to cause the walls to restrict very much the area which the conducting column would cover if it met with no interference from the walls.

Suppose a tube to be operating, and let the current be gradually decreased. When the current reaches a certain value the tube suddenly ceases to operate; repeat the experiment, and the same thing happens at approximately the same value of the current. This value of the current is called the "minimum value," and is designated by I_m. Now I_m is a function of the external inductance and also of the external resistance and moreover not only of the will an inductance of equal magnitude with a straight iron core. The latter determines a smaller value of I_m. The general nature of the functional relation between I_m and the inductance, L, is as follows: When the inductance, L, is quite small, a few milli-henrys, say, and the resistance in series with the tube is not very large, say over thirty ohms, then I_m will be somewhere in the neighborhood of three amperes. As the value of L is increased the value of I_m will decrease very rapidly at first, then more slowly. When L has a value of one henry, then I_m is about .5 of an ampere. An increase in L to four henrys causes but a slight decrease in I_m. In fact if the functional relation be plotted, the curve appears to become asymptotic to a line $I_m = .4$, approximately. The "minimum value" of the current seems to vary somewhat with the diameter of the tube, being smaller for tubes of smaller diameter.

The increase of external resistance, the E.M.F. of supply being increased to correspond, effects a decrease in I_m. But in practice this is not so good a method of producing the desired result, namely a small value of I_m, which means steadiness of operation.

Certain Voltage Current Relations.

Before proceeding to the consideration of the specific factors which determine the conductivity of a mercury vapor column, certain relations between the voltage over a tube and the current passing through it will be presented. The remarks immediately following apply to the case of a tube operating in air at ordinary room temperature. The diagrams comprehending the results are from a paper by Dr. Hewitt.[1] These results are introduced here, not alone for the sake of completeness, but also because a study of them suggested methods of attack upon the more difficult problem of the segregation of the factors which determine the conductivity of the vapor column itself.

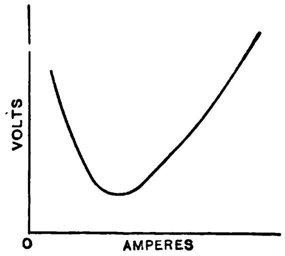

Fig. 4.

Fig. 4 shows the general nature of the relation between the voltage over a tube and the current passing through it.

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[1] Electrical World and Engineer, April 27, 1901, p. 679.

The form and position of this curve may easily be changed by altering the radiating power of the tube, and this is more especially true of the right hand limb of the curve. This fact points towards the conclusion that the density of the vapor plays an important rôle in determining the conductivity of a vapor column.

Fig. 5 is another diagram taken from Dr. Hewitt's earlier paper. The currents employed and the diameter of the tube were such that the vapor density was an important factor in determining the conductivity of the vapor column. The points fixing the curve were obtained as follows. The tube was started with a current of three amperes and the voltage over the tube taken, then the external resistance was suddenly decreased, causing an increase of current; the voltage did not immediately increase but soon began to do so and the current simultaneously decreased somewhat. In a few minutes the current and voltage became stationary; these were noted and determine the second point on the curve. The other points were obtained in the same way. The points thus represent steady states. The interesting thing in connection with this experiment is the fact that considerable time is required to reach a new

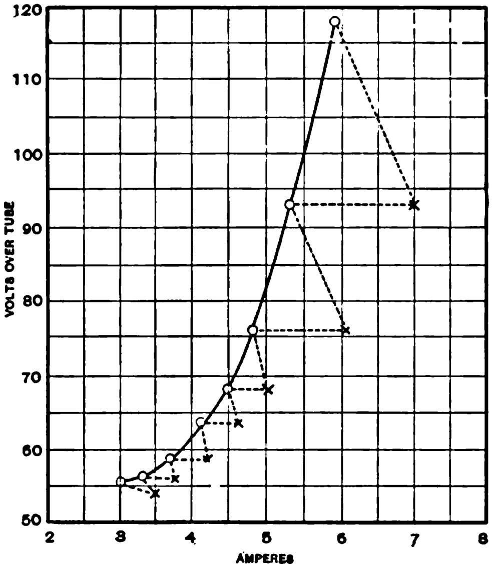


Fig. 5.

steady state after the current is changed. This is attributed to the sluggish variation in the vapor density following the increase in temperature due to the increase in current. Upon the fact that the density variation follows sluggishly the current variation is based the method for finding the relation between the conductivity of the vapor column and the vapor density. This method will now be described.

DESCRIPTION OF THE METHOD OF EXPERIMENT.

To discover experimentally the exact rôle which the density of the vapor plays in determining the conductivity of the vapor was difficult. Perhaps the most difficult thing of all was to discover what vapor density exists in a tube carrying a large current of electricity. Owing to large temperature differences between points along the axis of the tube and points near the walls there is probably a very considerable variation in the vapor density over a cross-section of the vapor column. So far, what has been done is to find the effect of varying the mean density in a tube upon the electromotive intensity. The experiments about to be described show how this was accomplished. It was finally decided to adopt a method of experimentation which would ensure the vapor to be in a state of saturation at every point in the tube at any desired temperature while no current was flowing. Then, if the temperature corresponding to this state be known, the vapor pressure and the vapor density may be deduced with the aid of tables. Now, if a current be sent through the tube and the drop in potential over a known length of vapor column be taken immediately, it will be that corresponding to a mean density of the vapor equal to the vapor density existing in the tube just before it was started, which is determined as described above. By repeating this proceedure over a range of temperatures, the effect of variation of the vapor density upon the electromotive intensity may be found. In the application of this method, it is assumed that a variation of the mean vapor density follows but slowly the introduction of the current. That this is the case, has been already pointed out.

The device employed to secure different vapor densities by filling the tube with saturated vapor at different temperatures is shown in Fig. 6. The upright hollow cylindrical columns shown in the figure were of very thick asbestos. At the top and bottom of these columns are wooden boxes lined with asbestos. These boxes had openings into the columns so that a free circulation of air could be produced throughout the interior of the arrangement. The tube is seen hanging in the right hand column, and in the left is seen the electric heating coil. The fan, seen just above this coil, was operated by an electric motor and caused the hot air produced by the action of the coil to circulate rapidly in the direction indicated by the arrows. By regulating the resistance in series with the heating coil, any desired temperature of the air surrounding the tube could be maintained. The diagram of connections is clearly shown in the figure. The temperature of the saturated vapor was found by the thermo-couple in connection with the D'Arsonval galvanometer. A Kelvin electrostatic voltmeter was employed to determine the drop in potential between the two thermo-couples shown in the figure; these latter were about 15 centimeters apart in all cases. The tubes employed in the experiments to be now described were all about 60 centimeters in length. The diameters of these tubes varied from 1.25 centimeters to 5.08 centimeters.

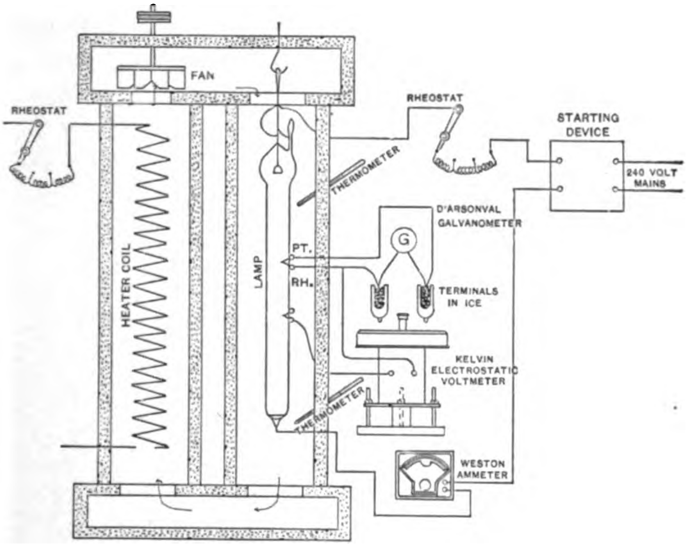


Fig. 6.

In making an experiment, the usual procedure was to raise the temperature to about 170° C., and to keep it stationary there for about ten minutes before starting the tube. The desired current was started in the tube and a reading on the voltmeter taken as soon as possible, after which, the current was turned off. Then the temperature was very gradually lowered and further readings taken at the temperatures selected and with the same current. The time occupied in making a set of experiments upon a given tube with a given current was usually about six hours. The vapor pressures corresponding to the temperatures selected were taken from a curve plotted from the experimental results given by Hertz. The galvanometer used in connection with the thermo-couple had a bad habit of changing its zero position in the midst of a set of experiments, and in spite of the care used to overcome this trouble it is probable that some slight errors crept in from this cause.

Results.

The results of the experiments thus made are to be found in the following tables.

Table V.

Diameter of Tube = 1.25 *cms.* *Current* = .5 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
158.0	4.05	1.90	140.7	2.02	1.35
155.3	3.62	1.79	138.7	1.84	1.33
151.2	3.09	1.67	136.7	1.68	1.26
149.0	2.84	1.60	132.4	1.38	1.19
147.0	2.63	1.54	128.0	1.14	1.14
145.0	2.43	1.48	119.3	.75	1.03
143.0	2.23	1.46	101.0	.30	.98

Table VI.

Diameter of Tube = 1.25 *cms.* *Current* = 1 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
167.0	5.60	2.04	149.3	2.90	1.47
163.5	4.92	1.89	145.0	2.43	1.33
159.5	4.30	1.75	140.7	2.02	1.24
157.5	3.97	1.70	132.4	1.38	1.08
155.5	3.65	1.66	123.3	.91	.973
153.5	3.39	1.60	104.5	.37	.897
151.3	3.10	1.52			

TABLE VII.

Diameter of Tube = 2.54 *cms.* *Current* = 1 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
163.0	4.90	1.33	145.0	2.43	.945
158.0	4.05	1.24	140.7	2.02	.864
155.3	3.62	1.19	136.7	1.68	.790
151.2	3.09	1.11	128.0	1.14	.688
149.0	2.84	1.06	119.3	.75	.620
147.0	2.63	1.00	101.0	.30	.550

TABLE VIII.

Diameter of Tube = 2.54 *cms.* *Current* = 2 *Amperes.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
167.0	5.60	1.31	149.3	2.90	.878
163.5	4.92	1.20	145.0	2.43	.824
159.5	4.30	1.14	140.7	2.02	.770
157.5	3.97	1.09	132.4	1.38	.633
155.5	3.65	1.03	123.3	.91	.578
153.5	3.39	.966	104.5	.37	.503
151.3	3.10	.920	84	.12	.490

TABLE IX.

Diameter of Tube = 3.80 *cms.* *Current* = .5 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
167.0	5.60	1.47	151.3	3.10	1.07
163.5	4.92	1.35	149.3	2.90	1.03
159.5	4.30	1.28	145.0	2.43	.947
157.5	3.97	1.22	140.7	2.02	.893
155.5	3.65	1.17	132.3	1.37	.718
153.5	3.39	1.12	123.3	.91	.630

TABLE X.

Diameter of Tube = 3.80 *cms.* *Current* = 1 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
171.0	6.40	1.32	144.7	2.40	.852
167.0	5.60	1.20	136.8	1.69	.723
163.5	4.92	1.10	130.0	1.24	.608
159.5	4.30	1.04	120.0	.78	.486
155.5	3.65	.974	108.0	.43	.426
151.3	3.10	.913	81.4	.10	.400

TABLE XI.

Diameter of Tube = 3.80 *cms.* *Current* = 3 *Amperes.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
167.0	5.60	.987	149.3	2.90	.744
163.5	4.92	.926	145.0	2.43	.676
159.5	4.30	.865	140.7	2.02	.607
157.5	3.97	.812	132.3	1.37	.534
155.5	3.65	.790	123.3	.91	.452
153.3	3.39	.778	104.5	.37	.365

TABLE XII.

Diameter of Tube = 5.08 *cms.* *Current* = 1 *Ampere.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
170.1	6.40	1.19	145.0	2.43	.723
167.0	5.60	1.13	143.0	2.23	.710
163.5	4.92	1.03	138.6	1.83	.648
159.5	4.30	.960	134.3	1.52	.675
155.5	3.65	.872	130.0	1.24	.533
151.3	3.10	.818	125.5	.94	.520
149.3	2.90	.798	104.5	.37	.398
147.3	2.65	.757			

TABLE XIII.

Diameter of Tube = 5.08 *cms.* *Current* = 2 *Amperes.*

Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.	Temperature in C°.	Pressure in mm. Hg.	Electromotive Intensity in Volts per cm.
170.1	6.40	1.013	147.3	2.65	.690
167.0	5.60	.960	145.0	2.43	.662
163.5	4.92	.879	143.0	2.23	.628
161.7	4.62	.859	140.7	2.02	.580
159.5	4.30	.818	138.6	1.83	.540
157.5	3.97	.805	134.3	1.52	.493
155.5	3.65	.778	130.0	1.24	.460
153.5	3.39	.757	121.0	.82	.385
151.3	3.10	.710	111.7	.52	.345
149.3	2.90	.703	101.8	.31	.338

These tabulated results are exhibited graphically in the following diagrams.

Discussion of the Results.

It is evident from these diagrams that, with the exception of values of the electromotive intensity corresponding to pressures less than half a millimeter of mercury, the relation between electromotive intensity and pressure may be fairly well represented by two intersecting straight lines. The pressure corresponding to the point of intersection of these lines depends upon both the diameter

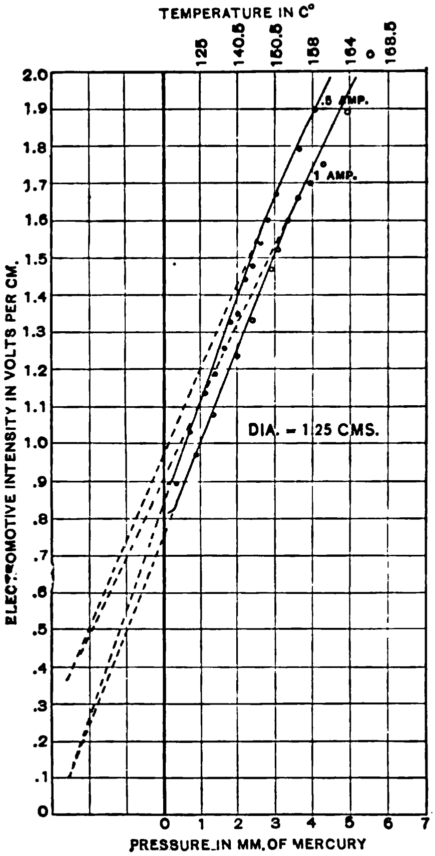


Fig. 7.

numerically, wherein I stands for the current in amperes, and D for the diameter of the tube in centimeters. At a value of the pressure about equal to $\varDelta_Q$, the walls of the tube seem to cease to exert a restrictive action upon the conducting vapor column; the luminous column

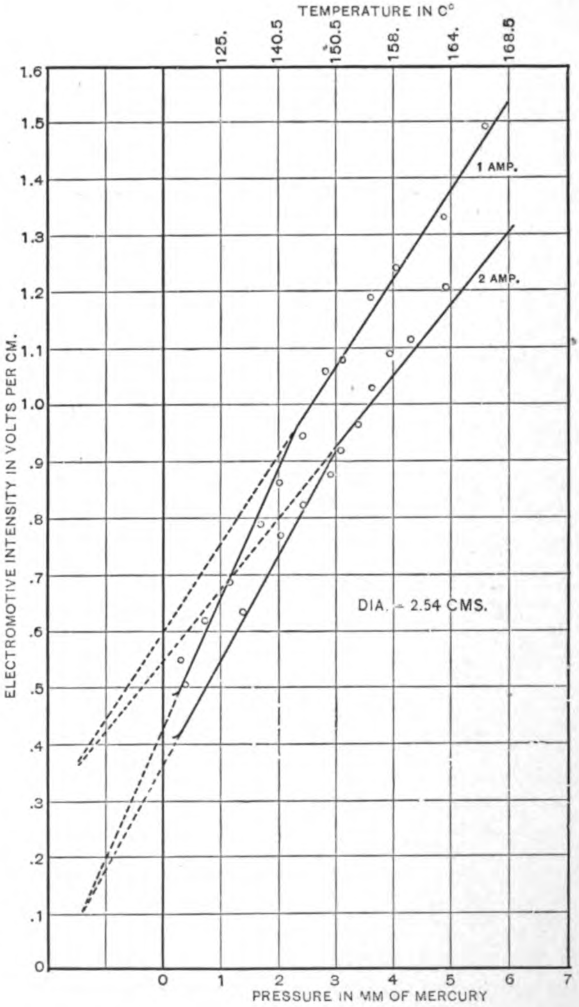


Fig. 8.

seems to leave the walls of the tube about this point and to grow density, in affecting the conductivity of the vapor contained in the tube, it is necessary to consider two cases, *viz*: Case I., in which the walls of the tube exert a restrictive action upon the luminous column; Case II., in which the luminous column is practically free of the walls of the tube. It will be noticed in the diagrams that the slopes of the lines representing the latter case are less than those representing the former. The slopes in both cases are dependent upon both the diameter of the tube and the current.

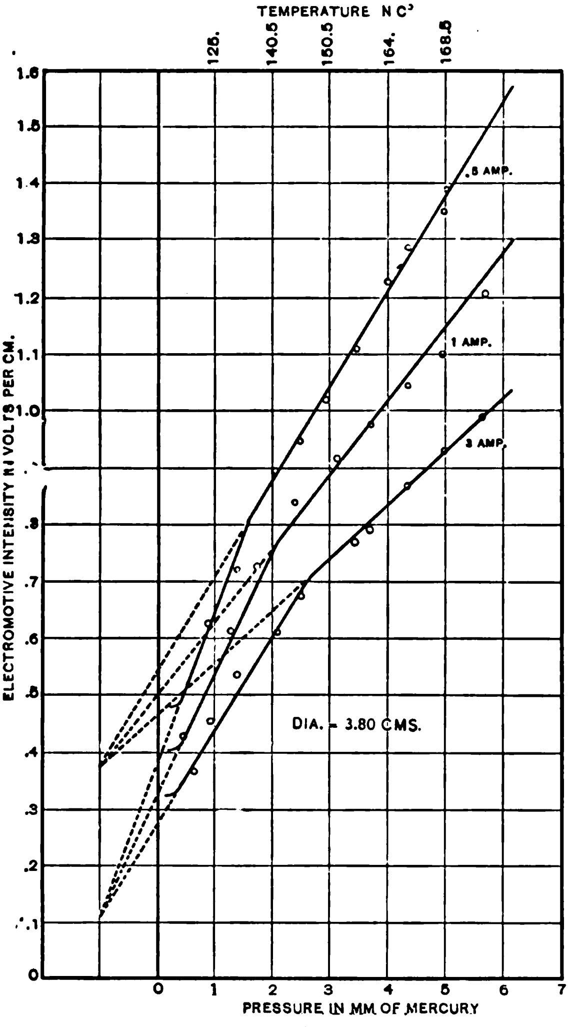


Fig. 9.

An attempt was made to find an empirical formula which would represent the results given in the above tables and diagrams. It was found that the results are represented with considerable accuracy by the formula

$$X = a\left(b + \frac{1}{\sqrt{I}}\right)\left(c + \frac{1}{\sqrt{D}}\right)\left\{\Delta + \frac{2}{D} + \frac{1}{\sqrt{D}}\right\} + \alpha$$

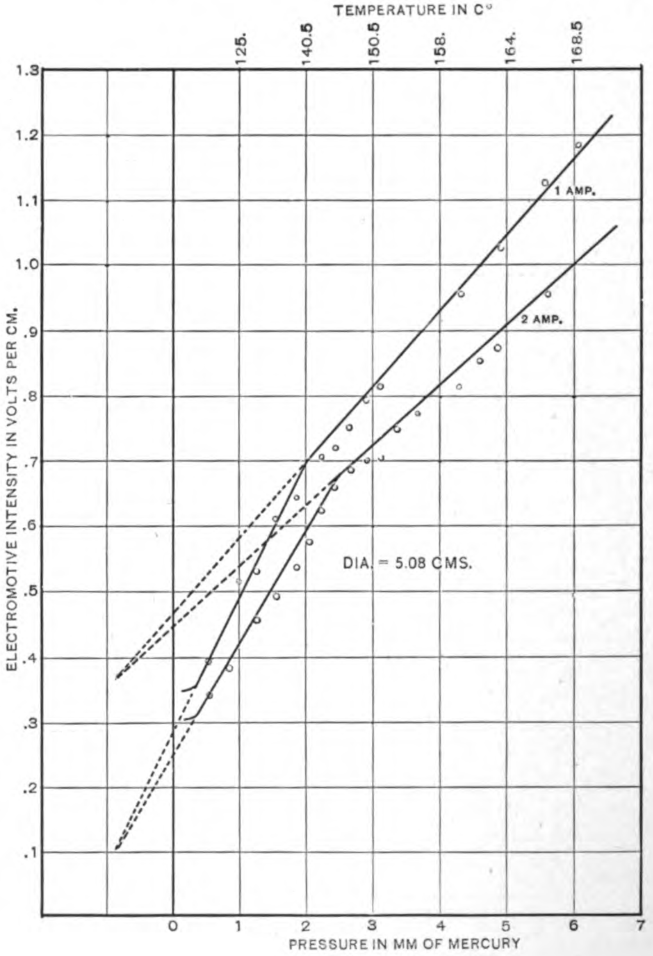


Fig. 10.

wherein X represents the electromotive intensity in volts per centimeter, I the current in amperes, D the diameter of the tube in and wherein a, b, c and α are constants which suddenly jump in value when the pressure Δ_Q is reached; the values of the constants which do not change in value when this point is reached are inserted in the formula. The numerical values of these constants are

	Below Δ_Q.	Above Δ_Q.
a	.0545	.150
b	.775	.398
c	1.71	.122
α	.100	.370

An idea of the accuracy with which the formula represents the observations is obtained from the consideration of the fact that the lines on the diagrams are plotted from the above formula, while the small circles represent the observations; the case of the tube with the diameter 1.25 cms. and current .5 of an ampere is an exception and the only one, to this statement. In this case the values given by the formula are about 8 per cent. too high. This empirical formula was suggested by certain properties which the curves plotted to the observations possess, namely — for a given diameter, the lines for different currents representing the observations below Δ_Q, when produced, meet in a point r; and likewise, the

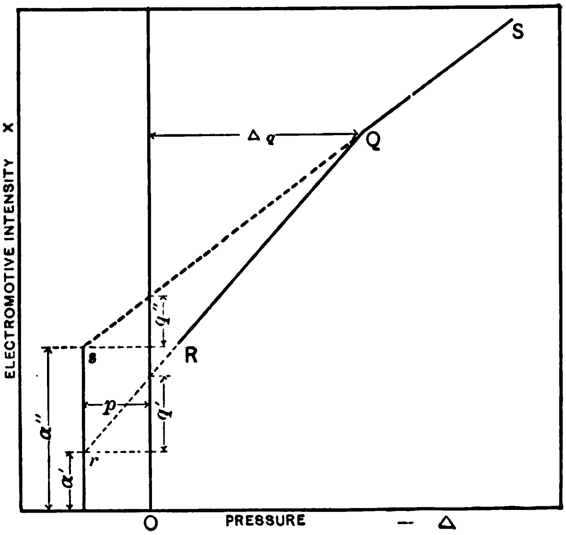


Fig. 11.

lines for different currents representing the observations above Δ_Q, meet in a point s; and the points r and s are both to the left of the axis of electromotive intensity and at the same distance from it; and furthermore, if the diameter of the tube be changed, the points r and s move along lines which are *parallel* to the axis of pressure. These properties are shown by the points of intersection of the dotted lines in the various diagrams above. The lines along which r and s move as the diameter is varied are $X = .10$ and $X = .37$, for r and s respectively, and these numbers are those which a assumes in the formula according as we are dealing with the restricted or unrestricted conducting column. A reference to Fig. 11 will serve perhaps, to make the matter clearer. A pair of lines representing the observations on a given tube with a given current is shown. The points r and s are determined as the points of intersection of the lines RQ and $X = a' = .10$, and of the lines QS and $X = a'' = .37$. Now the equation of either of the lines RQ, QS is of the type

$$X = \frac{q}{p}(\Delta + p) + a.$$

The observations show that q is a function of both I and D, and that p is a function of D alone; and it was found that

$$\frac{q}{p} = a\left(b + \frac{1}{\sqrt{I}}\right)\left(c + \frac{1}{\sqrt{D}}\right)$$

and

$$p = \frac{2}{D} + \frac{1}{\sqrt{D}}$$

represent the facts quite accurately within the limits of experiment; and so the empirical formula given above was derived.

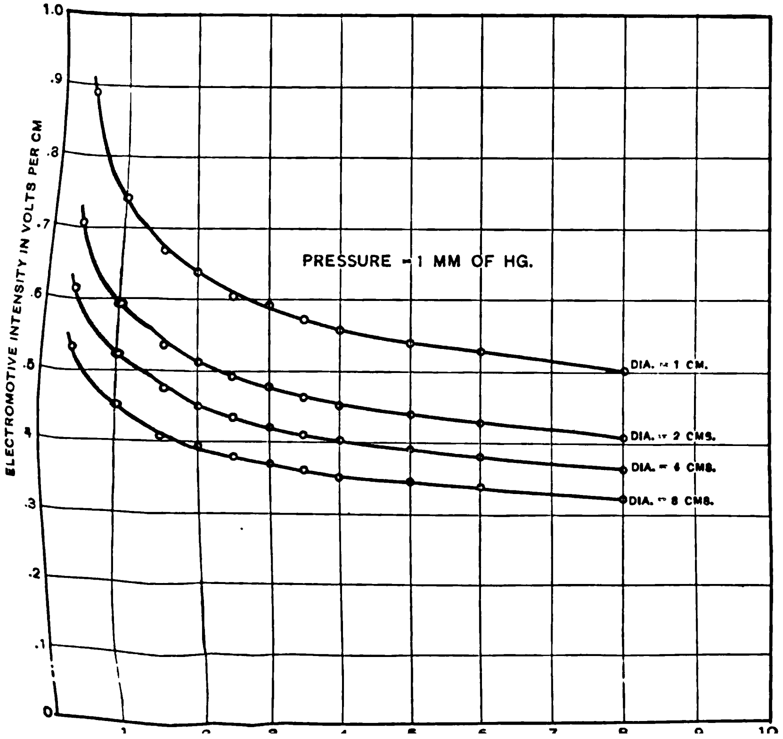
The formula does not represent the results for pressures below .3 mm. of mercury. At about this pressure, the electromotive intensity assumes a value which remains nearly constant as the pressure is still further decreased. In the present experiments, the pressures employed were not small enough to indicate the probable large values of the electromotive intensity which obtain for *very* low densities. [The investigation of the conducting properties of mercury vapor under these conditions would furnish interesting results.

Instead of the heating chamber employed in the present experiments, a cooling chamber would probably be required.]

There is one objection to the method employed in the present investigation which applies particularly to small tubes, in that the method prohibits the use of very large currents, which would cause such rapid heating as to produce a considerable change in the vapor density existing in the tube, before the necessary readings could be taken.

THE RELATION BETWEEN ELECTROMOTIVE INTENSITY AND CURRENT.

In order to exhibit the relation between electromotive intensity and current, the curves shown in Fig. 12 were plotted from the formula. The curves shown all relate to a pressure of one milli-



meter of mercury. The effect of current in increasing the conductivity of a tube is clearly shown. It would appear, however, as though this effect of the current may be largely due to a change in the distribution of the vapor density radially from the axis of the tube. For, as an increase in the current means more heating of the vapor in the path of the current and a consequent higher temperature, the vapor density in the path of the current will therefore be decreased, with the result that the conductivity of the tube is thereby increased. The current, as it were, throws the obstructing matter out of its path. This view seems to be supported by the fact that when the current is small the effect is greatest. For, in this case, there is more relatively non-conducting space near the walls into which the obstructing matter may be pushed without being still in the path of the current, which is for the most part in the neighborhood of the axis of the tube, where the heating is greatest. As the current gets larger, more and more of the available cross-section of the vapor column becomes conducting, with the result that the space into which the superfluous material may be thrust without interfering with the passage of the current becomes less. The effect of increasing the vapor density is to cause the current to become more concentrated in the neighborhood of the axis of the tube, so the effect just described should be larger at higher densities. That this is the case is evident if we consider the derivative of X with respect to I, namely,

$$\frac{\partial X}{\partial I} = -\frac{a}{2}\left(c + \frac{1}{\sqrt{D}}\right)\left\{\Delta + \frac{2}{D} + \frac{1}{\sqrt{D}}\right\}\frac{1}{I^{3/2}}.$$

Another cause operates also to assist the action of the current in reducing the electromotive intensity, namely, the increase in conducting area which follows an increase of current. This is easily observed in a large tube by viewing it from either end; upon mercury. When the diameter is small, the effect of increasing it upon the electromotive intensity is greater than when it is large, owing to the fact that the restrictive action of the walls upon the diameter of the conducting column of vapor is greater in the former case.

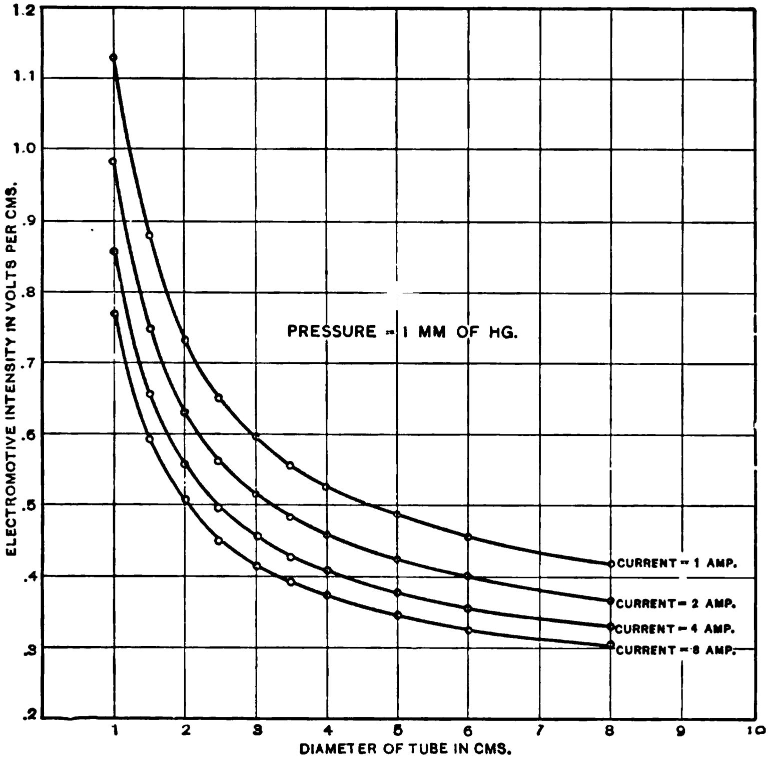


Fig. 13.

Approximate Equations.

Case in Which the Density is Large. — In this case we have approximately, if the diameter is not very small,

$$X = a\left(b + \frac{1}{\sqrt{I}}\right)\left(c + \frac{1}{\sqrt{D}}\right)\Delta + a, \quad \Delta \text{ large}$$

in which, of course, the appropriate values of the constants a, b and c must be selected. (See above.)

Case in Which the Diameter is Large. — If the general equation holds for sufficiently large diameters, then we may replace it by the approximate equation,

$$X = ac\left(b + \frac{1}{\sqrt{I}}\right)\Delta + \alpha, \quad D \text{ large.}$$

Case in Which the Current is Large. — Under the assumption, again, that the general equation includes the case,

$$X = ab\left(c + \frac{1}{\sqrt{D}}\right)\left\{\Delta + \frac{2}{D} + \frac{1}{\sqrt{D}}\right\} + \alpha, \quad I \text{ large.}$$

Hence, when the current is very large, the electromotive intensity is independent of current.

Case in Which the Diameter and Current are Both Large. — The general equation reduces to

$$X = abc \cdot \Delta + \alpha, \quad I \text{ and } D \text{ large.}$$

It would appear from this equation that, if the general empirical formula given above may be assumed to hold for very large currents and diameters, then the least electromotive intensity which will maintain a current in mercury vapor is somewhat greater than .1 of a volt per centimeter when the current is restricted, and somewhat greater than .37 of a volt per centimeter when it is unrestricted. It should be remembered that for values of Δ less than .3 millimeters of mercury the formula does not hold, as the electromotive intensity then ceases to fall with the vapor pressure. When $\Delta = .3$ we have

$$X = .022 + .10 = .122, \quad \text{below } \Delta_Q$$

and

$$X = .022 + .37 = .392, \quad \text{above } \Delta_Q.$$

It is questionable, however, if the formula be capable of such wide application. But it is most probable that a current of electricity may be maintained in mercury vapor under an electromotive intensity of less than .5 of a volt per centimeter.

OPTICAL NOTES.

By Wm. W. Coblentz.

I. Reflection and Refraction at the Interface of two Media Having Intersecting Dispersion Curves.

OCCASIONALLY there appear notices of methods for making objects invisible by selecting combinations of media having about the same index of refraction as the immersed solid. This subject was first investigated by Christiansen[1] in 1884. His object was to show that white powders are transparent. For this purpose he used a mixture of ground glass, benzene and carbondisulphide. The transmitted light varied in color from violet to red, depending upon the size of the glass particles, upon the quantity of carbondisulphide mixed with the benzene, and also upon the temperature. That wave-length, for which there is an equality of the indices of refraction of the mixture, is transmitted without reflection or refraction. It varies with the temperature, is fairly homogeneous, and is called "Monochrome" by Christiansen. The reflected and refracted colors bear his name.

The most recent notice on this subject is by Wood,[1] who dissolved chloralhydrate in glycerine. This solution has almost the same dispersion as glass, and finely powdered glass, in it, is transparent without showing a trace of Christiansen's colors.

The object of the present communication is to call attention to a phenomenon connected with media having intersecting dispersion curves, hence the same index of refraction for a certain wave-length of light.

While investigating the infra-red absorption spectrum of eugenol, $C_{11}H_{14}O_2$, it was found that the liquid had decomposed on standing exposed to the light, the color having changed from a light to a deep reddish brown. As a film between two plates of rock salt, observed at a varying angle of almost grazing emergence of the light, one sees a brilliant series of spectral colors — reddish brown for the reflected, and violet for the transmitted rays. Safrol, $C_{10}H_{10}O_2$, which is light straw-color also shows this effect while carvacrol $C_{10}H_{14}O$, also straw-color, does not. At first this seemed likely to be due to "Resonance." But the directly transmitted light showed no peculiarities as viewed in a direct-vision spectroscope. Furthermore, the same films between glass plates lost this property. It was then suggested by Professor Nichols that this is probably due to the near equality of the refractive indices of the two media.

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[1] Christiansen, Ann. der Phys., 23, p. 298, 1884.

[2] Wood, Phys. Rev., Vol. XV., p. 123, 1902

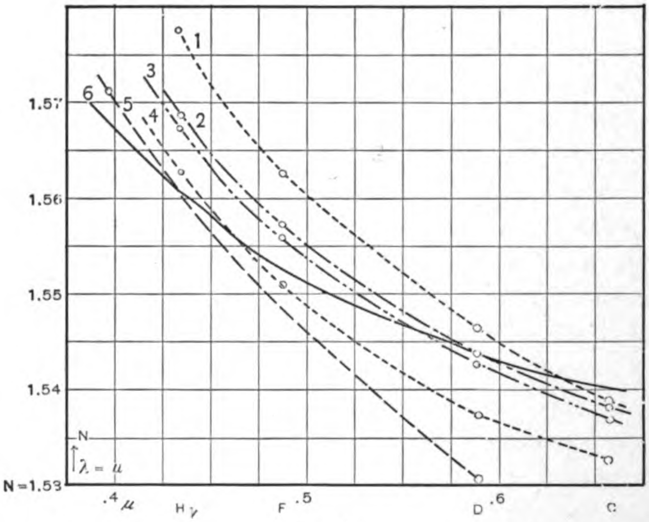

Fig. 1.

Dispersion curves: 1 = benzaldehyde; 2 = eugenol; 3 = safrol; 4 = methyl eugenol; 5 = cuminol; 6 = rock salt.

A glance at the dispersion curves[1] of rock salt and the liquids, in Fig. 1, explains the question. In fact they contain more information than was first anticipated. In the region to the *right* of a ray, traveling in a given medium, falls upon the boundary separating it from a less refracting medium at an angle of incidence greater than the critical angle. To the *left* of the intersection, in the same manner, the rays pass from a less to a greater refracting medium, and we have the ordinary case of refraction.

However the occurrence of total reflection, and of refraction at the boundary of two media, at the same time, by breaking up and dispersing a non-homogeneous ray of light is rather unique, and is so easily observed that it seems worth mentioning. It impresses us the more with the question of what occurs at the interface of two media, of optical density, etc., which will be considered presently.

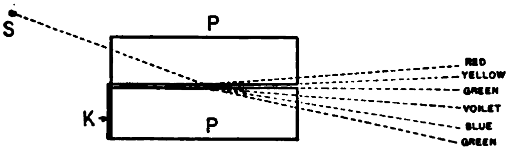

Fig. 2.

Let us first consider what has really been observed when non-homogeneous light passes into a combination of media like a film of eugenol (from oil of cloves or benzaldehyde from bitter almond-oil; the essential oils might answer the purpose), held between the two plates of rock salt, as indicated in Fig. 2, where p, p, = plates, s = the source of light, k = an opaque screen to prevent light from entering from below and thus confusing the resultant distribution of light. It was found, on viewing the surface of the films of eugenol at an angle of almost grazing emergence, as shown in Fig. 2, that the totally reflected light was a reddish brown, which changed to a greenish yellow as the angle with the normal was increased. Viewed end on a light green was visible. Moving the eye farther along so as to get the transmitted light, a quite deep blue appeared which changed to a blue-green, and finally light green as the eye was moved farther around the edge. For benzoldehyde, following in the same order, the reflected light was a deep red, cardinal, reddish brown to orange, then blue to green for the transmitted rays. In the same manner the colors for safrol were reddish brown, orange to light green for reflection, then violet, blue green to light green for the transmitted ray. Most interesting of all is cuminol, $C_{10}H_{13}O$, which, as will be noticed in the dispersion curves, Fig. 1, intersects the rock salt curve towards the violet. For this compound, the reflected rays, observed in the usual order, appeared in succession brownish red, yellow, green, and light green, while the transmitted rays were deep violet then blue to deep green.

It is to be noticed that the arrangement of the spectral colors appears anomalous, *i. e.*, red, yellow, green, then violet, blue, green.

However, since we are *looking at* the transmitted light we see the virtual spectrum in which case the violet appears the least refracted.

A quotation from Rayleigh[1] seems in order there, since the media in question show continuity as well as discontinuity when we consider their dispersion curves. This combination might possibly be used to study the question of a finite reflection at the interface of two media having different dispersive powers, at the point where the indices are the same.

The present cursory examination shows qualitatively how dependent the reflected wave-lengths are upon the relative difference in the refractive indices. Thus for benzaldehyde where the intersection of the dispersion curves is toward the red no green is reflected while for cuminol all wave-lengths up to the deep green are reflected.

The experiment is simple and easily performed. Rock salt is easily obtained. The natural cleavage surfaces last the longest, and are usually just as plane as the hand polished. The few striations will do no harm if the plates are put together at their common cleavage face. The splitting of the plates is most easily done by applying a thin, sharp-bladed knife, and tapping it gently with a light hammer. The size of the plate produced depends upon the dexterity of the manipulator as well as upon the homogeneity of the crystal. A plate 2 x 3 x .2 cm. will answer the purpose, unless a lantern projection is desired. A few drops of the liquid is placed on a plate, which is then covered with a second plate and the two held together in a clamp or by means of a wire wrapped around them. The liquids mentioned have high boiling points, hence are convenient to manipulate. The incident light should not be too intense. Daylight has sometimes been found most serviceable. The bottom, one end, and the two sides of the lower plate should be protected from stray light. The incident light is then permitted to enter from above, and viewed from the other end, as already described. Such plates will last for several months provided the surfaces have not been rubbed hard, nor scratched in cleaning.

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[1] Rayleigh, Light, Encyc. Britt., 9th ed., 24, p. 458, 1888. "The whole problem of reflexion is so much concerned with the condition of things at the interface of two media about which we know so little. . . . Reflection depends entirely upon an approach to *discontinuity* in the properties of the medium. If the thickness of the transitional layer amounted to a few wave-lengths there would be no sensible reflexion at all. Our theories take no account of the fact that one at least of the media is *dispersive*. . . . We may thus expect a finite reflexion at the interface of two media if the dispersive powers are *different* even though the *indices* be absolutely the same for the waves under consideration, in which case there is no refraction."

The disappearance of an object when immersed in a medium having the same refractive index is less exactly shown by placing powdered rock salt in one of the liquids mentioned. (Ordinary salt will not behave thus.) A blue color is visible for the transmitted light, while patches of totally reflected red are also apparent — "Christiansen colors." Applying heat the whole becomes opaque, but again becomes transparent on cooling. For a lantern projection it is best to have the rock salt split into parallelopipeds having edges, say .5 to 1 cm. These are placed in a flat, glass cell and covered with benzaldehyde. In this case the reflected and refracted rays (for light passing through the liquid) at the interface, are just the reverse of the case already considered when the liquid was between the rock salt plates. Here we have violet, blue, green for reflection, then green, yellow, red for refraction, observed in the order already described.

At the same time that one observes this effect at the upper surface of the rock salt cube, the phenomena already described for a film of liquid between the plates can be seen at the lower surface if precaution has been taken to avoid stray light from entering the sides and the bottom of the cell. To demonstrate this most successfully it is best to have one large parallelopiped of rock salt in the liquid, covering its sides and the upper half of one end with tinfoil to avoid confusion from stray light. Even then it is difficult to repeat one's observations, and the method of placing the liquid between plates of rock salt seems preferable. However, for the spectacular, the display of colors from the parallelopipeds of rock salt in benzaldehyde rivals that obtained by interference of polarized light in piles of mica plates. Rock salt is about the only transparent mineral from which such regular solids with plane, highly polished faces, can be obtained cheaply. The effect is rendered still more conspicuous by placing ground rock salt in the bottom of the cell and the parallelepipeds above, thus projecting both upon the screen at the same time. As the cell grows warmer from the lantern the wave-lengths transmitted without reflection or refraction will shift in a very striking manner.

II. The Infrared Absorption Spectrum of Selenium.

About a year and a half ago the absorption spectrum of a film of selenium, melted between rock salt plates, was found. Commercial vitreous selenium was used. It was found that the transmission varied with the age of the film, for the first few days, hence it seemed wise to repeat the work. Since then, through the courtesy of Dr. Carveth, of the Chemistry Department, an opportunity was granted to repeat the observations on very pure selenium prepared by A. P. Saunders.[1] The results seem worth recording, since they complete the exploration of the spectrum from the ultraviolet to 14 μ in the infrared.[2]

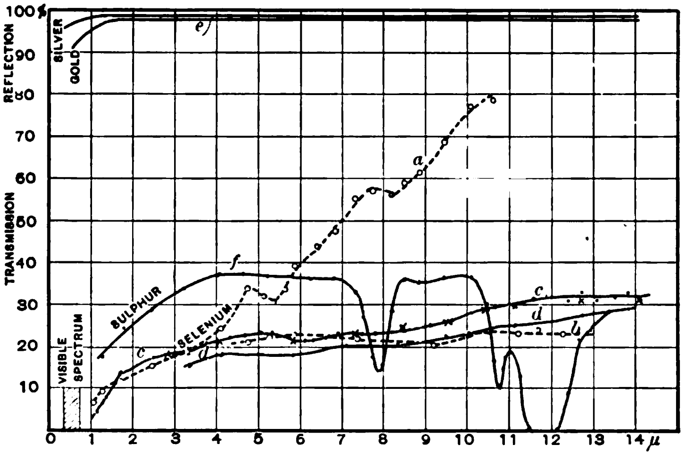

Fig. 3.

The results are shown in Fig. 3, in which curve *a* shows the transmission for the commercial material. The transparency at this time increased with increase in wave-length. Two days later the film was reëxamined, curve *b*, when a constant transmission of about 20 per cent. was found throughout the spectrum. This it continued to do, as was found on subsequent examination.

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[1] Saunders, Allotropic Forms of Selenium, Jour. Phys. Chem., Vol. 4, p. 423, 1900.

[2] A mirror spectrometer, rock salt prism, and radiometer were used. Described in the Phys. Rev., Vol. XVI., Nos. 1 and 2, 1903.

Ordinary selenium contains sulphur, and since the latter changes from one crystallographic form to another soon after it has been melted and permitted to cool, this may perhaps explain this change in transparency.

Curve *c* gives the transmission of pure selenium, examined like the preceding. The dots show the first examination just after melting while the crosses show the second one made two days later. It is to be noticed that the two curves coincide.

Curve *d* gives the transmission for this film thirty days later. It likewise is fairly constant in its transmission, being parallel but about 3 per cent. less than for the preceding dates. This, no doubt, is due to the fact that the film had to be remounted on the carrier before the spectrometer slit, while the surfaces of the rock salt plates may have changed by that amount. The results show that the pure material did not change from a varying to a constant transmission, as was found for the commercial article. The film of pure selenium was made by melting between very clear rock salt plates, heated just above the melting point. The film was homogeneous, and its color was a deep ruby-red. Thickness, 0.07 mm.; area, 8 × 14 mm.

It would have been interesting to examine the metallic modification of selenium. But no film could be made procured that was free from rills, so the examination was never made.

The absorption and refraction indices have been determined in the visible spectrum by Quincke,[1] in the visible and ultraviolet by Cornu,[2] and by Wood.[3] The ultraviolet reflecting power has been measured by Nutting.[4] He found that the reflecting power increases abruptly from a low value in the red to a maximum in the yellow, falls off slightly toward the violet and then more rapidly in the ultraviolet, which agrees with Wood's observations. Wood thinks

[1] Quincke, Wied. Ann., Jubelband, p. 336, 1874.

[2] Cornu, Compt Rend., 108, 917 and 1211, 1889.

[3] Wood, Phil. Mag. (6), 3, p. 607, 1902.

[4] Nutting, PHYS. REV., XVI., p. 129, 1903.

that the violet and ultraviolet regions have several bands with maxima beyond .22 μ. This point is of considerable interest, because of the high dielectric constant, 10.2, of selenium; for according to the electromagnetic dispersion theory, the dielectric constant of any substance is the sum of that of the ions, which by acting on the ether waves cause absorption and dispersion, plus the dielectric constant of the ether. It has been noticed that selenium is quite transparent in the infrared and has no broad absorption bands. Hence, on the above hypothesis we must attribute this high dielectric constant to the ions whose free periods correspond in frequency to the green, blue and ultraviolet rays, as surmised by Wood. He also found that, for a film of selenium which absorbs but little of the red, the violet and ultraviolet is absorbed nearly as strongly as a metal film of the same thickness. This is one peculiarity of selenium, occupying, as it does, a position midway between the metals and non-metals. Like the metals (curve *e*, Fig. 3, from Hagen and Rubens[1]) it has a high reflecting and absorbing power for the visible and ultraviolet rays. Unlike the metals but like the non-metals (except that it has no defined selective absorption bands) it is quite transparent in the infrared. Hagen and Rubens have examined a large number of metals and alloys to 14 μ and found a uniform, high reflecting power throughout the infrared region. They call attention to the enormous influence of the molecular free periods of the metals upon the optical region. The curves impress us with the complexity of the phenomenon of total (of metallic) reflection and the depth of penetration of the same, especially for a metallic film so thin that it is optically transparent in which case the penetration might be considered infinite, *i. e.*, the disturbance is never turned back but is transmitted.

Curve *f*, Fig. 3, gives the transmission of a plate of sulphur 3.6 mm. in thickness. Nothing was known about the direction of the optic axes. Since this is a problem by itself, in pleochroism, it will cent. As is well known, sulphur and selenium have an extraordinary similarity in their properties, but no such similarity is to be found in their transmission curves. Like the metals, selenium is more opaque than sulphur and its absorption is general, while unlike the non-metals it shows no selective absorption as is true of sulphur.

PHYSICAL LABORATORY, CORNELL UNIVERSITY
March 1, 1904.

SOME NEW THEOREMS ANALOGOUS TO GREEN'S.

By P. G. Nutting.

STARTING with Gauss' theorem

$$\iiint \operatorname{div}\, \omega dv = \iint \omega \cdot d\sigma = 4\pi M,$$

or with Stokes' theorem

$$\iint \operatorname{curl}\, \omega \cdot d\sigma = \int \omega \cdot d\rho,$$

it is possible, by making some simple substitutions, to arrive at a number of useful theorems analogous to these and to Green's. This investigation was originally undertaken to discover a relation similar to Green's which should be directly applicable to vector functions in the same way that Green's applies to scalar functions of position in space. Green's celebrated equation,

$$\iiint \nabla u \cdot \nabla v dv = \iint u \nabla v \cdot d\sigma - \iiint u \nabla^2 v dv,$$

is very useful in dealing with problems in gravitation, hydrodynamics and electrostatics where scalar functions are to be dealt with or where potential functions may conveniently be resorted to, but in magnetism, electromagnetism and optics it is more convenient to deal directly with vector functions and above all to have an analogous equation to which Maxwell's equations may be directly applied. In other words, Green's theorem applies best when force lines or stream lines run off to infinity or from positive to negative sources, the new theorem when these are closed in small circuits.

Along with the theorem sought, several other useful theorems were obtained and are here presented. The vector notation is employed throughout on account of the simple form which the equations take and the ease with which properties and limitations are kept in mind. The notation is essentially that of Gibbs, Heaviside, Bucherer and others. Greek letters represent vectors, Roman scalars or scalar functions of position. The Hamilton operator ∇ is the *del* of Gibbs, the *slope* of Heaviside and the *gradient* of Planck and others. The compound operator div ∇ is equivalent to the ordinary Laplacian, here written ∇^2. Vectors which have a divergence (not zero) are called for brevity *polar* vectors and are said to have a polar field. Vectors which have a curl are *axial* vectors and have a *solenoidal* field.[1]

Evidently then Gauss' equation can be of service only in polar fields, Stokes' in solenoidal fields. Gauss' equation says that the surface integral of a vector function must be zero over every closed surface in a solenoidal field while in a polar field it is not zero in general. Similarly Stokes' equation shows that the line integral of a vector function must be zero around every closed path in every polar field.

Consider the compound vectors of simple form that may be substituted for the vector ω in Gauss' or Stokes' equation. These are: $\omega = \nabla u$, curl τ, $u\tau$, $u\nabla v$, u curl τ, $\varphi \times \tau$, $\varphi \times$ curl τ, $\varphi \times \nabla u$ and τ div φ.

1. The substitution $\omega = \nabla u$ in Gauss' equation, *i. e.*, considering the vector ω to be the gradient of some scalar point function, gives

$$\int\!\!\int\!\!\int \nabla^2 u\, dv = \int\!\!\int \nabla u \cdot d\sigma, \tag{1}$$

the volume integral of the Laplacian equals the surface integral of the gradient of a scalar function, the surface being the closed surface enclosing the volume integrated over. Corresponding relations between vector functions are developed later. The same substitution in Stokes' equation yields nothing since curl ∇ is a zero operator, *i. e.*, the gradient of a scalar function, although a vector, cannot have a solenoidal field. Equation (1) has already been obtained as a particular case of Green's.

2. The substitution $\omega =$ curl τ in Gauss' equation brings the first member to zero, since the curl of a vector can have no divergence. The same substitution in Stokes' equation gives

$$\int\!\!\int \nabla^2 \tau \cdot d\sigma = \int\!\!\int \nabla \operatorname{div} \tau \cdot d\sigma - \int \operatorname{curl} \tau \cdot d\rho, \tag{2}$$

[1] On the properties of vector functions see M. Abraham, Enzykl. d. Math. Wiss., Band IV., 2, Heft 1.

since the operator curl curl = ∇ div — ∇^2. This general equation breaks up into two. In polar fields

$$(2a)\qquad \iint \nabla^2\tau \cdot d\sigma = \iint \nabla \text{ div } \tau \cdot d\sigma,$$

while in solenoidal fields

$$(2b)\qquad \iint \nabla^2\tau . d\sigma = -\int \text{curl } \tau \cdot d\rho,$$

i. e., the surface integral of the Laplacian of an axial vector equals the line integral of the curl of the vector along the curve bounding the surface integrated over. (2*a*) amounts to an identity in polar fields and does not apply elsewhere.

3. The substitution $\omega = u\tau$ in Gauss' and Stokes' equations gives two formulas of purely mathematical interest. They have already been obtained by Gibbs by integrating by parts the expressions div $(u\tau)$ and curl $(u\tau)$.

4. The substitution $\omega = u\nabla v$ transforms Gauss' equation directly into Green's

$$\text{(Green)}\qquad \begin{aligned} \iiint \nabla u \cdot \nabla v dv &= \iint u\nabla v \cdot d\sigma - \iiint u\nabla^2 v dv \\ &= \iint v\nabla u \cdot d\sigma - \iiint v\nabla^2 u dv, \end{aligned}$$

since div $(u\nabla v) = u$ div $\nabla v + \nabla u \cdot \nabla v$. Green's equation then includes Gauss' and Gauss' implies Green's whenever the vector function of Gauss' equation is the product of one scalar point function into the gradient of another, but the fact that this transformation exists does not of course imply that the two equations are not entirely independent. Evidently Gauss' equation cannot be applied to solenoidal fields since the gradient of a scaler function (such as u and v) cannot have a curl other than zero. The vectors ∇u and ∇v are in general polar, hence we may apply Green's theorem to polar fields provided the potential u or v of one of the vectors ∇u

since curl$(u\nabla v) = u$ curl$\nabla v + \nabla u \times \nabla v$ and curl$\nabla v = 0$. This equation has already been obtained by Gibbs by an integration by parts (Wilson-Gibbs, p. 199). It appears to be widely applicable to physical problems. Suppose for example, u and v are electric and magnetic potential, then ∇u and ∇v are electric and magnetic force. Each member of (4) is a maximum when these forces are normal to each other, as in electromagnetic radiation. In this case the first integrand is Poynting's Energy Flow function. The theorem states that the integral of this function over the surface of a plane of the electric and magnetic forces equals the line integral of the product of say electric potential into magnetic force around the boundary to the surface chosen. Over an equipotential surface (say $\nabla v =$ const.), equation (4) reduces to one of Föppl's

$$\int\int \nabla u \times d\sigma = -\int u\, d\rho.$$

5. The substitution $\omega = u$ curl τ in Gauss' equation gives

$$(5a) \qquad \int\int\int \nabla u \cdot \text{curl}\, \tau\, dv = \int\int u \,\text{curl}\, \tau \cdot d\sigma.$$

This relation is perhaps most useful as an energy equation. Suppose for example, u is electric potential and τ magnetic force. Then within a conductor where $4\pi i =$ curl τ, the first member becomes the volume integral of the transformed energy Ei. The equation states that this is equal to the surface integral of the product of the electric potential and electric force. With an insulator where $K\dot{E} =$ curl τ and $\dot{E} = pE$, p being a complex factor, the first term of (5a) reduces to the volume integral of E^2. This equals the same surface integral as in the case of a conductor. In a uniform τ field, (5a) reduces to another equation of Föppl's

$$\int\int\int \nabla u\, dv = \int\int u\, d\sigma.$$

In Stokes' equation the same substitution $\omega = u$ curl τ gives

$$(5b) \qquad \int\int u\nabla\, \text{div}\, \tau \cdot d\sigma + \int\int \nabla u \times \text{curl}\, \tau \cdot d\sigma$$
$$= \int u\, \text{curl}\, \tau \cdot d\rho + \int\int u \nabla^2 \tau \cdot d\sigma.$$

which reduces to ($2a$) when $u =$ const. In solenoidal fields

$$(5c) \qquad \iint \nabla u \times \operatorname{curl} \tau \cdot d\sigma = \int u \operatorname{curl} \tau \cdot d\rho + \iint u \nabla^2 \tau \cdot d\sigma,$$

an equation complementary to ($5a$). On a surface over which the scalar function u is constant, ($5c$) gives the important relation ($2b$). This part of ($5c$) being independent of u is of course like ($2a$), a general equation in the vector function τ.

6. The substitution $\omega = \varphi \times \tau$ in Gauss' equation gives a convenient transformation formula

$$(6) \qquad \iiint \tau \cdot \operatorname{curl} \varphi dv - \iiint \varphi \cdot \operatorname{curl} \tau dv = \iint \varphi \times \tau \cdot d\sigma,$$

apparently of little direct importance in physics. The same substitution in Stokes' equation gives a long and complicated expression apparently of little value.

7. The substitution $\omega = \varphi \times \operatorname{curl} \tau$ in Gauss' equation gives the theorem complementary to Green's originally sought, namely

$$\begin{aligned} &\iiint \operatorname{curl} \varphi \cdot \operatorname{curl} \tau dv \\ (7) \qquad &= \iint \varphi \times \operatorname{curl} \tau \cdot d\sigma + \iiint \varphi \cdot \nabla \operatorname{div} \tau dv + \iiint \varphi \cdot \nabla^2 \tau dv \\ &= \iint \tau \times \operatorname{curl} \varphi \cdot d\sigma + \iiint \tau \cdot \nabla \operatorname{div} \varphi dv + \iiint \tau \cdot \nabla^2 \tau dv. \end{aligned}$$

In most problems $\operatorname{div} \tau$ and $\operatorname{div} \varphi$ will be zero and (7) takes a form very similar to Green's. This will be the case in all electromagnetic problems for $\operatorname{div} E = 0$ as well as $\operatorname{div} H$ where E is electric and H magnetic field.

Maxwell's equations $4\pi i + K\dot{E} = \operatorname{curl} H$ and $\mu \dot{H} = - \operatorname{curl} E$ may be applied directly to (7) in many different ways. For instance, we may substitute so as to eliminate the space operators or H or E separately and thus obtain relations between the time operators or functions of E or H alone.

a relation already obtained by Gibbs. It appears to be a limited form of (6) in which the vector τ has been replaced by the gradient of a scalar ∇u.

9. The substitution $\omega = \tau \operatorname{div} \varphi$ in Gauss' equation gives another important relation similar to Green's and complementary to (7), namely

$$(9)\quad \iiint \operatorname{div} \varphi \operatorname{div} \tau dv = \iint \tau \operatorname{div} \varphi \cdot d\sigma - \iiint \tau \cdot \nabla \operatorname{div} \varphi dv$$
$$= \iint \varphi \operatorname{div} \tau \cdot d\sigma - \iiint \varphi \cdot \nabla \operatorname{div} \tau dv$$

This equation applies to polar fields in the same way that (7) applies to solenoidal fields. It should prove very useful in the treatment of problems in gravitation and fluid motion. Within a region in which one of the vector functions, say φ, has no divergence, the relation

$$(9a)\qquad \iiint \nabla \operatorname{div} \tau dv = \iint \operatorname{div} \tau \cdot d\sigma$$

holds for the other.

The same substitution $\omega = \tau \operatorname{div} \varphi$ in Stokes' equation gives

$$(9b)\quad \iint \operatorname{div} \varphi \operatorname{curl} \tau \cdot d\sigma = \int \tau \operatorname{div} \varphi \cdot d\rho + \iint \tau \times \nabla \operatorname{div} \varphi \cdot d\sigma,$$

applying to fields which are polar for one vector function and solenoidal for another.

To complete the set of Green equations to cover every possible kind of field would require one more relation to be derived from Stokes' containing the vector product of the curls of vector functions. This equation I have not yet been able to obtain. The substitution $\omega = \operatorname{curl} \varphi \times \operatorname{curl} \tau$ in Gauss' equation gives the relation

$$(10a)\qquad \iint \operatorname{curl} \varphi \times \operatorname{curl} \tau \cdot d\sigma = 0$$

over every closed surface not including a source or sink within its boundary. The same substitution in Stokes' equation gives

$$(10b)\quad \int \operatorname{curl} \tau \times \operatorname{curl} \varphi \cdot d\rho = \iint (\operatorname{curl} \tau \cdot \nabla)\varphi \cdot d\sigma -$$

Another relation of this type that promises great usefulness in electromagnetic problems may be obtained from (7) by substituting curl φ for φ namely:

$$\iint \text{curl}\,\varphi \times \text{curl}\,\tau \cdot d\sigma$$

$$(10c)\qquad = \iiint \text{curl}\,\varphi \cdot \nabla\,\text{div}\,\tau dv - \iiint \text{curl}\,\tau \cdot \nabla\,\text{div}\,\varphi dv + \iiint \text{curl}\,\varphi \cdot \nabla^2 \tau dv + \iiint \text{curl}\,\tau \cdot \nabla^2 \varphi dv = 0.$$

If the vector functions φ and τ represent electric and magnetic force, we find by substituting Maxwell's relations that (10*a*) states that the integral of Poynting's energy flow function over a closed surface is zero unless the surface encloses a source or sink when it is 4π times the strength of the source or sink. This is true of conductors as well as insulators. Equation (10*b*) shows that the integral of Poynting's function is zero around every closed path in the field in which Maxwell's relations hold, for each term of the second member reduces to zero when the electric and magnetic forces are the form $\dot{E} = (A + Bi)E$, where A and B are functions of only space coördinates. Equation (10*c*) reduces in an electromagnetic field to

$$\iiint \text{curl}\,E \cdot \nabla^2 H dv + \iiint \text{curl}\,H \cdot \nabla^2 E = 0,$$

since E and H have no divergence. Eliminating space operators in this by means of the Maxwell relations, we get for insulators the relation

$$q^3 \iiint \mu H^2 dv = p^3 \iiint K E^2 dv,$$

assuming H and E to be of the form $E = E_0 e^{ipt}$, p being a complex function of space coördinates, say $p = A + Bi$ and similarly for H, $q = C + Di$. This equation then gives the ratio of the magnetic field energy to the electric as $p^3 : q^3$ or $A^3 : B^3 = C^3 : D^3$, which are of course in terms of wave-lengths and damping factors. Similarly for conductors we obtain for the ratio of the field energies $ip : 2q^2$, or $A : 4CD = B : 2(C^2 - D^2)$.

(4) (Gibbs) $\iint \nabla u \times \nabla v \cdot d\sigma = -\int u \nabla v \cdot d\rho$

(9) $\iiint \text{div}\, \varphi \, \text{div}\, \tau dv = \iint \tau \, \text{div}\, \varphi \cdot d\sigma - \iiint \tau \cdot \nabla \, \text{div}\, \varphi dv$

(7) $\iiint \text{curl}\, \varphi \cdot \text{curl}\, \tau dv = \iint \varphi \times \text{curl}\, \tau \cdot d\sigma$

$$+ \iiint \varphi \cdot \nabla \, \text{div}\, \tau dv + \iiint \varphi \cdot \nabla^2 \tau dv$$

(10*a*) $\iint \text{curl}\, \varphi \times \text{curl}\, \tau \cdot d\sigma = 0$

(10*b*) $\int \text{curl}\, \varphi \times \text{curl}\, \tau \cdot d\rho = \iint (\text{curl}\, \varphi \cdot \nabla)\, \tau \cdot d\sigma$

$$- \iint (\text{curl}\, \tau \cdot \nabla)\, \varphi \cdot d\sigma.$$

The left-hand members of the above being symmetrical in the arguments, the arguments may be interchanged in the right-hand members, care being taken to interchange the signs at the same time in equation (10*b*). The remaining Green equations are not symmetrical.

(5*a*) $\iiint \nabla u \cdot \text{curl}\, \tau \, dv = \iint u \, \text{curl}\, \tau \cdot d\sigma = \iint \tau \times \nabla u \cdot d\sigma$

(5*c*) $\iint \nabla u \times \text{curl}\, \tau d\sigma = \int u \, \text{curl}\, \tau \cdot d\rho + \iint u \nabla^2 \tau \cdot d\sigma$

(9*b*) $\iint \text{div}\, \varphi \, \text{curl}\, \tau d\sigma = \int \tau \, \text{div}\, \varphi \cdot d\rho + \iint \tau \times \nabla \, \text{div}\, \varphi \cdot d\sigma.$

To complete the list would require three more unsymmetrical equations, namely those whose left member integrands are $\nabla u \, \text{div}\, \tau$, $\nabla u \cdot \nabla \, \text{div}\, \tau$ and $\nabla u \times \nabla \, \text{div}\, \tau$, but these may be obtained at once from (4) and Green's equation by replacing v by div τ.

Any of the above equations may be easily rendered into ordinary cartesian notation, but they are so long and unwieldly in this form that they cannot be rewritten here. The substitution to be made are given below. Let ω be any vector, then it will be of the form

$$\omega \equiv iX + jY + kZ, \quad \nabla \equiv i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z},$$

$$\nabla \cdot \omega \equiv \text{div}\, \omega \equiv \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z},$$

$$\nabla \times \omega \equiv \text{curl}\, \omega \equiv \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ X & Y & Z \end{vmatrix}$$

$$\begin{array}{c|ccc} \cdot & i & j & k \\ \hline i & 1 & 0 & 0 \\ j & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{array} \qquad \begin{array}{c|ccc} \times & i & j & k \\ \hline i & 0 & k & -j \\ j & -k & 0 & i \\ k & j & -i & 0 \end{array}$$

$$d\rho \equiv idx + jdy + kdz, \quad d\sigma = idydz + jdzdx + kdxdy, \quad dv = dxdydz$$

the dot and cross multiplication tables indicating the scalar and vector products of the unit vectors so difficult to keep in mind.

In conclusion it may be noted that the integrals discussed in this paper remain invariant with a transformation of coördinates. Many very useful theorems relating to this class of invariants are given by Ricci and Levi-Civita, Math. Ann., 54, 128–201, 1901, with several applications to physical and mechanical problems.

Washington, D. C, 1904.

THE ELASTIC MODULUS AND ELASTIC LIMIT OF RUBBER AND THEIR RELATION TO CHANGE OF TEMPERATURE.

By J. C. Shedd and R. L. Ingersol.

THE fact that a loaded rubber tube will shorten when steam is passed through it has long been a striking experiment; it has also proven a somewhat puzzling one.

The first explanation suggested was that of a negative coefficient of expansion. Direct experiment upon short solid pieces of rubber have failed to support this view. Under all conditions the volumetric coefficient has been found to be positive.[1] A. E. Lundal[2] found the cubical coefficient to be positive and very large; also to increase rapidly with rise of temperature. He also confirmed the previous conclusion of W. Röntgen that the volumetric change due to stretching is very small. Further he found that the cubical coefficient of stretched rubber is identical with that of rubber not under stress. The linear coefficient of expansion as determined in the experiments above noted was found to be positive for rubber not under stress, but for rubber under stress seemed to be abnormal, now positive, now negative, and always dependent upon the stress. Lundel concludes: (I.) The temperature coefficient of linear expansion is positive for small values of the stretching force, but decreases and becomes negative as this force increases. (II.) That this coefficient increases rapidly with rise of temperature, is negative for low values of stretching force but positive for higher values. There is therefore an inversion temperature where the coefficient is zero, also this inversion temperature is higher as the stretching force increases. A review of these experiments leads one to conclude that the

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[1] P. Joule, J. Pierre and P. Lebedeff; A. Winkelmann, Handbuch d. Physik, 2, p. 66, 1896.

[2] Wiedemann Annalen, Dec., 1898, p. 741.

A second explanation was advanced by G. Schmulervitsch.[1] He suggested that the linear coefficient like the cubical is always positive, but that a rise of temperature increases the elastic modulus. G. R. Dallander went further and gave an equation for the relation of the elastic modulus and the temperature.[2] Lundal[3] confirmed the equation of Dallander, finding an increase of about 11 per cent. between 0° C. and 60° C. He also found that the increase is smaller when the stretching force is increased. In arriving at this conclusion Lundal introduces no correction for the temperature dilatation of the rubber, and he significantly remarks in this connection that "it is strictly speaking uncertain whether the elastic modulus increases or decreases with rises of temperature." In the experiments for determining the elastic modulus a peculiar source of error is present which Lundal calls the "elastic-after-effect" (Nachwirkung). This seems to be somewhat of the nature of viscosity and is found to diminish as the temperature rises. According to Lundal this diminution goes on up to 50° C. and then increases. He does not, however, give much weight to his determination above 60° C.

The purpose of the present experiments was to extend the stretching force over a much greater range than heretofore and to study the elastic-after-effect. The first apparatus used was a simple hook support for one end of a common rubber band, with a weight-pan mounted at the other end. For load increments stamped rifle bullets were used which were found to be remarkably uniform in mass and of approximately 10 grams mass each. The elongation was read with a cathetometer. A large number of readings were taken upon a single rubber band at room temperature; from 10 to 100 grams being used as load increments. Also various loads were placed on the *weight pan* and left for a number of hours or days and the readings then taken for both rising and falling loads. The following conclusions were reached:

1. The same rising curves are traced whether the interval be one half minute, one minute or two minutes. The falling curves approximately coincide.

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[1] Pogg. Ann., 144, p. 280, 1871.

[2] Ofvers af. Vet. AK. Forh., 28, p. 703, 1871.

[3] Loc. cit.

behavior of rubber cannot be adequately explained on the supposition of a negative cubical or linear coefficient of expansion.

2. The return curve is never the same as the rising curve even for a small load range.

3. Hence the "elastic-after-effect" or viscosity factor cannot be eliminated by merely making an extra time allowance while taking the readings.

4. If the load exceed a certain limit the same cycle cannot be repeated, *i. e.*, the rubber seems to receive a permanent set.

In order to vary the temperature the following simple scheme was used: A stream of water from the faucet *F* (Fig. 1) was passed through a glass tube bent back and forth and under which was placed a gas flame. The water after being heated was passed through the chamber *c* which surrounded the inner chamber *d*. By controlling the volume of water the temperature of the chamber *d* could be regulated quite closely. The range of temperature available was from 11° C. to 93° C. (the boiling point at this altitude being 94° C.) A large number of curves were obtained at different temperatures and with different load limits; also different load increments were used as well as initial loads of various values, which were left on the weight pan for longer or shorter periods. In all of this work the conclusions already given were confirmed to the fullest extent.

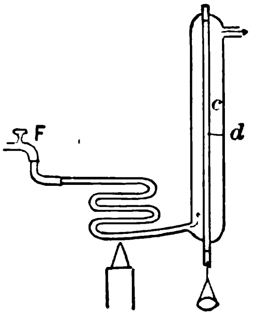

Fig. 1.

It was next thought worth while to try to eliminate the *per saltem* feature of the method of loading and unloading. This was done by mounting a Mariotte flask in such a way as to give a constant flow of water into a beaker which replaced the weight pan. A steady discharge rate was secured by mounting a siphon on a cork which floated within the beaker. Readings were taken at minute intervals both rising and falling curves being secured. The results thus obtained showed that the curves gotten by the two methods do not materially differ. The easier method of fixed load increments was therefore again adopted.

The Elastic Modulus.

It was now apparent that the load limit above which a given cycle could not be repeated with a given specimen is very small. Data therefore obtained from the same specimen at different temperatures and for different load limits would not be strictly comparable and hence of no great value. It was also apparent that it would be practicably impossible to obtain a number of single rubbers exactly alike. A number of rubbers were therefore tested by means of a small load and their relative cross-sections determined.

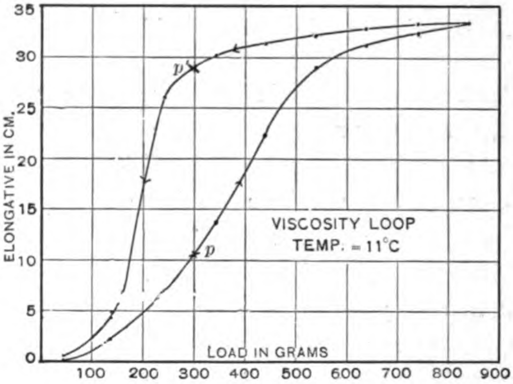

Fig. 2.

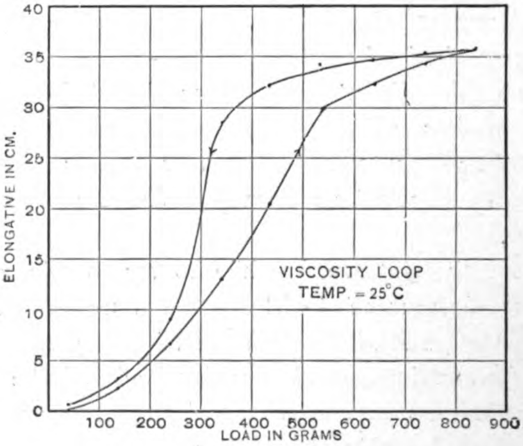

Fig. 3.

Combinations of rubbers were then made which were regarded as of the same cross-section. Care was also taken that the load limit should be low enough so that the rising and falling curves should be as nearly as possible parallel.

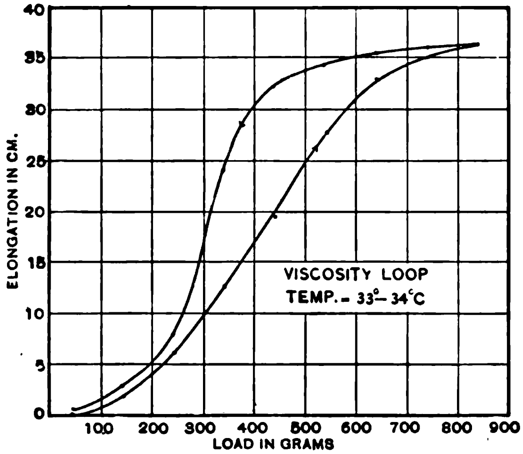

Fig. 4.

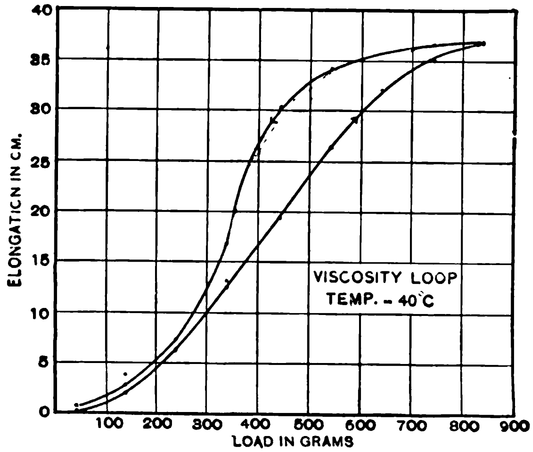

Fig. 5.

To the conclusions already noted may now be added those stated below.

6. For a given load the value of E seems to increase slightly with increase of temperature.

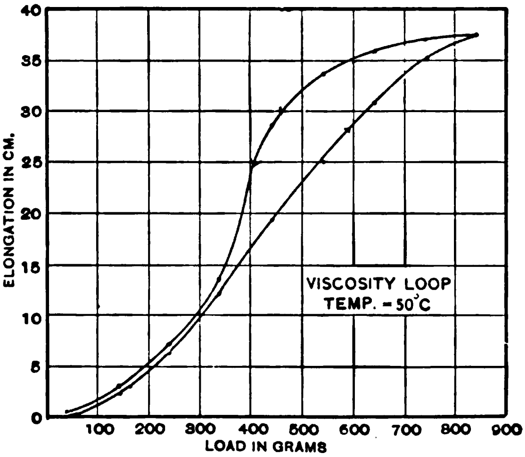

Fig. 6.

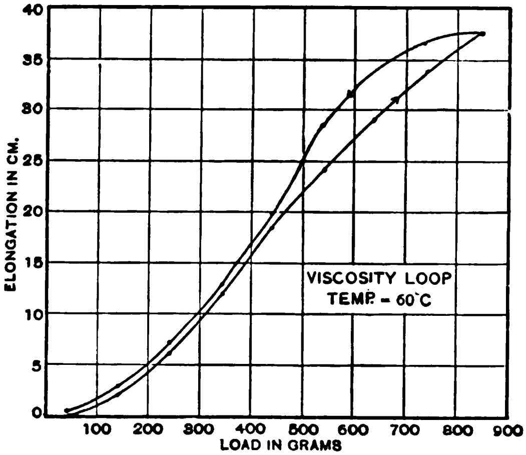

Fig. 7.

7. For a given temperature the value of E, beginning with zero load, slowly diminishes, then remains constant for a considerable range and finally sharply increases in value. The first stage would seem to correspond to initial viscosity or static friction and the last to a permanent set. In the falling load the third stage is prolonged at the expense of the second.

8. In case the third stage is not entered upon the return curve is more nearly parallel to the rising curve.

Energy Absorbed Due to Viscosity. — The fact that rising and falling curves never coincide must be due to the irreversible action of viscosity. It is also apparent that the area of the curve, like that of the hysteresis loop, is a function of the energy absorbed. If then we compare the loops having the same range of load but taken at different temperatures we shall be able to compare the viscosities at these temperatures. Two series of such comparative areas are given in the following table :

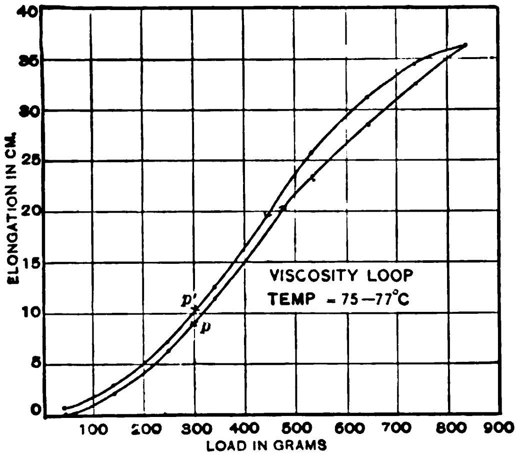

Fig. 8.

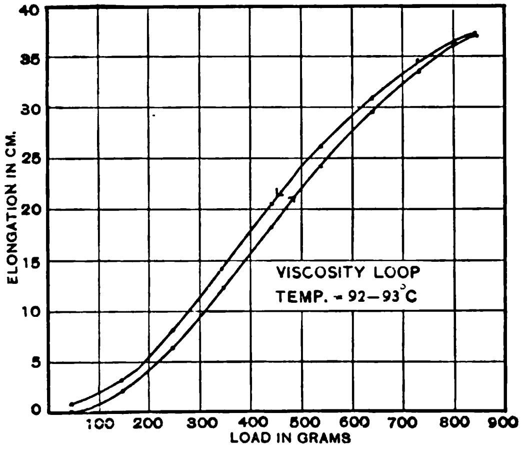

Fig. 9.

Series I, Load limit 840 grams.

Temp. C.	Energy Absorbed per Cycle (comparative).	Temp. C.	Energy Absorbed per Cycle (comparative).
11°	662	50°	338
25°	475	60°	179
34°	449	76°	129
40°	377	92°	145

Series II, Load limit 1080 grams.

Temp.	Energy Absorbed per Cycle.
18°	842
29°	535
36°	488
85°	152

The plat of these tables (Fig. 9) would lead to the conclusion that:

9. The energy dissipated per cycle due to viscosity depends upon the load limit, falls rapidly with rise of temperature, and becomes asymptotic to a fixed value.

ELASTIC LIMIT.

A study of the viscosity loops would seems to make two definitions of the elastic limit possible. In the first place on the rising curve there is seen to be a more or less well defined "knee" where the relation of stress to strain undergoes a marked change. This "knee" might be said to mark the elastic limit. The elastic limit so defined is found to rise as the temperature rises.

In the second place it is probable that a more comprehensive definition would be had by considering the descending curve also. The elastic limit would then be defined as that point above which the ascending and descending curves are no longer parallel. This point lies much lower down on the curve than does the first mentioned point, and it also rises as the temperature rises and at a somewhat faster rate. This second definition of the elastic limit could be given as follows: *The elastic limit is the limiting value of the stress (or strain) for which the body upon being released will repeat its previous history in exactly reversed order.*

tures, falls rapidly as the temperatures rises, and reaches an approximately constant value at about 80° C.

11. The elastic limit of rubber rises with rise of temperature.

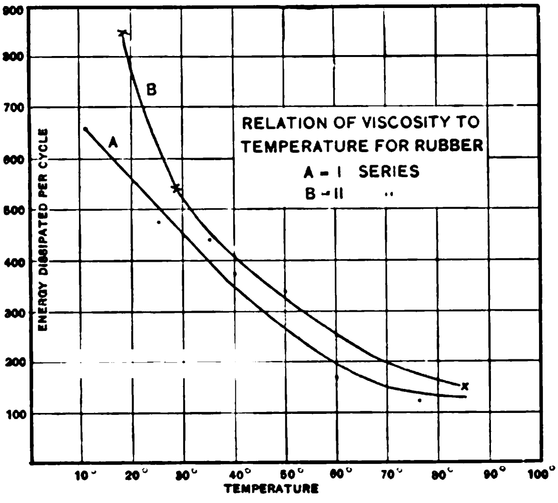

Fig. 10.

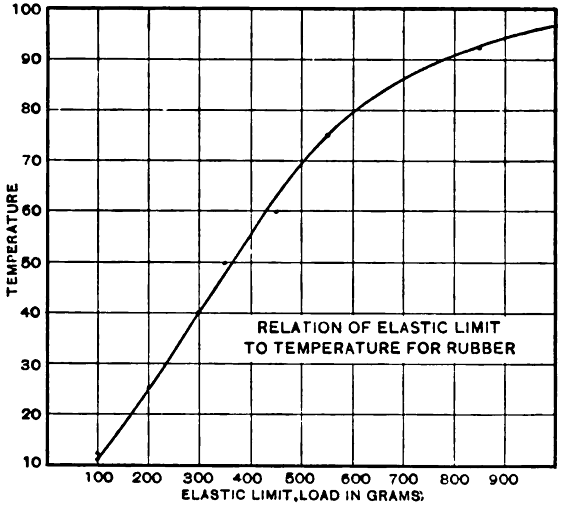

Fig. 11.

The fact that the viscosity loop approached a constant area would indicate that the rise of the elastic limit is indefinite, limited only by the change of state of the rubber from a viscous solid to a viscous fluid (see plat 10).

A consideration of the viscosity loop affords a partial explanation of the behavior of the rubber tube spoken of at the beginning of this paper. Let us suppose that the temperature is 11° C., and that the load is 300 grams as shown on Fig. 2. On the rising curve this would be represented by the point p. However, if by chance, the rubber has been so handled as to have passed around the cycle then the point p' might more nearly represent the state of the rubber. Now if the temperature be raised to 75° C., points p and p' on Fig. 8 would approximately represent the state of the rubber. In the latter case the contraction would be 17 cm. as against 1.75 cm. in the former case, and would be due, not so much to an increase in the elastic modulus, as to a rise in the elastic limit.

SUMMARY.

We may conclude by dividing the history of a cycle into the following parts:

A. The first part of the curve has a small slope showing an apparently high value for E. This in a measure corresponds to the early stage of the magnetization curve of iron.

B. The second stage shows a comparatively constant but smaller value of E.

C. In the third stage the curve bends sharply showing a very high value for E. During this stage the rubber is in an abnormal state behaving very much like a metallic wire. This stage ends abruptly by the breaking of the rubber, or else the rubber receives a permanent set as is shown by the descending curve. As the temperature rises stage B is extended in both directions D. The form of the descending curve will depend upon whether or not stage "C" has been reached.

In conclusion it is apparent that all experimental work upon rubber should be conducted upon that part of stage B which lies below the elastic limit as defined in the second instance above. Also that the previous history of the specimen under test should be known.

PHYSICAL LABORATORY, COLORADO COLLEGE,
April, 1904.

THE ELECTRIC ARC.

By C. D. Child.

THE following article consists of measurements of the electric arc made at pressures less than one atmosphere and of a discussion of some explanations which have been suggested. The drop in potential at the anode and that at the cathode, the electric force through the arc, and the rate of discharge from the arc to a surrounding cylinder were studied.

Apparatus.—There were three difficulties to be overcome in making a tube for the investigation; first, one of the carbons and also the carbon pencil or the wire used to test the potential of the arc must be movable; second, when the current was led into the tube through a platinum wire sealed into the glass, the glass cracked with large currents; third, it was necessary to have the apparatus so that it could be easily taken apart. These difficulties were met with fair success with the apparatus shown in Fig. 1. This consists of a tube having three openings which were closed with rubber stoppers. It was found that with pressures of over 2 mm. of mercury rubber stoppers fitted in securely could be used to advantage. With less pressure they

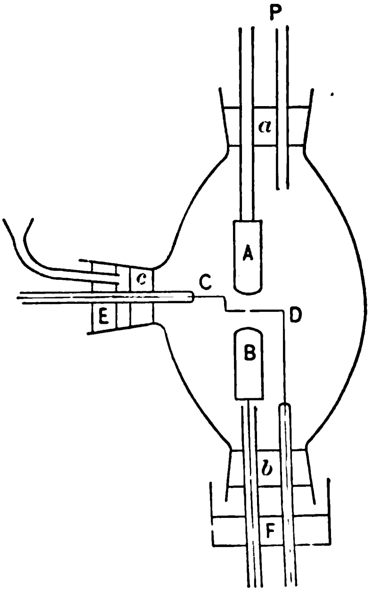

Fig. 1.

will be given later no attempt was made to reach very low pressures. The rubber was indeed liable to soften because of the heat, but by using a large tube and by running the arc only a little while at a time the rubber did not become excessively heated.

The tube used the greater part of the time was 24 cm. in length and 17 cm. in diameter at the middle. An iron rod passing through the stopper *a* brought the current to the carbon *A*. The movable carbon *B* was fastened to a second iron rod. This was placed inside a glass tube about 80 cm. long, of which only the top is shown in the diagram. The bottom of the tube was immersed in mercury. The bottom of the rod was bent up and passed out of the mercury, so that it could be used as a handle with which to move *B*. *P* is a glass tube making connections with the air pump.

For testing the potential of the arc the pencil at *D* was used. This could be raised or lowered the same as *B*. By rotating the rod the pencil could be placed in the arc or entirely removed from it. In some of the work the pencil *C* was used. This was fastened to a plantinum wire sealed into the glass rod *CE*. This wire was so bent that the pencil *C* could be moved in and out of the arc. This, however, was not used except in one or two special cases.

When a high vacuum was desired *a* was covered with shellac and the spaces between *c* and *E* and *b* and *F* were filled with the same. Some of the vapor from the shellac passed into the tube when the air was first pumped out, but this soon ceased and slight impurities from this source and also from the rubber are of little consequence as compared with unknown vapors which are continually given off from the carbon.

A cathetometer was used to determine the distance between the carbons and also the position of the exploring pencil. The pressure was measured by a manometer which could be read to millimeters at all points and could be estimated to fifths of millimeter with the lower pressures.

In studying the changes in the drop at the anode a carbon pencil 1.6 mm. in diameter was placed as near to the anode as possible and the air was pumped from the tube, the current through the arc being kept constant by changing the resistance in series with the arc. Solid carbons 12 mm. in diameter were used.

Use of Voltmeter Allowable. — The potential difference between the electrodes and the exploring carbon was read by a Weston voltmeter of 5,000 ohms resistance. An electrometer is theoretically better, but it is not so convenient and cannot be read so quickly. That it was allowable to use a voltmeter was shown by connecting an electrometer between one of the electrodes and the carbon pencil placed in the arc, and also connecting the voltmeter in multiple with it. A key for opening and closing the circuit was placed in series with the voltmeter. It was found that both at atmospheric pressure and at one of 2 mm. the reading of the electrometer did not change more than one-fourth volt when the circuit through the voltmeter was completed.

The use of the voltmeter was also tested by switching a known E.M.F. into the circuit. The reading was changed by the amount of the E.M.F. Thus in one case when measuring the drop in potential at the anode the reading was 44 volts. On switching four volts from the battery into the circuit the reading was 48 volts. On reversing the battery the reading was 40 volts. A similar test was made when measuring the drop at the cathode with the same result.

This behavior of the current flowing between the carbon pencils and the electrodes is apparently not in agreement with the description of Fleming's work given by Thomson.[1] I have not at hand Fleming's account of the work and can not give the cause of the apparent discrepancy.

The Exploring Pencil may become the Electrode. — It may be noticed in passing that when too much current is taken from the exploring carbon it no longer gives the potential of the gas about it. Thus when a shunt of 100 ohms is placed about the voltmeter, the reading was materially changed, especially if the pencil was not white hot. The amount of the change was affected both by the amount of resistance in shunt with the voltmeter and by the temperature of the pencil.

When a variable resistance was placed between the anode and the pencil, the current through it continually increased as the resistance was made smaller until about 2 amperes was reached. On further decreasing the resistance, the main current passed through the pencil making this the anode. When the same pencil was connected with the cathode, it was found that the pencil had to be much hotter in order to cause the cathode end of the arc to jump to it than it had previously been to cause the anode end to do so.

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[1] Conduction of Electricity through Gases, p. 423.

At a pressure of 2 mm. it was found impossible to cause the cathode to jump to the pencil in any case. At this pressure the gas was not hot enough to give the exploring carbon the necessary temperature. On the other hand the pencil could easily be made the anode at this pressure. In fact a large fraction of the current would come from the exploring pencil, even when it was at a temperature lower than red heat. The same glow surrounds the pencil as the anode proper, a glow similar to that seen at the anode of a mercury arc. This shows conclusively that the anode does not require as high a temperature as the cathode. Weintraube[1] has observed a similar phenomenon with a mercury arc. He found it impossible to cause a current to flow from the anode of an arc already formed to a new cathode, while it was simple to start a current from a new anode to the cathode of an existing arc.

With pressures less than 3 mm. a platinum wire was used instead of a carbon pencil, since it produced less disturbance in the arc and could be used at that pressure without melting.

Table I.

Pressure.	Drop at Anode.	Drop at Cathode.	Pressure.	Drop at Anode.	Drop at Cathode.
730	50	14.1	30	30	16
500	45	14	20	28	13.4
300	43	14.1	10	26.8	10
200	43.5	15.6	5	25.5	8
100	36.5	17.2	2	22	8
50	32	16.5			

Changes at Anode and at Cathode. — Data are given in Table I. showing the effect of changing the pressure. Column 1 gives the pressure in millimeters of mercury, column 2 the drop at the anode in volts, column 3 that at the cathode. The current used when taking this data was 10 amperes. The distance between the electrodes was 7 mm. The distance of the middle of the exploring pencil from the electrode was approximately 1.5 mm. The current came from a dynamo of 110 volts.

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[1] Phil. Mag. (6), 7, p. 96.

The measurements below 10 mm. are not at all accurate. This was especially the case with the cathode drop. It was continually fluctuating, being at times much greater than the value given in the table and again somewhat less. In general it would slowly increase and then suddenly decrease. The anode showed the same behavior, but not to the same extent. For this reason no attempt was made to secure measurements lower than those given in the table.

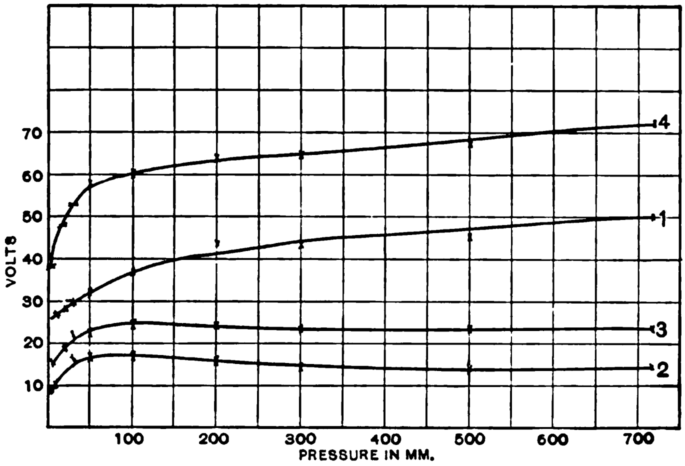

Fig. 2.

These data are plotted in Fig. 2. Curve 1 represents the drop at the anode and 2 that at the cathode.

The surprising fact here shown is that the cathode drop should for a time increase when the pressure is decreased. But besides taking several sets of observations the drop at 730 mm. was compared many times with that at 100 mm. At times the reading at 730 mm. was taken first, at others that at 100 mm., and in every case the latter was found to be the greater. This was true whether the cathode was above or below the arc.

On the other hand the drop at the anode decreases gradually as the pressure is decreased. This difference in behavior between the anode and the cathode may be compared with the difference found by Mrs. Ayrton when varying the length of the arc.[1] She found that when the length of the arc was increased the anode drop invariably increased, while there was little, if any, change in that at the cathode. Evidently the conditions controlling the drop at the cathode are quite different from those at the anode.

Electric Force Through the Arc. — In finding the electric force through the arc the tube running through *Cc* was divided into two branches as in Fig. 3, and platinum wire was sealed into each branch and a carbon pencil fastened to it. The two wires were insulated from each other at all points. By distorting the cork at *c* the pencils could be kept out of the way of the carbon until the arc had been formed and then allowed to swing back into the arc. When the second cork was inserted at *E* the pencils could not be thus displaced and the last two data given in Table II. were taken by using a platinum wire and observing the potential at different points through the arc.

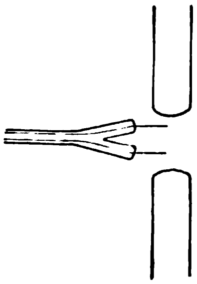

Fig. 3.

In using the two carbon pencils the distance between them was not altered during any set of observations, so that changes in the readings could not be due to errors in the measurement of this distance. In making observations the pencils were always allowed to become white hot before readings were taken.

Table II.

Pressure.	Drop Through Gas.	Electric Force.	Pressure.	Drop Through Gas.	Electric Force.
730	10.5	23.4	30	10	22.4
500	10.5	23.4	20	8.5	18.9
300	10	22.4	10	7	15.5
200	11	24.5	5	7	15.5
100	11	24.5	2	6	13.3
50	10	22.4			

Several sets of observations were taken. Those given in Table

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[1] The Electric Arc, pp. 222 and 224.

II. were with the pencils 4.5 mm. apart, the current being 10 amperes, and the distance between the electrodes 9 mm. The pressures are given in column 1, the observed potential difference in volts in column 2, and the electric force in volts per cm. as computed from this in column 3.

The variation in the data down to the last four is, no doubt, due to errors in observations, but between 40 and 2 mm. there is a decrease in the electric force. These data are plotted in Fig. 2, Curve 3.

Voltage Between Carbons.—In Table III. the whole potential difference between the carbons is given in column 2 and the corresponding values computed from the proceeding data in column 3. The current was again 10 amperes and distance between the carbons 7 mm.

TABLE III.

Pressure.	Observed Potential Between Carbons.	Computed Potential Between Carbons.	Pressure.	Observed Potential Between Carbons.	Computed Potential Between Carbons.
730	72	73	30	53	52
500	67	68	20	48	48
300	65	66	10	44	42
200	64	68	5	38	36.5
100	60	62	2	35	33
50	58	57			

The observed potential differences are plotted in Fig. 2, Curve 4.

Temperature of the Arc.—In taking these data it was observed that platinum wire did not melt when inserted in the arc at low pressure. It, therefore, seemed possible to get some idea of the changes in the temperature of the arc. It was found that a carbon filament from an incandescent lamp was consumed in the arc until a pressure of about 5 cm. was reached. At lower pressures it gathered a deposit of carbon. A platinum wire .31 mm. in diameter was melted until a pressure of about 4 mm. was reached. At lower pressures this also gathered a coating of lamp black. But a platinum wire .11 mm. in diameter was melted at 2 mm. pressure and one which was said by the makers to be .01 mm. was melted at approximately .5 mm. of pressure, which was the lowest pressure used. It is thus apparent that the temperature of the arc is very high even at low pressures and also that any determinations of the arc made at low pressures by inserting substances to be melted are of little value. In the immediate neighborhood of the bright spot of the cathode the largest size of platinum wire was melted at the lowest pressures attained.

Discharge to Surrounding Cylinder. — A series of observations was also taken on the rate of discharge from the arc to a surrounding cylinder. It has been shown[1] in a previous article that such discharge at atmospheric pressure was greater when the cylinder was charged negatively than when charged positively. In the former case the discharge is carried by the positive ions coming from the arc and we, therefore, call this the positive discharge. The observations there indicated that the positive ions moved the more rapidly.

A similar phenomenon was observed when a hot carbon pencil or platinum wire was used instead of the arc.[2] In those cases it was shown that the smaller velocities of the negative was due to their becoming loaded with particles given off from the hot substance. It was also shown with hot carbon and platinum that the negative discharge became the greater at low pressures.

With the arc the phenomena were in all essentials the same. The data here given are, however, only approximations. The rate of discharge from the arc depends on the length of the arc, and apparently on the amount of the carbons that has become hot and on the condition of the gas about the arc. The longer the arc has been running the greater the number of particles held in suspension ready to collect on the ions. Yet it was not possible to run the arc in the apparatus for a sufficient length of time to reach a steady condition without overheating. At lower pressures and with the cylinder used, the positive end of the arc would jump to the cylinder, if this was kept at 50 volts or more above the potential of the arc, so that no data were taken below a pressure of 10 mm. of mercury.

However, the essential changes were shown and it was not thought that the phenomena were of sufficient importance to justify the great amount of work necessary to secure accurate results.

[1] PHYS. REV., XIV., p. 65.

[2] PHYS. REV., XIV., p. 265; XV., p. 345.

In making measurements a shunt was placed about the arc and the cylinder was connected with such a point on the shunt that no current passed between the arc and the cylinder. This was done when the pressure of the gas about the arc was 50 mm. This point on the shunt was afterwards assumed to have the same potential as the arc.

In taking the data given in Table IV. the cylinder was made of sheet iron which was 3.6 cm. in diameter and 2.3 cm. in height. The potential difference between the arc and the cylinder was 54 volts, as measured by a Weston voltmeter. Column 1 gives the pressure of the gas in millimeters of mercury, column 2 the rate of discharge in amperes when the cylinder was charged negatively, column 3 that when charged positively.

TABLE IV.

Pressure of Gas.	Positive Discharge.	Negative Discharge.	Pressure of Gas.	Positive Discharge.	Negative Discharge.
730	1×10^{-7}	$.7 \times 10^{-7}$	100	1.8×10^{-6}	3×10^{-4}
600	2.1×10^{-7}	$.8 \times 10^{-7}$	40	3.6×10^{-6}	1.8×10^{-2}
500	2.5×10^{-7}	1.5×10^{-7}	30	6.6×10^{-6}	5×10^{-2}
400	2×10^{-7}	5×10^{-7}	20	5.4×10^{-5}	1.6×10^{-1}
300	3.6×10^{-7}	4.5×10^{-7}	10	$.1 \times 10^{-4}$	1.6×10^{-1}
200	7×10^{-7}	7×10^{-5}			

As has been stated the data here given are only approximations. But they are sufficiently accurate to show first, that the positive discharge is the greater at atmospheric pressure and the negative at lower; second, that the rate of discharge at the lower pressure is very much greater than at the higher. The behavior with the discharge from the arc is thus much the same as that from hot platinum or carbon. The observations at low pressure do not, however, confirm the idea which I once held that there was some connection between the velocity of the positive ions about the arc and the greater fall of potential at the anode.

Theory of the Arc. — Let us now consider the bearing which these data have on the theory of the arc. There have been three explanations offered which are based on the conception that the current is carried by ions. One was given in 1900 by the present writer.[1] Less was known at that time than now about the discharge from hot metals and ionization by impact, and unfortunately no account was taken of the part which they play. Quite recently two other explanations based on the ionic theory have been given, one by Stark [2] and one by J. J. Thomson.[3] These both consider the ionization produced within the cathode as the essential phenomenon.

The Cathode. — As has been shown by several experimenters,[4] a hot platinum wire discharges negative electricity with great rapidity when in a vacuum, and the rate of discharge increases rapidly with rise of temperature. Basing his explanation on these experiments Thomson says (p. 425): "It will be seen that the essential feature in the discharge is the hot cathode, as this has to supply the carriers for the greater part of the current in the arc." Stark says (p. 687): "Hohe Temperatur in der kathodischen Strombasis des Lichtbogens ist also eine hinreichende und, als wir annehmen wollen, die notwendige Bedingung dafür, dass aus der Lichtbogenkathode in den angrenzenden Dampfraum negative Elektronionen in grosser Zahl übertreten."

There is a difficulty with this explanation which at least appears to be serious. It is possible to have an arc between substances which boil at comparatively low temperatures. For example an arc may be formed in a vacuum between mercury terminals, and when the pressure is .29 mm. mercury boils at 100° C. This is a temperature very much too low to drive the negative ions from the mercury.

On the other hand experiments tend to show that no arc can be formed without a hot cathode. Stark and Cassuto [5] show that an arc is impossible, if the cathode is kept at a low temperature by rotating it, and that an arc is also impossible when using an electrolyte for the cathode. It has been shown in this article that a third carbon inserted in the arc cannot become a cathode unless it is heated first to a high temperature. The experiments of Weintraube [6] point to a similar conclusion.

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[1] PHYS. REV. X., p. 151.

[2] Drude's Ann., 12, p. 673.

[3] Conduction of Electricity through Gases, p. 456

It may possibly be that the boundary surface between the cathode and the gas must be very hot rather than the cathode itself. However, it is not my intention to discuss this question at this time. Further experiments must be performed before any decision can be reached. There are two divisions of this subject, however, which may be considered: first, the necessary conditions at the anode; second, those between the electrodes.

The Anode. — Thomson's idea is that the positive ions come from the boundary surface of the anode, and that for this purpose the anode must be very hot. Thus he says (p. 425): "The conditions that determine the current are that the work supplied to the cathode and anode should be sufficient to maintain them at incandescence." On the other hand Stark does not consider it necessary that the anode should be thus heated. He says (p. 677): "Dass auch die Anode des Lichtbogens Dampf aussende, ist nicht notwendig. Sie kann künstlich auf niedriger Temperatur gehalten werden, oder infolge ihrer Grosse von selbst auf mässiger Temberatur bleiben."

This latter would appear to be the correct view. The anode of the mercury arc may remain cool. It was found also in my own work that with pressures of 1 mm. the anode was not red hot until after the current had been passing for some time. It usually became hot, but this was not a necessary condition. This fact was noticed with carbon, iron and copper. In fact the cause of the drop in potential at the anode appears to be the same with the arc as with the usual form of discharge in a vacuum tube, modified to a certain extent by the temperature of the anode. The manner in which it is modified will also be considered when discussing the cause of the drop at the cathode.

Ionization near the Electrodes. — Thomson considers ionization to occur within the cathode and at the boundary surface of the anode, but none between these places. He considers the ions leaving either electrode to pass directly to the other. Thus he says (p. 425): "The anode has in general to be hot, otherwise it could not supply the positive ions which keep the cathode hot." In the equations applying to this work (p. 426) he assumes that the number of positive or negative ions leaving an electrode is the same as the number passing any cross section of the arc.

Stark on the other hand assumes that ionization takes place through the arc, being caused by impact of the negative ions (p. 681). "Wie in leuchtenden Anodeschicht und in den positiven Lichtsäule des Glimmstromes, so hat auch in der Anodenschicht und in der positiven Säule des Lichtbogens Ionizirung durch den Stoss der negativen Elektronionen statt."

In this connection there are two questions to be discussed, first, whether ionization occurs in the immediate neighborhood of the electrodes; and second whether it occurs through the length of the arc. In regard to the first question the evidence seems conclusive that ionization does occur. In the first place it is improbable that the number of ions coming from the cathode is in any case sufficient to account for all of those in the arc. Richardson[1] found the maximum current from a hot carbon filament to be 2 amperes per q. cm. of surface. In attempting to find the current density of the arc at the cathode I caused a current of 10 amperes to pass from a carbon 12 mm. in diameter to one 2 mm. in diameter. The negative end of the arc then covered all of the end of the carbon, but it did not appear to extend down the sides. If we may assume that at least 8 amperes passed into the end of the carbon, the current density was 255 amperes per cm. If one were to exterpolate from the curves given by Richardson, this would require a temperature 140° higher than the highest that he could use without vaporizing the carbon filament. It, therefore, seems probable that the negative ions do not all come from within the carbon, but that some are produced by impact.

Further it seems most reasonable to expect ionization to occur, at least, near the cathode. To take again the case of the mercury lamp we have the dark space about the cathode corresponding to the ordinary space with discharge in high vacuum, and we should expect new ions to be formed by impact at the end of the dark space.

But we can show that there must be an increase in the number of ions in order to account for the drop in potential at the electrodes. In general the fall of potential through the arc has the form indicated in Fig. 4, where *C* is the cathode and *A* the anode. There

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[1] Phil. Trans. Roy. Soc. Lon., 210, p. 497.

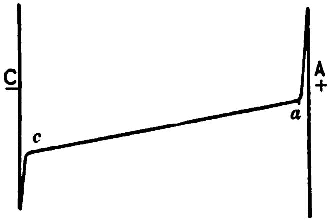

Fig. 4.

is a large rise in potential between C and c, a uniform increase through the arc and a large rise near A which in most cases is larger than that at C. If we are to offer any explanation which is based on the ionic theory, we must assume the existence of the positive charge between C and c. Thus if we let X equal the electric force, x the distance from C, V the electric potential, and ρ the density of electricity,

$$\frac{d^2V}{dx^2} = -4\pi\rho.$$

(Since there is no appreciable flow at right angles to x, $d^2V/dy^2 = d^2V/dz = 0$)

$$\therefore X_1 - X_2 = -4\pi\int_C^c \rho dx$$

where X_1 and X_2, are the values of X at C and c.

But evidently X_2 is less than X_1. Therefore, there must be a positive charge between C and c. Moreover from c to a X is constant, so that ρ must be zero, that is, the number of positive ions per c.c., must there equal the number of negative ones.

The distribution of positive electricity between C and c can be accounted for in two ways; first, by assuming that more new ions are there produced than recombine; second, by assuming that the velocity of the negative ions per unit potential gradient is greater when they first leave C.

To put this in mathematical form, where ρ equals 0 let

X_g be the value of X.

n_1 be the number of positive ions per cm.

n_2 be the number of negative ions per cm. $(n_1 = n_2.)$

k_1 be the velocity of the positive ions per unit X.

k_2 be the velocity of the negative ions per unit X.

N_1 be the number of positive ions passing any cross section per sec.

N_2 be the number of negative ions.

Let X', n_1', n_2', k_1', k_2', N_1', N_2', be the corresponding values at some point between C and c where ρ is positive.

$$\text{Then } \frac{N_1}{N_2} = \frac{X_g n_1 k_1}{X_g n_2 k_2} \quad \therefore \quad \frac{N_1 K_2}{N_2 K_1} = \frac{N_1}{N_2} = 1$$

$$\frac{N_1'}{N_2'} = \frac{X' n_1' k_1'}{X' n_2' k_2'} \quad \therefore \quad \frac{N_1' K_2'}{N_2' K_1'} = \frac{n_1'}{n_2'} > 1,$$

since ρ is + and $n_1' > n_2'$,

$$\therefore \frac{N_1 K_2}{N_2 K_1} < \frac{N_1' K_2'}{N_2' K_1'}.$$

As a result either

$$N_1 < N_1', \quad N_2 > N_2',$$
$$K_2 < K_2', \quad K_1 > K_1',$$

or some combination of these inequalities must exist. But if

$$N_1 < N_1' \text{ then } N_2 > N_2',$$

for there can be no production of positive ions without as great a production of negative, and as we go from left to right there must be as great an increase in the negative ions as there is in the number of positive in going from right to left.

There is no probability whatever that k_2 is less than k_2'. There is no reason to suppose that the mass of the positive ion would be greater when near the cathode than when away from it, or that there would be any change in the ions so that the velocity per unit electric force would decrease.

We are, therefore, reduced to one of two possibilities, either that there is a production of new ions between C and c, or that the velocity of the negative ion per unit X is very much greater near the cathode than at a distance from it. Either of these hypotheses would appear entirely reasonable. The negative ions are, no doubt, driven out with great velocities, and we should expect new ones to

This last suggestion is in accordance with the explanation of Thomson, but while this would in general explain the drop in potential at the electrodes, it does not explain the fact that the greater drop is at the anode. It may indeed be said that it is necessary that there should be a greater drop there in order that the anode may be very hot, but the fact that it is needed there does not explain how it can be there. As we see from the equations which we have given, in order to have a drop at the anode greater than that at the cathode there must either be an increase in the number of ions through the arc or else a greater change in the velocity of the positive ions as they are leaving the anode than there is in that of the negative ones when they are leaving the cathode. In fact the change must be much greater, since according to the explanation which we are now considering, the number of the positive ions is much smaller than that of the negative.

Now in every case which has thus far been studied the difference in the velocity of the negative ions when first produced and that which they afterwards have is much greater than any change in the velocity of the positive ions. The negative ions are at first electrons with masses very much smaller than the masses of the atoms and with velocities correspondingly great, while the positive ions never have masses appreciably less than those of atoms, and never have velocities as great as that of electrons. It would, therefore, appear that the only explanation which will account for the change in the electric force near the electrodes and also for the fact that that at the anode is the greater is that there is an increase in the number of ions through the body of the gas.

I have thus given two reasons for believing that the number of ions coming from the electrodes cannot be the same as that passing the center of the arc; first, that few, if any, cathodes ever become hot enough to give out ions in sufficient abundance; second, that we cannot account for the greater drop at the anode without assuming a production of ions between the electrodes.

Ionization through the Arc. — Passing now to parts of the arc where the potential gradient is small the following reasons may be given for believing that ionization occurs. If we assume that no ionization occurs through the arc, then we must also assume that no recombination occurs, at least, in appreciable amount. This follows from the fact that the potential gradient through the arc is uniform and consequently that the number of positive ions at all points must equal that of the negative. But that there should be recombination of the ions in every other case, even where the mean free path is much longer and where the number of ions is very much smaller, and yet not in this case is hardly to be expected. But even granting that this is possible, how could it be possible that the potential gradient between the electrodes can remain the same when the pressure of the gas is changed from 730 to 50 mm. As will be shown below, the mean free path of the ion in the first place is approximately .001 cm.; and in the second 14.6 times as long. If the ions starting from the cathode pass directly through to the anode, we should expect them to be drawn through the gas more easily in the latter case. But they are not. It requires as strong a field in one case as in the other.

The cause for this ionization may be some unknown action, possibly some form of radiation, as ultra-violet light, or "Entladungstrahlen."[1] There is, however, an objection to such an explanation as shown by the arc between mercury terminals when the tube is heated above 100° C. At these temperatures the arc no longer fills all the tube. It extends rather as a pencil of light through the center. If any radiation were given off, we should expect all of the gas to take part in the discharge. It does not, however, do so. The gas is a good conductor only where a large amount of energy is being expended.

But one does not need to go to an arc in a vacuum to see this illustrated. If two large carbons are used with the two opposite ends blunt, the arc does not fill all of the space between them. It only fills that part of the space which it can heat very hot. If radiation is the cause of the ionization, it must be some kind of radiation which can only exist where a large amount of energy is being expended. Now all known forms of radiation penetrate, at view. First, the mean free path of the ions is not sufficiently great. The negative ions, at least at ordinary temperatures, must move through 25 volts, in order that new ones may be produced in appreciable numbers.[1] If we assume the electric force in the arc to be 27 volts per cm.,[2] the mean free path of the negative ions in the air at a pressure of 1 mm. and at a temperature of 20° C. to be .0657 cm. as found by Townsend,[1] and the temperature of the arc to be 3500° C. as given by Violle,[3] then the mean free path of the ions in the arc at atmospheric pressure will be through only .03 volts. This is much too small to produce ionization.

Second, the fact that the fall of potential through the arc does not decrease at all in proportion to the length of the mean free path is an argument against this view as well as against one of the preceding.

Third, an increase in the current does not require an increase in the current through the arc. When ionization is caused by impact the current increases very rapidly with increasing electric force, but with the arc the opposite of this occurs. The data given by Mrs. Ayrton[4] shows that in general the greater the current between carbon terminals the smaller the electric force. With the mercury arc Arons considers that within wide limits the electric force is independent of the current[5] while Hewitt finds the electric force to decrease with increasing current.[6]

Thus we have again reason to believe that the ionization is caused by the change of electrical energy into some other form of energy. There is then the possibility that the high temperature is the cause of the ionization. With all forms of the arc it is apparently necessary to have a high temperature and except when the mean free path is very long, it is high enough to vaporize a carbon filament. Even an arc between carbons at a pressure of .5 mm. has a temperature sufficient to melt platinum.

As to the temperature of the mercury arc, there appears to be some difference of opinion among experimenters. Hewitt[1] finds the temperature as measured by a thermo-junction to vary from 115° to 179° C.[2] But a thermo-junction must lose heat rapidly both from radiation and conduction, while it must receive heat from a gas as rare as mercury vapor very slowly, and we have already shown how a small platinum wire is melted in an arc between carbons when larger ones are not.

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[1] Phil. Mag. (6), 5, p. 395.

[2] Mrs. Ayrton's The Electric Arc, p. 231.

[3] C. R., 115, p. 1273.

[4] The Electric Arc, p. 225.

[5] Wied. Ann., 58, 77.

[6] Elec. World, 42, p. 1001.

Arons[3] considers the temperature to be very high. He found platinum to be melted in the mercury arc, and theoretical considerations led him to expect even higher temperatures than that needed to melt plantium. He worked with a greater density of current than any given by Hewitt and the temperature, no doubt, varies greatly with the current. It may also have been that there were gases other than mercury vapor in Arons' lamp, but his value for the potential gradient is small and this would indicate the absence of such gases. We may, therefore, assume for the present that a temperature of a thousand or so degrees Centigrade is needed even for the most perfect mercury lamp.

It has not been proven that a high temperature alone will produce ionization. In all cases the phenomena have been complicated either by chemical action or by the presence of solid substances. To be sure, if we assume that ionization may be produced by impact, then ionization must occur at sufficiently high temperature. For if a negative ion, moving with a given velocity, produces new ions when it hits a molecule at rest, then a molecule having the same velocity will be divided into ions on hitting an ion at rest. But on the basis of the data which we now have this would require exceedingly high temperature. In the cases studied by Townsend[4] the ratio between the velocity of the negative ions needed to produce new ions and the velocity of the molecules at 20° C. was shown to be $\sqrt{25P}$ where P is the fall of potential measured in volts through which the ions must pass. Since the mean velocity of the molecules varies as the square root of the absolute temperature, it velocity of the molecules as great as that of the ions which ionize by impact.

However, this would not be essential. Probably if a comparatively few of the ions had such velocities it would be sufficient. If we further assume that at high temperatures the molecules are more easily ionized, the facts are readily explained, and this is a reasonable assumption to make, for at 3,500° C. the internal energy of a molecule is thirteen times that at 20° C. At this temperature many of the molecules become dissociated and in the arc the motion of the molecules is sufficient to produce light.

This effect would generally be helped by more or less chemical action, and whenever this occurs at high temperature we have an abundance of ionization. For example, I find that the electric force between the electrodes is much decreased when NaCl is introduced into the arc. Thus the potential difference between two exploring pencils was 10 volts when the electrodes were ordinary carbons and 7 volts when one of the carbons was hollowed out and filled with NaCl. Indeed it is a kind of chemical action when a molecule is dissociated only to recombine, and this no doubt occurs even with the enclosed arc.

A very probable explanation then would be that at ordinary pressures the high temperature is the essential condition, that the ionization is then produced by the impact of the molecules on the ions modified more or less by chemical action, and that the motion of the ions themselves have little effect; while at low pressures the motion of the ion comes to be an important factor and the temperature less important.

Such an assumption would explain very easily the relation of the electric force through the arc with the current. When the current is increased there is always an increase in the heat developed, and this, no doubt, is the cause of the increased ionization which then occurs.

This would also explain the phenomena at different pressures. When the mean free path is sufficiently large ionization would be produced without so high temperatures, and we then have the arc at lower temperatures. As the mean free path becomes less, the average velocity of the ions becomes less, and in order to have ionization the velocity of the molecules becomes comparable with that of the ions, that is, the temperature must be raised. At still higher pressures the mean velocities of the ions become very small and the temperature with its rapid motion of the molecules is the determining factor. Then it makes no difference what the mean free path of the ions may be, if only the temperature is sufficiently high.

Thus we are not able to detect any change in the temperature of the arc when the pressure of the gas is decreased, until a pressure of 50 mm. is reached. Here carbon begins to be deposited on the filament placed in the arc and here also the electric force begins to diminish. Assuming that the temperature is still 3500° C., and that the electric force has not changed, the mean free path is then through .45 volts. Many of the ions would have paths longer than this and ionization might begin to occur without so much motion of the molecules. In other words the temperature might decrease.

At 4 mm. of pressure the larger platinum wire no longer melted. Assuming the temperature to be that of melting platinum, namely 1775° C. and the electric force to be 13 volts per cm. the mean free path would be through 1.5 volts. In reality the temperature must have been higher than this and the mean free path longer, but even this is quite comparable with the potential difference necessary to produce ionization at low temperatures. The electric force here is also quite comparable with that in discharge at low pressure.[1]

Summary. — The potential gradient through the arc at different pressures was studied. The drop at the cathode was found to pass through a maximum at about 100 mm. of pressure and then to decrease, the larger part of the change occurring below 30 mm. pressure. The drop at the anode was found to decrease quite uniformly as the pressure was decreased, the rate of decrease, however, being greater below 50 mm. No change in the electric force between the electrodes was shown until a pressure of 50 mm. was reached. Below this it decreased, being approximately one half as great at 2 mm. as at atmospheric pressure.

The temperature of the arc is apparently less at low pressures than at high, although at .5 mm. pressure platinum wire is still melted.

The discharge from the arc to a surrounding cylinder was found to increase very rapidly as the pressure was decreased. The discharge carried by the positive ions is the greater at atmospheric pressure, that carried by negative the greater at low pressures.

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[1] Wied. Ann., 64, p. 67, and Phil. Mag. (5), 49, p. 512.

In regard to the theory of the arc it was pointed out that a hot anode is not an essential requirement. Reasons were also given for believing that the number of ions leaving the electrodes is not as great as that passing through the center of the arc. A more complete discussion of the phenomena at the electrodes is to be given at a later date.

A high temperature is considered to be an essential requirement for ionization within the arc except when the pressure is very low. There the free path of the negative ions may be sufficiently great to allow ionization by impact.

Colgate University,
May, 1904.

A STUDY OF THE SILVER VOLTAMETER.[1]

By K. E. Guthe.

CONSIDERING the important rôle which the silver voltameter plays in electrical measurements, relatively few investigations have recently been made with a view of improving the accuracy of this instrument. The older investigations of Kohlrausch,[2] Rayleigh and Sidgwick,[3] Gray,[4] Schuster and Crossley[5] and of Glazebrook and Skinner[6] proved the silver voltameter to be more reliable than the copper voltameter and this led to the general use of the well known form, the specifications of which have been adopted in connection with the legal definition of the ampere and are practically identical in the United States,[7] England[8] and Germany.[9] This type has consequently been used, sometimes with only slight modifications, in a large number of investigations; but as the demands upon the accuracy of the results increased it became more and more apparent that it presented a number of peculiarities, and the thorough researches on the behavior of this voltameter by Kahle[10] and others soon created the impression that the results obtained with a silver voltameter cannot lay any claim to a high degree of accuracy.

Until recently the attempts to find the source of trouble in the voltameter have been rather unsuccessful, different investigators arriving at different conclusions regarding the influence and nature of the disturbing factors. While it must be conceded that in two voltameters of the usual form when arranged in series and treated in exactly the same manner the same quantity of electricity will yield deposits differing in weight not more than 1 in 5,000, or even 1 in 10,000, these same instruments when used under slightly differing and frequently under apparently the same conditions will show much larger differences in a series of independent experiments.

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[1] Communication from the National Bureau of Standards.

[2] F. and W. Kohlrausch, Wied. Ann., Vol. 27, p. 1, 1886.

[3] Rayleigh and Sidgwick, Phil. Trans., Vol., 175, p. 411, 1884.

[4] Th. Gray, Phil. Mag., Vol. 22, p. 389, 1886.

[5] Schuster and Crossley, Proc. Roy. Soc., Vol. 50, p. 344, 1892.

[6] Glazebrook and Skinner, Phil. Trans., Vol. 183, p. 567, 1892.

[7] National Academy of Sciences, Feb. 20, 1895.

[8] Electrician, Vol. 27, p. 325, 1891.

[9] Ztschr. f. Instr.-K., Vol. 21, p. 180, 1901.

[10] Kahle, Ztschr. f. Instrum., Vol. 18, pp. 229 and 267, 1898.

At this point Richards, Collins and Heimrod[1] and Leduc[2] took up the problem of improving the silver voltameter or coulometer, a name which Richards proposes for the instrument and which, I think, would be much more appropriate.

Since so far the research of Richards and his students is the only one in which a systematic comparison of various types has been made, it seemed of sufficient importance to partly repeat the work with such modifications as seemed advisable, and, after having found the most reliable form, to subject it to a severe test in order to determine the degree of accuracy which may be expected from it.

DESCRIPTION OF THE VOLTAMETERS AND THEIR TREATMENT.

The following types were investigated:

1. *The usual type*, consisting of a platinum bowl 10 cm. in diameter and 5 cm. deep as kathode and a silver plate held by a silver rod at its center as anode. The anode was surrounded by filter paper secured at the back with a little sealing wax. The electrolyte, unless otherwise stated, was a neutral freshly prepared 20 per cent. solution of silver nitrate.

To test the neutrality of the solution, the silver was precipitated by NaCl solution and the filtrate tested with methyl-orange. Silver nitrate, obtained from Eimer & Amend as chemically pure, contained no acid. In a few cases the crystals were first fused to insure neutrality, but since it was found that the results obtained with a solution of the fused salt agreed closely with those where the solution was made from crystals, the latter was used in almost all experiments.

2. *The Silver Oxide Voltameter.* — Patterson and Guthe[3] employed as electrolyte a solution, saturated with Ag_2O. To allow

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[1] Richards, Collins and Heimrod, Proc. Am. Ac., Vol. 35, p. 123, 1899. Richards and Heimrod, Proc. Am. Ac., Vol. 37, p. 415, 1902.

[2] Leduc, Journ. de Phys., Vol. 1, p. 561, 1902.

[3] Patterson and Guthe, PHYS. REV., Vol. 7, p. 257, 1898.

the oxide to have as great an influence as possible the voltameter was constructed in such a manner that the current had to pass through a layer of the oxide. For this purpose the anode was placed inside a porous cup glazed at the side, at the bottom of which a layer of carefully prepared Ag_2O was placed. Since the glazing showed a tendency to crack, an ordinary porous cup was used in the later experiments, and a tight fitting glass tube inserted in it, in order to prevent the current from passing through the sides. The liquid was a 20 per cent. solution of silver nitrate kept in a bottle containing silver oxide. Either platinum bowls or platinum crucibles served as kathodes.

3. *The Richards voltameter*, consisting of a platinum crucible 10 cm. high and 6 cm. wide as kathode. The anode, a silver rod, was surrounded by a fine grained porous cup, made from the lower end of a Pasteur filter. The electrolyte was the same as in type 1. During the experiment the liquid in the porous cup was always kept at a lower level than that outside by removing about every ten or fifteen minutes some of the liquid in the cup by means of a pipette. Once a siphon was employed to produce this difference in level, without, however, changing the result.

A slight and somewhat more convenient modification of this type will be described later.

4. *Large Silver Anode Type.* — A platinum bowl formed the kathode. In the 20 per cent. solution of silver nitrate a porous cup, 7 cm. in diameter, was suspended and, at the bottom of this, finely divided silver was placed, the current being led into the voltameter by means of a plate, pressed upon the silver. At first the fine crystals deposited at the kathode in previous runs were used, but it was soon found that the silver became packed so closely that it acted like an ordinary plate; therefore, in the later, final experiments, granulated silver, prepared from the crystals, was substituted and upon this the plate was pressed.

5. *Leduc's form* differs from type 4 principally by the use of filter paper and muslin to hold the granules on the anode. Granulated silver

The silver was either pure silver obtained from the United States Mint or silver bought from Eimer & Amend as chemically pure. While the results obtained with the two kinds agreed well with each other, the appearance of the anodes after the experiment showed a striking difference. While the finely divided silver which covers the pure anode after electrolysis is whitish and crystalline, the less pure silver from Eimer & Amend is perfectly black, the layer consisting mainly of silver. This difference in the appearance of the anode allows a good estimate of the purity of the silver used and has been noticed before in the work of the Reichsanstalt.[1]

The porous cups were treated with aqua regia, potassium cyanide solution, nitric acid and finally with boiling water before being used in the voltameters. After a few runs the cups become dark on the outside, due to a thin layer of finely divided silver which can, however, easily be removed by potassium cyanide. For the large silver anode type, closely grained cups of very convenient shape were obtained from the Akron Insulator and Marble Company.

The use of filter paper was carefully avoided in the preparation of the solution since it seems that the organic substances act chemically upon neutral silver nitrate solution.[2] The solution was not filtered, but transferred from its bottle to the voltameters by means of a clean pipette. Only in types 1 and 5 was filter paper used during the experiment since it is characteristic for these forms.

The treatment of the silver voltameters was in all cases identical. The platinum bowls — or crucibles — were washed successively with nitric acid, water, then heated to 160° C. in an air bath, and after having cooled in a desiccator for an hour or two, carefully weighed. After the experiment the deposits were washed four or five times with distilled water until the last wash water did not show a bluish color on addition of NaCl solution. Then the vessels were left over night filled with distilled water well covering the deposit, were emptied in the morning, heated to 160° C., cooled and weighed as before. The weight of the deposit for each run was about 4 grams. Usually silver was deposited twice in the same vessel, after which most of it was removed by scraping and the rest dissolved in nitric acid.

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[1] Ztschr. f. Instr.-kunde, Vol. 22, p. 155, 1902.

[2] Ztschr. f. Instr.-kunde, Vol. 22, p. 156, 1902.

It was not deemed necessary to wash the deposit with water at 80° C., as has been recommended by the Reichsanstalt. Though Kahle[1] and Merrill[2] have found that silver is slightly soluble in hot water, Richards, Collins and Heimrod[3] as well as Leduc[4] have shown that after proper washing the weight of the deposit does not change upon continued digestion with water of room temperature, a conclusion fully corroborated by my experience.

Great care was taken to guard against any loss of silver during the washing of the deposit. The solution as well as the wash water was always first emptied into a clean beaker. With proper illumination even minute particles of silver could be seen very distinctly. If any were present they were carefully collected in a small filter and later added to the deposit.

Lord Rayleigh and Richards have found that in spite of careful washing some of the mother liquid remains included in the interstices between the crystals as shown by a slight loss of weight on heating to dull redness, but Gray[5] was unable to observe any loss in deposits properly washed. At any rate the amount included can only be very slight and must be rather constant, considering how accurate the results are with properly constructed voltameters. The weighings were made with a good chemical balance, having a sensitiveness of about twenty divisions for the load used. It was found advisable to keep two shallow dishes with calcium chloride in the balance case in order to prevent condensation of moisture on the deposit. In each experiment four voltameters were arranged in series, namely, a set of two bowls agreeing in weight within 4 mg. (55.24 grams each) and a set of two crucibles agreeing to within 7 mg. (39.10 grams each). Each vessel of a set was first weighed separately by the method of double weighing and then the difference between the two was found by the same method. The latter always agreed with the difference between the two separate determinations within 0.06 mg. From these data the probable weight of each vessel was calculated.

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[1] L. c., p. 233.

[2] Merrill, PHYS. REV., vol. 10, p. 170, 1900.

[3] L. c., p. 139.

[4] Leduc, C. R., vol. 135, p. 24, 1902.

[5] Gray, Phil. Mag., Vol. 22, p. 399, 1886.

The vessels were counterpoised by platinum so as to avoid troublesome corrections for the buoyancy of air, and the weights used were platinum weights, standardized by the section of weights and measures of the Bureau.

Description of the Method.

The arrangement of the apparatus is represented by the figure. The current was furnished by a storage battery of 40 cells, *B*, and in series with it were placed a large variable resistance, r_1, the four voltameters, *V*, and standard resistance coil, *St. R.* The small resistance r_2 was shunted by a Kelvin rheostat. This consists of a wire wound upon two cylinders, of which one is of metal and the other one insulating. By rotating the cylinders the wire is transferred from one cylinder to the other, and the resistance of the rheostat varied in a continuous manner. By the use of this instrument slow variations of the current can easily be avoided. In a shunt to the standard resistance a sensitive galvanometer, *G*, and a Weston cell, *St. C.*, were placed, the latter so as to oppose the potential difference at the terminals of the standard resistance; equality between the two is indicated by the galvanometer remaining at rest. An unsaturated Weston cell was selected on account of its negligible temperature coefficient. The same cell was used throughout the investigation. For the determination of its E.M.F. in terms of the standard of the Bureau and for the measurement of the standard resistances, and their temperature coefficient I am indebted to the section of resistance and electromotive force.

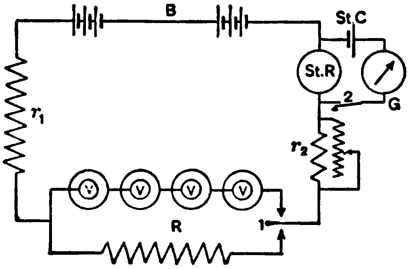

Fig. 1.

The current is first closed through the resistance *R*, which was made as nearly equal as could be estimated to that of the voltameters, then adjustment was made for no deflection of the galvanometers by varying the resistance of the circuit and finally, after the current had become practically constant, it was sent for a definite time through the voltameters by throwing the switch 1. By rotating the Kelvin rheostat the current was kept constantly adjusted to its proper value until the experiment was completed.

Standard resistances of 1, 2, 5 and 10 ohms were used in *St. R.* and once the 1 and the 2 ohms were placed in parallel. Thus the current was varied in the different experiments from 0.1 to 1.5 amperes. The current was usually kept closed long enough to allow one ampere-hour to pass through the voltameters, *i. e.*, each time about 4 grams of silver were deposited. A change of the current as small as 1 in 100,000 could easily be seen on the scale. In the final determinations of the electrochemical equivalent only such experiments were used in which the variations of the current did not exceed 1 in 25,000.

The time was measured by an astronomical clock, connected with a chronograph, which also recorded the making and breaking of the current through the voltameters. The rate of the clock was determined by means of the noon signals received from the Naval Observatory.

In a number of experiments the variations of the current exceeded the required limit, due partly to the inconstancy of the storage battery, partly to variations in the resistance of the voltameters or the circuit. In some cases the rate of the clock was not known to a sufficient degree of accuracy or the resistance of the voltameters was not estimated correctly. Experiments in which such troubles occurred could not be used for the final calculation, but they allow relative comparisons of the different types of voltameters.

For convenience the Weston cell was not placed directly in the shunt from the standard resistance, but a potentiometer was introduced, the Weston cell balanced over a resistance of 10,190 ohms and then the terminal potential difference of the standard resistance substituted for the cell. Of course, the constancy of the current through the potentiometer was frequently checked by reconnecting with the standard cell.

COMPARISON OF DIFFERENT VOLTAMETERS.

In the following tables the results obtained are given. Table I. shows that two voltameters of the same type and treated in precisely the same manner will yield deposits which will in general not differ from each other by more than one in 10,000.

Table I.

Richard's Type.			Usual Type.		
1	2	Difference, per cent.	1	2	Difference, per cent.
4.09927	4.09911	0.004	4.09973	4.09945	0.007
4.09955	4.09959	0.001	4.10003	4.09961	0.010
4.09851	4.09870	0.007	4.10154	4.10098	0.014
4.09927	4.09941	0.003	4.10100	4.10247	0.036
Ag_2O Type.			**Large Anode Type.**		
1	2	Difference, per cent.	1	2	Difference, per cent.
4.09913	4.09935	0.006	4.10082	4.09985	0.001
4.10382	4.10392	0.002	4.09891	4.09888	0.001
			4.10172	4.10167	0.001

The usual form does not show nearly as constant results as the other three. The last large difference may however be due to loss of silver in one bowl.

Tables II.–VIII. allow a comparison between the different types, when the same quantity of electricity is sent through them. In the

Table II.

Current.	Richard's Type.	Usual Type.	Difference, per cent.
0.1	4.09863	4.10150*	+ 0.070
0.5	4.09870	4.10126*	0.062
1.0	4.09800	4.09973	0.036
1.0	4.09927*	4.10100	0.042
1.0	4.09941*	4.10247	0.050
1.0	4.09881	4.10016	0.030
1.0	4.09896	4.10056	0.039
0.2	4.09993*	4.10225	0.057
		Average,	+ 0.048

Table III.

Current.	Richard's Type.	Ag_2O Type.	Difference, per cent.
0.1	4.09863	4.10147	+ 0.069
0.5	4.09881*	4.10053	0.042
1.0	4.09881	4.10167	0.070
1.5	4.09897	4.10091*	0.047
		Average,	+ 0.055

Table IV.

Current.	Richard's Type.	Leduc's Type.	Difference, per cent.
0.2	4.09920	4.10245	+ 0.057
0.5	4.09906*	4.10141*	+ 0.057
		Average,	+ 0.057

Table V.

Current.	Large Anode Type.	Ag_2O Type.	Difference, per cent.
1.0	4.10172	4.10382*	+ 0.051
1.0	4.10167	4.10392*	0.055
		Average,	+ 0.053

Table VI.

Current.	Usual Type.	Ag_2O Type.	Difference, per cent.
0.1	4.10150	4.10147	− 0.001
0.2	4.10003*	4.09913	− 0.022
0.2	4.09961*	4.09935	− 0.006
1.0	4.10016	4.10167	+ 0.038
		Average,	+ 0.002

Table VII.

Current.	Richard's Type.	Large Anode Type.	Difference, per cent.
0.2	4.09993*	4.09962	− 0.007
0.5	4.09881*	4.09889	+ 0.002
0.5	4.09906*	4.09919*	+ 0.003
1.0	4.10449	4.10468	+ 0.005
1.0	4.10461	4.10510	+ 0.012
1.0	4.09919	4.09912	− 0.002
1.0	4.11888	4.11902	+ 0.004
1.0	3.47004*	3.47053	+ 0.014
1.5	4.09897	4.09880*	− 0.004
		Average,	+ 0.003

Table VIII.

Current.	Richard's Type.	Richard's Type (Modified).	Difference, per cent.
0.2	4.09920	4.09942	+ 0.005
0.5	4.09906*	4.09896*	− 0.002
1.0	3.47004*	3.46998*	− 0.003

first column of each table the current is given and in the last the percentage difference between the two types. The asterisks denote that the deposit is on silver, *i. e.*, that it is second deposit.

From these tables can be seen that there are two distinct classes of voltameters; on one side the usual, Leduc's and the silver oxide type, on the other Richard's and the large anode type, the latter class yielding deposits about 0.05 per cent. smaller than the former.

The difference between the usual form and Richards' agrees very well with the final average of 0.050 per cent. given by Richards.[1] A voltameter in which the electrolyte was always kept saturated with Ag_2O has not been used before. Patterson and Guthe employed a solution saturated before the experiment was made, but in all other respects adhered to the legal specifications. Richards' comparison between his type and Patterson and Guthe's showed for the latter a deposit 0.112 per cent. larger than for the former, besides a considerable variation in the percentage difference (from 0.039 per cent. to 0.230 per cent.) We see from Table III. that the form of Ag_2O voltameter used in the present investigation gives somewhat more concordant results and a much smaller difference between the two types. Possibly the presence of filter paper in the Patterson and Guthe voltameter may explain the greater value. A porous cup has doubtless the effect of decreasing the weight of the deposit.

It must be confessed that the results obtained with Leduc's voltameter were not expected. He has repeatedly asserted that no trouble arises when small anodic current density is employed. As will be seen from Table IV., in these comparisons small currents were used, but the deposit was in each case 0.057 per cent. larger than that obtained with the Richard's type. A comparison of Tables III. and IV. verifies Leduc's statement[2] that a solution saturated with the oxide and a normal solution will give identical results. It was also noticed that even with these small currents the heavy anode liquid sinking to the bottom of the platinum bowl and there spreading produced a starlike figure of the deposit. This peculiar arrangement of the crystals, very characteristic for the usual type, has been studied by Behn,[2] who showed it to be due to the motion of the heavy anode liquid along the surface of the kathode.

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[1] Richards and Heimrod, l. c., p. 439.

[2] Leduc, Rapp. Congr. intern., Paris, 1900, Vol. II., p. 444.

With a porous cup around the anode the crystals are distributed perfectly evenly over the kathode, without showing the striæ spoken of. Filter paper allows an easy passage to the anode solution and should therefore be avoided, even if the chemical action upon neutral nitrate solution, to which attention was drawn above, will not affect the electrochemical equivalent.

All previous researches on the subject have led to the conclusion that the source of trouble is to be looked for at the anode. The solution becomes acid during electrolysis and therefore the presence of acid has been suggested as the reason for the inconstancy of the voltameter. Leduc[3] has shown however that a fresh solution, originally neutral, but slightly acidified before electrolysis, gives smaller deposits than a neutral solution. According to this observation the presence of acid alone has not the effect of increasing the weight of the deposit. Another interesting observation by the same author shows that if the amount of acid surpasses a certain limit in the original solution, its density will decrease during electrolysis, a fact observed before by Rodger and Watson.[1] These investigators seem to be the first to have pointed out that during electrolysis a complex silver ion may be formed and that this may produce too heavy deposits. The thorough work of Richards and his students has strengthened this view and my own results seem all to point to the same explanation. The existence of such a complex ion, whatever it may be,[2] would easily enough explain the formation of the acid at the anode. It is well known that during electrolysis of silver nitrate solution, crystals are formed at a platinum anode, which according to Sulc[3] and Mulder and Heringa[4] have the formula Ag_7NO_{11}. I have been able to obtain these bluish black crystals mixed with silver at the *kathode* from a solution, saturated with Ag_2O, using a large current density. In this case the deposit was abnormally heavy, 0.23 per cent. and 0.32 per cent. greater than that obtained in a large silver anode voltameter. Complex ions are therefore present in such a solution and it seems reasonable to suppose that if they reach the kathode, as they will in voltameters in which the anode is surrounded by filter paper or in the Ag_2O type, the deposit will be too heavy, resulting in a wrong value for the electrochemical equivalent of silver.

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[1] Behn, Wied. Ann., Vol. 51, p. 105, 1894.

[2] Leduc, Journ. de Phys., Vol. 1, p. 573, 1902.

[3] Rodger and Watson, Phil. Trans., Vol. 186, p. 631, 1895.

[4] Richards suggests Ag_2O. See also the theoretical discussion between Revay and Küster and von Steinwehr: Ztschr. f. Electroch., Vol. 4, p. 313 and 451, 1898.

[5] Sulc, Ztschr. f. anorg. Chem., Vol. 12, p. 90, 1896.

[6] Mulder and Heringa, Verh. Kon. Ak. Wet., Vol. 5, p. 37, 1896.

Richards' method of removing the heavy anode liquid from the bottom of the porous cup and preventing its diffusion towards the kathode by keeping the level inside lower than the outside, is therefore correct and will not give too small an equivalent, as Leduc asserts.

The large silver anode voltameter gives the same results as Richards' form, in spite of the fact that the solution is at the same level inside and outside (see Table VII.). How can we explain this? A few experiments by Richards and Heimrod[1] show that a crucible on which silver had been deposited previously will increase in weight when filled with the anode solution. Apparently a chemical reaction takes place, resulting in a breaking up of the complex ion and a deposition of silver or some silver compound. In the large anode type the porous cup prevents the anode solution from diffusing rapidly and the silver at the bottom of the cup breaks up the troublesome ion. This secondary chemical reaction is the reason why the anode is always covered by a layer of finely divided silver, the "anode slime," the anode itself acting to a certain extent chemically on the ion produced by electrolysis. The large anode type will therefore give as good results as the Richards type and is more convenient to use.

The above considerations led the author to the construction of a modified Richards voltameter. Instead of removing the liquid from the bottom, some of silver crystals deposited by previous runs were placed in the bottom of the cup and the liquid was left undisturbed during the experiments. At first some inconvenience was experienced due to the rapid growth of the silver towards the anode and consequent sudden changes in the resistance of the voltameter, which prevented the proper adjustment of the current to a constant value. But this trouble was overcome entirely by surrounding the lower part of the porous cup by a small glass cup cut from a test tube or a small beaker. The excellent agreement with Richards' form (see Table VIII.) shows that this modification also will give the correct silver equivalent.

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[1] Richards and Heimrod, l. c., p. 431.

It seems therefore advisable to abandon in future experiments, especially for the absolute measurement of current, the usual form of the silver voltameter and employ either Richards' type or one of those proposed by the author. The differences in the weight of silver, obtained by two of these voltameters in series, show much smaller variations than those by the old form, and as will be seen in the following, determinations made by means of independent experiments agree very closely.

Silver Nitrate Solution. Deposit on Platinum and on Silver.

It has been shown by Kahle that an old solution will yield too heavy deposits, its effect increasing as more and more silver is electrolyzed. The Reichsanstalt advises therefore to use the solution only until 3 grams of silver are deposited from 100 c.c. of solution. It seemed to be of interest to see whether in the Richards form the solution outside of the porous cup could be used repeatedly. Eight grams of silver were deposited by two runs from the same solution, amounting to about 150 c.c., and the solution left standing for several weeks. In the experiments following two Richards voltameters were placed in series, one with a fresh solution, one with this old solution; the latter gave on the average 0.055 per cent. more silver. Now it was thought that it might be possible to obtain a good solution by leaving it in contact with silver crystals. After several weeks more this solution was tried again and gave on the average 0.025 per cent. heavier deposits than the Richards form. While an improvement is to be observed, it is clear that keeping the solution simply in contact with silver will not entirely eliminate the complex ion. It is therefore advisable to use always freshly prepared silver nitrate solution. Variations in concentration have been shown by former investigators to be of no influence upon the weight of the deposit. In our case the electrolyte was always a 20 per cent. solution.

Gore[1] and Kahle found a somewhat larger deposit when the kathode was used a second time before removing the silver collected in a previous run; and Richards and Heimrod corroborate their statement so far as the filter paper voltameter is concerned, but found no difference with the porous cup voltameter. Patterson and Guthe obtained the same deposit on platinum and on silver.

Table 9 gives the results of tests made with this point in view.

TABLE IX.

Current.	Richard's Type.			Large Anode Type.		
	On Platinum.	On Silver.	Difference Per Cent.	On Platinum.	On Silver.	Difference Per Cent.
1.0 A.	3.70837	3.70850	+ 0.003	3.70879	3.70839	− 0.011
"	4.10461	4.10449	− 0.003	4.10468	4.10510	+ 0.011

It is apparent that it makes no difference whether a clean platinum kathode is used or whether the silver from previous runs is left on it. Of course, the deposit should not become too heavy on account of the increasing danger of mother liquid remaining included in the interstices between the crystals.

The explanation for the heavier deposits on silver in the filter paper voltameter is to be sought in the action of the silver upon the anode liquid.

ELECTROCHEMICAL EQUIVALENT OF SILVER.

In conclusion the results are given of experiments which were suited for the calculation of the electrochemical equivalent of silver in terms of the electrical standards of the Bureau. According to legal specifications the E.M.F. of a Clark cell at 15° C. is 1.434 volts. The unsaturated Weston cell was found to have an E.M.F. of 1.01954 volt. The resistance standards have been repeatedly intercompared with those of the Reichsanstalt.

In Table 10 the amount of silver deposited by one ampere hour and under the conditon that the terminal potential difference of the resistance was equal to the E.M.F. of the Weston cell, is given for the different forms.

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[1] Gore, Nature, Vol. 27, p. 327, 1882.

TABLE X.

Current in Amperes.	Richard's Type, Grams.	Richard's Type (Modified), Grams.	Large Anode Types, Grams.	Usual Type, Grams.
0.5	4.09881		4.09888	4.10154
0.5	4.09906	4.09896	4.09891	4.10098
0.5	4.09870			
1.0	4.09881	4.09881		4.10016
1.0	4.09896			4.10056
1.5	4.09897		4.09880	
	Average,	4.09888 grams.		4.10081
0.2	4.09920	4.09942	4.09952	
	4.09955		4.09985	
	4.09959		4.09962	
	4.09993			

It is seen that the agreement between these independent experiments is a surprisingly good one, the largest differences amounting to less than 1 in 10,000 in the first three types, as long as the current is equal or larger than 0.5 ampere. For 0.2 ampere the deposits are invariably greater. This is not surprising. The current in these cases was closed five hours and in spite of all precautions some of the anode liquid probably diffuses through the porous cup during that time. This explanation seems the more plausible from the results obtained in the Richards form with 0.2 ampere. In the first experiment of the four the anode liquid was removed every ten minutes, in the next two every fifteen to twenty minutes and in the last every half hour. The increase of the weight of the deposit with the opportunity of the liquid to penetrate the porous partition is apparent. The current should therefore be closed not longer than two hours, this of course depending to a certain extent upon the grain of the porous cup.

To obtain the electrochemical equivalent of silver we have to divide the deposit, 4.09888 grams, by 1.01954×3600. This gives 1.11675 mg. per coulomb; but the silver was weighed with platinum weights and therefore a correction of $+ 0.0069$ per cent. should be applied for the buoyancy of air. Thus the final result for the silver equipment is

1.11683 mg.

With a filter paper voltameter the corresponding results would be 0.048 per cent. higher or 1.11736 mg. Kahle's comparison between the E.M.F. of the Clark cell and the electrochemical equivalent of silver reduced to the same value for the E.M.F. of the Clark cell would give 1.11726 mg.

In the following table we find a comparison of the values obtained by different observers, all being reduced to same units; Carhart's values are based on two observations, Rayleigh's on only one. In the case of Perot and Fabry,[1] who used a Clark cell at 0° C., and found its E.M.F. force to be 1.4522 volts, using 1.118 as electrochemical equivalent, the difference of 0.0164 volts, given by the Reichsanstalt, has been used to reduce to 15° C., instead of the ratio given by them. Assuming the latter to be correct, the values would be identical with von Ettinghausen's.

Table XI.

Name.	Year.	Filter Paper Voltameter (mg.).	Porous Cup Voltameter (mg.).
Carhart.	1882	1.1172	1.1167
Rayleigh and Sidgwick.	1884	1.1184	1.1179
von Ettinghausen.	1884	1.1180	1.1175
Glazebrook and Skinner.	1892	1.11831	1.11778
Perot and Fabry.	1898	1.1193	1.1188
Kahle.	1898	1.11726	1.11673
Guthe.	1904	1.11736	1.11683

The large differences in these results can hardly all be due to the silver voltameter alone. No doubt the Clark cell comes in for its share.

Richards has made an attempt to reduce the absolute measurements of an electric current to the same basis and finds good agreement between the results of Lord Rayleigh and Mrs. Sidgwick, Fr. and W. Kohlrausch, Kahle and Patterson and Guthe. They all agree within 3 in 10,000 and given the mean value of 1.1175 mg.

The recent determination by van Dijk & Kunst using the filter paper voltameter, also gives 1.1175 mg. for the silver equivalent; but Pellat and Leduc obtained 1.1195 mg. or reduced to the same basis 1.1188 mg. Thus the latest determinations of the electrochemical equivalent still show a difference larger than 1 in 1,000.

Washington, D. C.

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[1] Perot and Fabry, Annal. de fac. d. sci. Marseilles, Vol. 8, p. 201, 1898.

A NEW FORM OF MAGNETIC CONTACT-MAKER.

By John C. Shedd.

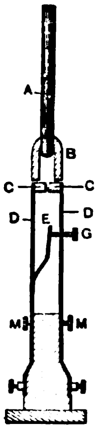
THE most common form of contact-maker is one where a platinum wire dips into a globule of mercury. This form requires adjustment each time it is used, as well as more or less frequent renewal of the mercury. The rate of the clock is intermittently affected, since the contact is thrown out when not in use.

Another form, free from some of the defects of the mercury contact, consists of a mechanical devise connected either with the upper part of the pendulum or with one of the wheels, whereby the circuit is closed each second. This form has the advantage of making it an easy matter to omit the sixtieth beat. It has the disadvantage of being inaccessible in case of repairs; it is also invisible to the student, who therefore generally fails to understand just what is going on.

Having in mind the objections to the present forms, the following form of contact was devised. It has worked so satisfactorily that I venture to describe it for the benefit of others.

Fig. 1 is taken from a drawing of the apparatus as it is mounted in the clock.

A is the lower end of the clock pendulum. *B* is a permanent magnet made up of 42 laminæ separately magnetized before assembling. The dimensions of this magnet are: height, 25 mm.; pole area, 6 x 6 mm.; distance between poles, 11 mm.; *CC* are soft iron armatures laminated and measuring 9 x 6 x 5 mm. They are mounted on springs, *DD*, which are adjustable by screws, *MM*. The contact point is at *E* and consists of a platinum point playing against a small gold plate. This is adjustable by the screw *G*. The clearance between the magnet and the armatures is about 1 mm. Magnet *B* has its plane perpendicular to the plane in which the pendulum swings.



The drawing together of the armatures at each swing of the pendulum is short and sharp. Since the action is continuous the rate of the clock, once adjusted, is not further affected; and since the action occurs at the middle of the swing the effect due to this cause is negligible.

When a signal is desired the circuit of a relay is closed. The switch for this purpose is located outside of the clock case, which need only be opened for winding the clock.

This contact has been in service for several months and has proven satisfactory in every particular. Its success has been largely due to the skill of Mr. Edward J. Hoff, instrument maker of this city, who developed the working plans of the apparatus.

Physical Laboratory, Colorado College,
March 15, 1904.

THE THERMO-ELECTROMOTIVE FORCE OF NICKEL NITRATE IN ORGANIC SOLVENTS.

By J. Frank Meyer.

IF two similar electrodes dip into an electrolyte and one of the electrodes is warmed, while the temperature of the other is kept constant, a difference of potential is in most cases produced. The magnitude of this potential difference depends on the condition of the surfaces of the electrodes, on chemical action that may result between the electrolyte and the electrodes, and on the difference of temperature.

The effect due to difference of temperature was probably first noticed by Walker[1] who as early as 1825 made some experiments by plunging a hot and a cold platinum wire into an electrolyte. His results give the direction of the flow of current for various solutions.

Faraday[2] repeated and varied the experiment, by placing the electrolyte in a U-tube, one arm of which was heated. Various metals were used as electrodes in solutions of their salts and in most cases the electrode in the hot tube was positive towards the cold one.

Lindig[3] found in 1864 that if he used copper electrodes in a solution of copper sulphate, the electromotive force decreased with increase in temperature, while for zinc in sulphuric acid no change in the electromotive force was observed for a change in the temperature. Gore (4)[4] in a series of investigations extending over many years, using nearly all available metals and solutions of their salts, showed that in most cases the electromotive force depends both on the difference of temperature of the cells, and on the concentration of the solution used as electrolyte.

Bouty's[1] results for various metals in solutions of their salts, show the electromotive force to be independent of the concentration, but directly proportional to the difference in temperature. He expresses his results by the formula $e = a + m(t' - t)$ where a is a constant and m the electromotive force for unit difference in temperature.

Carhart[2] has confirmed Bouty's results for copper and zinc in an investigation of the temperature coefficient of Daniell cells. He finds however that the electromotive force increases slightly more rapidly than the temperature.

Ebeling[3] also found the E.M.F. to rise more rapidly than the difference of temperature, and contrary to Bouty's results, that the E.M.F. is dependent on the concentration, showing maxima and minima at definite points, these points being the same for the same concentration of various electrolytes having the same acid radical.

Hagenbach[4] in two series of experiments, the one with platinum electrodes, the other with cadmium and lead amalgam, showed that with decreasing concentrations from 0.1 normal to about 0.0001 normal, the E.M.F. decreased.

Brander[5] in 1889 and Emery[6] in 1894 have also made experiments on the subject.

The salts of nickel in several of the above mentioned investigations gave very discordant results and McClellan[7] has recently shown that the electromotive force is very small, at least for temperature ranges of about 30° C.

The present investigation has to do with solutions of nickel nitrate in four different organic solvents.

APPARATUS.

The solutions were contained in two cells made by fusing side tubes into short test tubes and connecting them by a short piece of pure rubber tubing as shown in Fig. 1. *A* and *B* are the cells connected at *C*. The cells were supported by large corks in large beakers, and surrounded by water. The beakers were placed in wooden boxes and closely packed round with cotton. By these means it was possible to keep the temperature of the cells constant to 0.1° C. for several hours at a time. To heat the cells, each was surrounded by a coil of german silver wire, with leads coming through the corks and connected to a switch so that a current could be sent through either coil at will.

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[1] Bouty, Jour. de Phys , 9, p. 306, 1880; Comp. Rend., 90, p. 918, 1880.

[2] Carhart, Primary Batteries, p. 100.

[3] Ebelign, Wied. Ann., 30, p. 530, 1887.

[4] Hagenbach, Wied. Ann., 53, p. 447, 1894; 58, p. 21, 1896.

[5] Brander, Wied. Ann., 37, p. 457, 1889.

[6] Emery, Nature, 50, p. 236, 1894.

[7] McClellan, Phys. Rev., 17, p. 255, 1903.

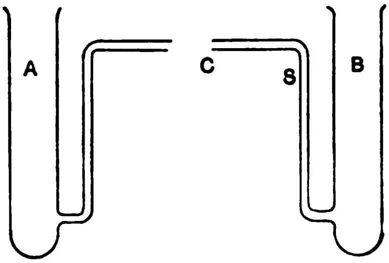

Fig. 1.

The cells were 10.5 cm. high, 2 cm. in diameter and the connecting tube *S* about 25 cm. long and 0.4 cm. internal diameter. The shape of *S* prevented convection currents between the two cells and the rubber connection at *C* allowed the cells to be easily cleaned.

Each cell was fitted with a two-hole rubber stopper, through which passed a glass tube into which the electrode was sealed, and a thermometer. The thermometers were graduated to fifths of degrees.

The electrodes finally used were long platinum wires sealed into glass tubes so that 15 cm. of wire projected below the glass tube. This wire was then coiled into a coil about 1.5 cm. long. The bulbs of the thermometers were placed at the same level in the cells with the electrodes. The symmetrical form of the cells allowed the interchange of the electrodes, and either cell could be heated, as before mentioned. The rather large internal diameter of the connecting tubes decreased the internal resistance of the cell and evaporation was prevented by the rubber stoppers. This was specially necessary in the case of acetone as a solvent, its boiling point being 56.3° C.

Various forms of electrodes were tried before the form above described was adopted. It was thought that polarization could be largely prevented by using platinum foil as electrodes. Pieces of foil 2 cm. square were rolled into cylinders and tried. Though the pieces had been cut from the same piece of foil, it was found impossible to get such large surfaces to the same condition exactly, and an initial E.M.F. of from .02 to .03 volts was always found. They were therefore not used. Platinized platinum wires were tried with the same hope,— to prevent polarization by increasing the surface. But again an initially large E.M.F. was always found. Nickel wires and nickel plated platinum wires gave no better results, though they were boiled in water, in concentrated potassium hydroxide and again in water before each trial. With the smaller surfaces of the platinum wire coils, by successive boiling in water, concentrated nitric acid and again water, and final heating to incandescence for several minutes in a Bunsen flame, it was possible to get the initial E.M.F. equal to zero. If at any time the electrodes were not alike when put into the cells, an hour's standing usually brought them to zero potential. If not, they were removed and recleaned.

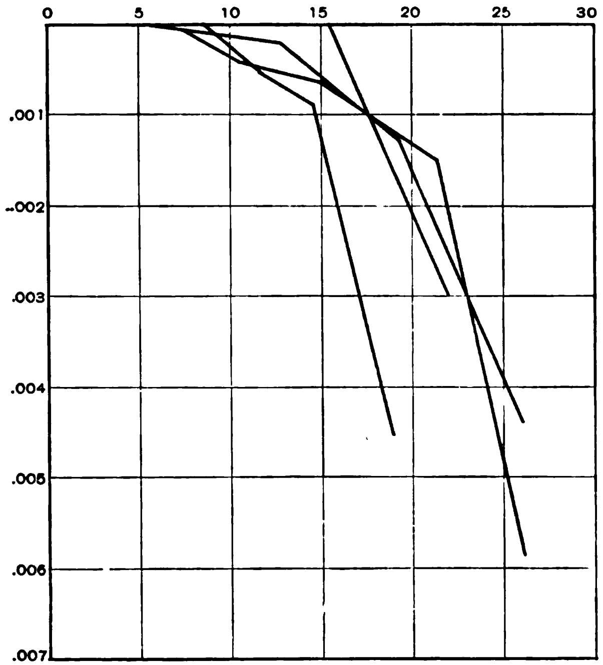

Fig. 2a.

Long platinum wires led from the electrodes and to these were fastened thin copper wires, which were in turn connected to a 4-way plug key, and thence to the potentiometer. The junctions of copper and platinum were far from the cells and were fully exposed to the air. No difference of temperature at the junctions was detected.

The potentiometer used was made by Morris E. Leeds & Co., and reads directly to .00001 volt. A Weston standard cell was used for calibration. By moving a single switch on the instrument, the unknown or the standard is put into the circuit. A Rowland D'Arsonval galvanometer was used for the measurements.

When the electrodes had been prepared, the cells were filled with the solution, electrodes and thermometers inserted, and the whole allowed to stand in the water baths until the temperatures were the same and constant. This often required an hour or more. The potentiometer was carefully balanced, the cells thrown into circuit and another balance obtained. By again shifting the double throw switch the original balance was tested. A current of about 2 amperes was then sent through the heating coil of one cell and the temperature gradually raised. Readings of the E.M.F. were taken about every 5 degrees. The following solvents were used: Acetone, absolute ethyl alcohol, aniline and pyridine. The nitrate of nickel is the only salt soluble in those organic substances, and the present paper gives results for this salt only.

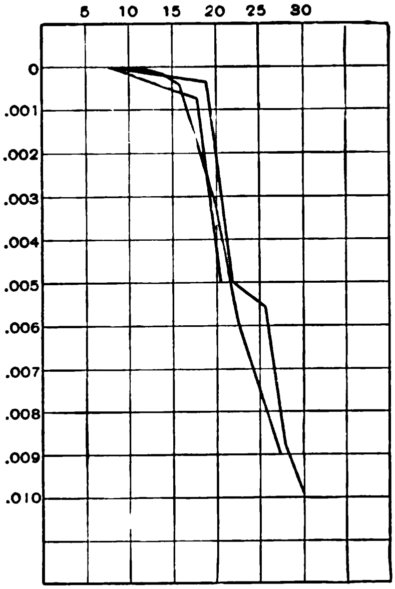

Fig. 2b.

Results.

The following tables (p. 162–3) give temperature differences between the hot and cold cell, and the corresponding electromotive force.

In the tables *To* denotes the temperature of the cold cell, which was kept constant during an experiment, *T* — *To* the difference in temperature of the cells, and $V - Vo$ the electromotive force corresponding to the temperature $T - To$. In the curves, temperature differences are plotted as abscissæ and E.M.F. in volts as ordinates. The positive sign in the tables and curves above the axis of abscissæ, denote an E.M.F. such as to cause a current to flow from the cold to the hot electrode through the external circuit.

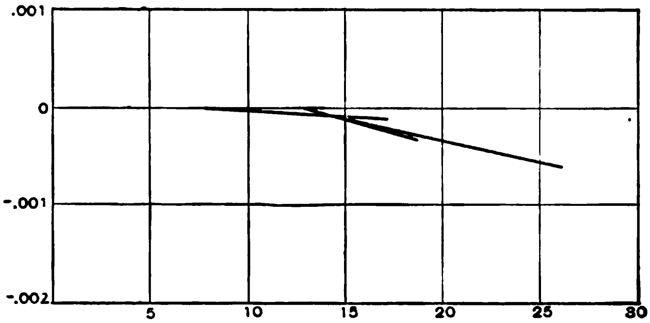

Fig. 2c.

The curves themselves simply connect points at which readings were made. It was thought better to show the variations in successive series of measurements, than to find averages and plot an average curve.

Nickel Nitrate in Acetone. — Tables I., II. and III. and Figs. 2, *a*, *b* and *c*, give results for one tenth, one twentieth, and one fiftieth normal solutions.

For the acetone solution the thermo-electromotive force is zero for nearly 10° for all concentrations used. There is an abrupt rise in the case of the tenth and twentieth normal solutions, the twentieth normal giving about .01 volt for a temperature difference of 30°. The fiftieth normal solution is nearly zero for all temperatures. A large number of curves show the same characteristics, and in all cases, the flow of current was from the hot electrode to the cold in the external circuit, denoted by the negative sign in the tables above. The difference in E.M.F. produced by changing the concentration from a tenth normal to a twentieth normal is inappreciable, showing the E.M.F. independent of the concentration.

Nickel Nitrate in Absolute Alcohol.—Tables IV. and V., and figures 3, *a* and *b*, give results.

TABLE I.

1/10 *Normal Nickel Nitrate in Acetone.*

To=18°	*Vo*=—.0006	*To*=23.8°	*Vo*=.0000	*To*=22.6°	*Vo*=.0000	*To*=20.9°	*Vo*=—.0160
T—To	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*
.0	0.0000	0.88	0.0000	0.0	0.0000	0.3	0.0100
5.7	.0000	1.4	.0000	0.4	.0000	3.1	.0000
12.8	—.0002	6.4	.0000	8.6	.0000	7.3	.0000
19.2	—.0013	10.6	—.0004	11.6	—.0005	10.9	.0000
26.0	—.0044	15.2	—.0006	14.4	—.0009	15.1	.0000
		21.6	—.0015	18.8	—.0045	21.9	.0030
		26.2	—.0059				

TABLE II.

1/20 *Normal Nickel Nitrate in Acetone.*

To=19.3°	*Vo*=.0000	*To*=14.9°	*Vo*=.0000	*To*=15.5°	*Vo*=.0000	*To*=22°	*Vo*=.0000
T—To	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*
3.5	0.0000	2.1	0.0000	2.8	0.0000	2.0	0.0000
4.7	.0000	4.6	.0000	6.1	—.00003	4.6	.00005
7.7	.0000	7.6	.0000	9.3	—.00008	7.0	.00005
11.4	—.00005	10.6	—.0002	14.3	—.0004	11.0	—.0006
13.7	—.0002	13.7	—.0004	18.8	—.0004	14.0	—.0008
15.7	—.0004	17.4	—.0007	20	—.0020		
19.7	—.0030	20.4	—.0050	21.7	—.0050		
22.6	—.0060			25.6	—.0056		
27.3	—.0090			27.5	—.0087		
				30	—.0099		

TABLE III.

1/50 *Normal Nickel Nitrate in Acetone.*

To=24.8°	*Vo*=.000	*To*=20°	*Vo*=.0000	*To*=26.6°	*Vo*=.0000
T—To	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*
3.2	0.0000	6.2	0.0000	4	0.0000
7.5	0.0000	12.5	0.0000	7.6	0.0000
8.6	0.0000	18.3	−0.00037	13.9	−0.00006
13.7	−0.00006	24.0	−0.0005	17.2	−0.0002
17.1	−0.0001			25.9	−0.0006

For the fiftieth normal solution only in one instance in many trials was any thermo-electromotive force obtained—a temperature difference of 30° gave —.0005 volts. In all other trials no E.M.F.

TABLE IV.

1/10 *Normal Nickel Nitrate in Absolute Alcohol.*

To=21.4°	*Vo*=.0000	*To*=12°	*Vo*=.0000	*To*=20°	*Vo*=.0000	*To*=15.8°	*Vo*=.0000
T—To	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*
3.1	0.00005	4.0	0.00006	3.3	0.00009	6.2	0.00006
8.8	−.00034	7.4	−.00009	7.8	−.0002	9.2	−.00009
12.6	−.0004	15.4	+.00016	16.1	−.0002	25.5	−.0005
16.2	−.00043	22.7	+.00039	22.8	-.0004	33.2	+.0036
19.8	−.00043	27.8	+.0013			40.2	+.0092
25.0	−.0002	33.2	+.0044				
29.4	+.0040	38.0	+.0056				
32.0	+.0052	42.5	+.0099				
34.4	+.0059						

TABLE V.

1/20 *Normal Nickel Nitrate in Absolute Alcohol.*

To=4°	*Vo*=.0000	*To*=20.3°	*Vo*=.0000	*To*=8.6°	*Vo*=.0000
T—To	*V—Vo*	*T—To*	*V—Vo*	*T—To*	*V—Vo*
6	0.0000	3.6	0.0000	4.4	0.0000
10	0.0000	11.7	+.0005	10.0	0.0000
12	0.0000	16.3	+.0045	15.4	+.0003
17.3	+.0004	21.5	+.0086	21.6	+.0036
21.0	+.0050	27.1	+.0175	24.6	+.0098
25.0	+.0096			31.6	+.0131
29.0	+.0160				

could be detected. The potentiometer was much reduced in sensibility. Shifting the keys so as to change the balance by 3 or 4 tenths of a volt hardly deflected the galvanometer. The resistance of the solution is evidently large.

The curves for the tenth normal solution are quite regular. For temperature differences of about 25° the hot electrode is positive, but above 25° there is a change in sign, the E.M.F. rising very rapidly. The twentieth normal solution is similar in behavior, the change from "hot to cold" to "cold to hot" coming at about 15° The E.M.F. becomes large in both cases when compared with acetone or a water solution.[1]

It is probable that if it were possible to eliminate all polarization effects, the heated electrode would be positive throughout the entire range of temperatures.

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[1] McClellan, loc. cit.

The organic bases, aniline ($NH_2C_6H_5$) and pyridine (C_5H_5N) have also been used as solvents. Nickel nitrate is readily soluble in aniline, but only very slightly soluble in pyridine.

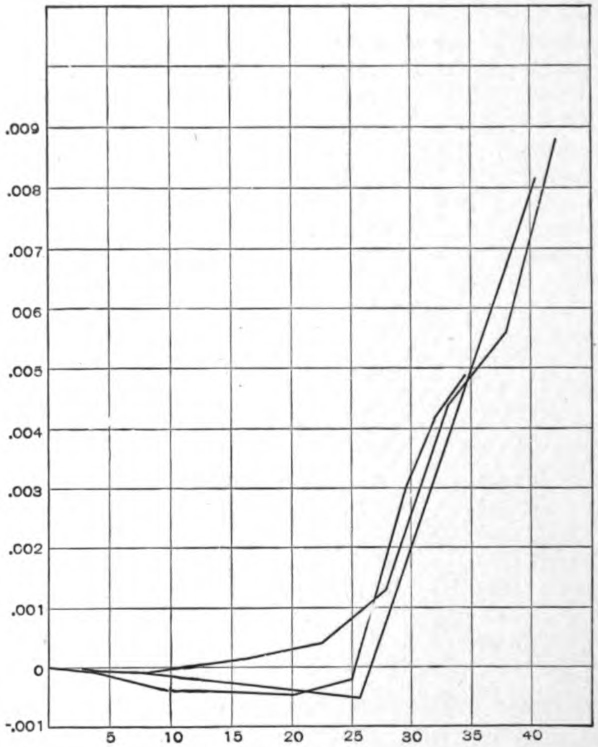

Fig. 3a.

A saturated solution in pyridine gave altogether negative results for temperature differences of nearly 100° C.

Aniline solutions, both one tenth and one twentieth normal, similarly gave negative results for wide ranges of temperature—from about 3° C. to 80° C.

CONCLUSIONS.

The thermo-electromotive force is independent of the concentration from one tenth to one fiftieth normal.

Polarization effects cannot be avoided and they are large enough to mask the thermal effects in the measurements. A method that would employ no current is the only one that seems entirely satisfactory for measuring the very small differences of potential. Only by the very greatest care can concordant results be obtained by the potentiometer method, and it is very desirable to check the results obtained by another method not using current.

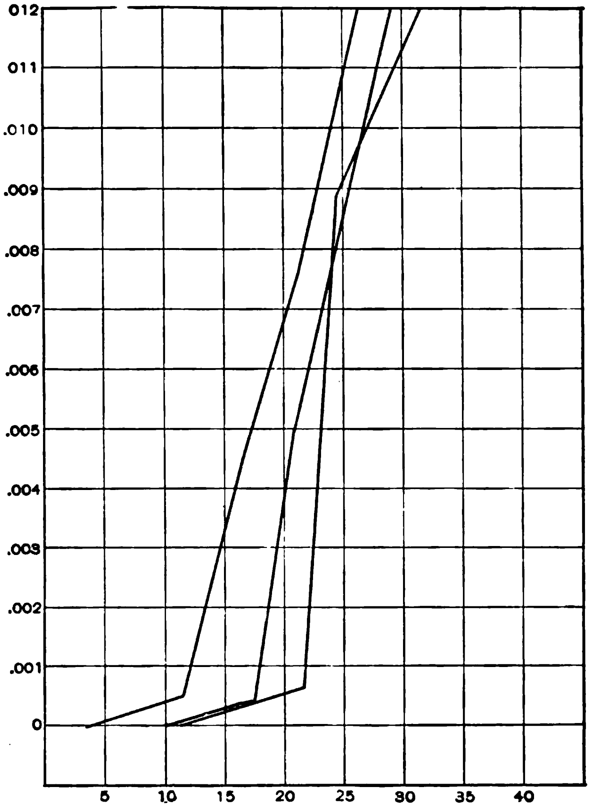

Fig. 3a.

It also remains a question of interest to measure the degree of dissociation in these organic solvents by determining the molecular conductivity. A possible relation between thermo-electromotive force and the dissociation might thus be quantitatively shown.

Randal Morgan Laboratory of Physics,
University of Pennsylvania, April 2, 1904.

ON THE DIFFERENTIAL TELEPHONE.

By H. Ho.

BEING of the opinion that the differential telephone must take an important place in any electrical laboratory, and that I was the first to show its utility in various measurements, I could not but be pleased to find that Messrs. Duane and Lory have recently employed the differential telephone in the comparison of inductances. An article describing the work of Messrs. Duane and Lory appeared in the April number of the Physical Review, in connection with which I take the liberty of making the following remarks:

The connection adopted by the authors is what may be called multiple connection, the inductances in series with the telephone coils being connected in multiple as represented by Fig. 1, in which m_1 and m_2 are the two coils of a differential telephone, in series with the known variable inductance L and the unknown inductance X respectively, and forming two parallel circuits between the points a and b, at which an alternating E.M.F. is applied. The inductance and the magnetic effect of the telephone coils and the total ohmic resistances are made exactly equal in both arms, and then L is varied till no sound is heard in the telephone, in which case $X = L$.

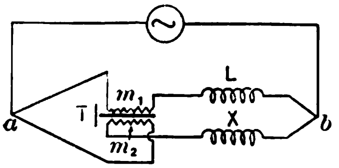

Fig. 1.

The principle is perfectly correct, and in fact this is the simplest method of using a differential instrument, at once suggested by the well-known method of comparing resistances by means of a differential galvanometer.

Before proceeding further, I have to speak of the method of winding a differential telephone. Messrs. Duane and Lory seem to have had some trouble in adjusting the two coils to be exactly equal in their inductance, resistance and magnetic effect. The difficulty can be easily avoided by adopting my method of winding. It is simply to wind two similar wires in twin, that is always side by side and as close together as possible from beginning to end. It is needless to say that the two coils thus made are perfectly equivalent in their magnetic effect, and the ohmic resistance also generally requires little adjustment; but the most important property of the coils thus made is that in the state of balance, that is when equal and opposite currents are flowing through these coils, their inductances are completely balanced by the mutual inductance and they act as non-inductive resistances. This latter property allows the coils to be treated in calculation as if they were simple non-inductive resistances, and allows them to be shunted with non-inductive resistance, in which case the ordinary calculation for shunt connections may be employed. This condition is vital in many applications of the differential telephone and the telephone used in my methods is always wound in this way.

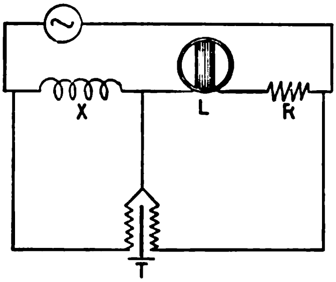

Fig. 2.

I will now explain why I did not recommend the method used by Messrs. Duane and Lory but adopted what may be called series connection, that is the method in which the inductances are connected in series instead of in multiple as in Fig. 2.

Suitable non-inductive resistance R is added either to L or to X, as the case requires, to make them perfectly equal in resistance; this adjustment is done with an ordinary mirror galvanometer. The differential telephone I is connected with proper consideration of the relative direction of currents in the coils, to the terminals of the known and the unknown inductance as shown in the figure. $\sim$ is a source which supplies a proper amount of alternating current. L is varied till no sound is heard in the telephone, in which condition $X = L$.

The principle is analogous to the ordinary method of comparing resistances by the fall of potential, and I will not enter into its explanation.

If, for example, *X* is below the range of *L*, then we make the resistance of *Lm* times larger than that of *X*, and also make the resistance of the telephone coil corresponding to *Lm* times larger than that of the other coil, by addition of suitable non-inductive resistances. In this case, the ratio of *X* and *L* that give balance is $1/m$, as may be easily proved. If *X* is above the range of *L*, we have only to do the reverse.

Now to come to the comparison of these two methods of connection. The division of current between the two parallel arms of the multiple connection is determined by the ohmic resistance and the inductance of the arms; the same things also determine the division of E.M.F. in the series connection. Therefore, the effect of the ohmic resistance of the inductive coils to be compared is to decrease the sensitiveness of both methods, the ideal favorable case being its entire absence. Now, in the multiple method, the resistance of the telephone coils, which is usually very large, is added to the resistance of the inductive coils to be compared, thus decreasing the sensitiveness of method, while the resistance of the telephone has nothing to do with the division of E.M.F. in the series method. In short, we may say that the multiple connection is in no case superior, and that in most ordinary cases it is inferior to the series connection. Messrs. Duane and Lory report that they obtained much more accuracy than I reported; but this is due, I think, to their experimental skill, and not at all to the superiority of method. I believe that they would have obtained still better results if they had adopted the series connection.

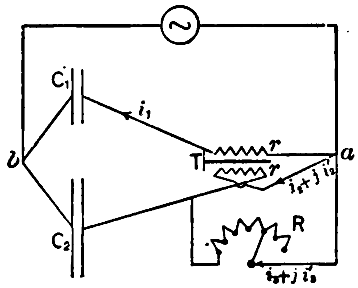

Fig. 3.

I add here a short description of the comparison of capacities by means of the differential telephone.

For this we must necessarily adopt multiple connection, as shown in Fig. 3, in which *R* is a variable resistance, preferably with sliding contact, which shunts one of the telephone coils in series with the larger capacity. *R* is varied till no sound is heard in the telephone *T*, in which condition we have

$$\frac{C_2}{C_1} = \frac{r+R}{R}.$$

This can be proved as follows: Let L_1 and L_2 denote the inductances of the telephone coils and M the mutual inductance between them, and let i_1, $i_2 + ji_2'$ and $i_3 + ji_3'$ be the currents through the two coils and R respectively as indicated in the figure, and $e + je'$ the potential difference between the two points a and b. Then we get the following three equations:

$$i_1\left(r - j\omega L_1 + j\frac{1}{\omega C_1}\right) + (i_2 + ji_2')j\omega M = e + je',$$

$$(i_2 + ji_2')(r - j\omega L_2) + i'j\omega M = (i_3 + ji_3')R,$$

$$[i_2 + i_3 + j(i_2' + i_3')]j\frac{1}{\omega C_2} + (i_3 + ji_3')R = e + je'.$$

Since in our case, $L_1 = L_2 = M$, we may put $\omega L_1 = \omega L_2 = \omega M = x$. Thus, putting for brevity, $1/\omega C_1 = x_1$ and $1/\omega C_2 = x_2$, the three equations become,

$$ji_1x_1 = e + je',$$

$$(i_2 + ji_2')(r - jx) + ji_1x = (i_3 + ji_3')R,$$

$$[(i_2 + i_3) + j(i_2' + i_3')]jx_2 = e + je'.$$

From these we get,

$$i_2' = \frac{i_1x_1xR - i_1xx_2(R+r)}{x^2x_2 + x_2(R+r)^2},$$

$$i_2 = \frac{i_1x_1xR - i_2'x^2x_2}{x_2(R+r)x}.$$

If we take such a value of R that

$$\frac{r+R}{R} = \frac{x_1}{x_2} = \frac{C_2}{C_1},$$

we get

$$i_2' = 0 \text{ and } i_2 = i_1,$$

By using an E.M.F. about 500 volts I could measure capacities less than one millionth of a microfarad in comparison with a parallel plate air condenser, in the laboratory of the Tokio Imperial University.

For the measurement of resistances, either series or multiple connection may be adopted, according to circumstances, and it was even found possible to measure a conductance and a capacity in parallel at the same time.

The agreeable sharpness of balance can never be realized until one has actually used a differential telephone, and I firmly believe that when the real value of the differential telephone is known it will become one of the most necessary instruments in many branches of electrical measurement.

A WAVE MACHINE.

By A. P. Carman.

MACHINES to illustrate wave motion have been divided into two kinds: first, those which show the forms of the wave at different instants but without any true wave motion; and second, those which have a real dynamical connection between the moving parts so that there is a true wave motion. Most of the wave apparatus found in physical cabinets belongs to this first class. Such are the various forms of apparatus used in the projection lantern, and also the devices with eccentrics for imitating wave motion. The machine described here belongs to the second kind, the elastic connection between the parts being by means of a spiral spring. It resembles in this respect the apparatus of Weinhold[1] and also that of Vincent.[2] The construction can be seen from the figures. There are sixty steel rods, each sixty centimeters long carrying on each end a lead ball about one centimeter in diameter. Each rod is supported and balanced in gimbals, so that free motion in every direction is secured. These gimbals are arranged uniformly in a horizontal frame three meters long. The lead balls on

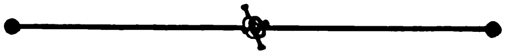

Fig. 1.

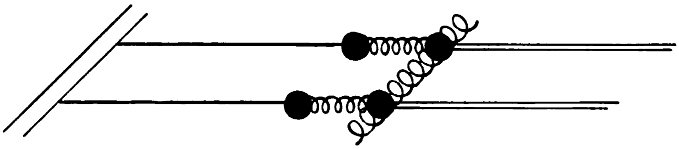

Fig. 2.

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[1] Weinhold's Physicalische Demonstration, p. 206.

[2] Phil. Mag., December, 1898.

one side are connected by a spiral spring made of fine steel piano wire. The spring is stretched horizontally on the same level as the gimbals, and the balls are spaced uniformly along the spring, being tied in place by fine binding wires. Each rod is balanced as regards gravity, so that its motion is determined wholly by the elastic connection of the spring with the neighboring rod and ball. It was found easy to make gimbals with very little friction, so that a motion travels along the whole line of balls and is reflected back. With this apparatus both transversal and longitudinal waves are shown, and the velocity of propagation is slow so that the eye can follow the motion without difficulty. Standing waves are also beautifully shown. To start the waves, the hand has been used, but it has been suggested to use a metronome or other mechanical means and that may be done.

The wave apparatus of Mr. J. H. Vincent, referred to above, was devised to illustrate Helmholtz's theory of dispersion. Mr. Vincent has a series of lead balls suspended by long threads (271 cm. long), and connected by a horizontal spiral spring. On this first series of pendulums there is suspended a second series of pendulums to represent the medium. He then shows how beautifully and completely the Helmholtz dispersion formula can be illustrated quantitatively by the model. It occurred to the writer that the apparatus described in this paper could be used for the same purpose. For this it is necessary to have a second series of lead balls which should be spaced in the same way as the balls on the wave model, and all balls of this second series should have the same natural period of vibration. This was accomplished by fixing a lead ball on the end of a stiff steel piano wire, and clamping the other end of the wire so that the spring pendulum thus formed was nearly horizontal. The series of these pendulums was arranged uniformly along a horizontal metal bar, and each ball was connected by a light spiral spring with the corresponding ball of the wave model. The period of the second set of balls could be easily altered by adjusting the length of the clamped steel wires. The wires used were about one millimeter in diameter and twenty-five centimeters long, and the lead balls were 1.5 cm. in diameter. As thus arranged the apparatus served well for illustrating the Helmholtz dispersion theory.

The figure explains the construction described above. The machine as constructed in the workshop of the department is mounted on a frame made of gas-pipe and is convenient to move about the laboratory. It serves better than any apparatus with which the writer is acquainted the purposes of lecture room wave apparatus.

Physical Laboratory,
University of Illinois, June, 1904.

NEW BOOKS

Electric and Magnetic Circuits. By ELLIS H. CRAPPER. Pp. i + 377. Longmans, Green & Co, New York, 1903.

The scope of this book is indicated in the preface as follows: "This, the introductory volume of a treatise on electrical engineering, deals with the fundamental principles of electricity and magnetism, and explains fully all the essential relationships of electric and magnetic circuits met with in continuous current working, and it is hoped that the subjects of generation, transmission, distribution and application of electrical energy may receive full and adequate treatment in the succeeding volumes without requiring so much introductory matter, devoted to electrical engineering principles, terms and definitions, as is customary in many of the standard technical treatises." The subject is treated with great care and accuracy under the following headings: Practical electrical units; electrical circuits and electrical effects; principles of distribution and design of mains; magnetism; the magnetic circuit; coil-winding design; the generation of electrical pressures; types of dynamos; the electric motor; efficiencies and dynamo calculations; systems of electrical units; useful tables and constants. The author's aim to acquaint the student with the quantitative side of the subject is certainly commendable, but seems overdone.

The treatment throughout is academic, and this is particularly true of the six or seven hundred problems which form an important and characteristic part of the book. Thus on p. 281 "In a shunt dynamo $r_a = 0.013267$ ohm, $r_{sh} = 32$ ohms. If the exciting current is 3.5 amperes, and its electrical efficiency is 96 per cent when $R = 0.7$ ohm, determine the E. M. F. generated." The answer is given as 114.1692 volts. If the data is assumed, one wonders why this particular value of armature resistance was selected. On the other hand if this be a real resistance, one marvels at the accuracy of the determination. It would seem that some method must be devised to have the armature run without heating to make it possible to compute the E. M. F. to one *ten thousandth of one per cent.*, an accuracy a thousand times greater than can be practically obtained. It is important for an engineer to know how far his figures are significant and such fictitious exactness leads the student in the wrong direction. Problems on the same page deal with 312.589 watts and 93.303 H. P. As horse power is made dependent upon the "force equivalent to the weight of one pound," the latitude and altitude of the dynamo in question, or some means of determining the force of gravity, must be included in the problem to determine horse-power to any such accuracy.

The author hopes the problems will appeal to the practical man. One who perseveres in working them all will certainly become a skilled arithmatician. F. B.

A Handbook for the Electrical Laboratory and Testing Room. By J. A. Fleming. Vol. II., pp. 1 + 616. London, Electrician Printing and Publishing Company, 1903.

Dr. Fleming has brought together much valuable material, arranging it in five chapters of about equal length as follows: (1) Measurement of electrical quantity and energy; (2) measurement of capacity and inductance; (3) photometry; (4) magnetic and iron testing; (5) dynamo, motor and transformer testing. Of these chapters the first four are the most valuable. The second chapter gives an admirable and perhaps the most complete text-book treatment of capacity and inductance measurements. Although, as is obvious, much of this cannot be new, the reader feels the benefit of the author's personality and laboratory experience.

Thus after a discussion of methods of Mawxell, Rayleigh, Anderson, Ayrton and Perry, etc., we read "After a wide experience of the various methods which have been proposed for the measurement of inductance, the author has found no method which is better adapted for the everyday purposes of the electrical laboratory than a combination of the Anderson method with the use of a battery and galvanometer circuit interrupter as proposed by Ayrton and Perry to increase its sensitiveness. The double interrupter, as made by most instrument makers, gives, however, a good deal of trouble with the contacts." After this a double-current contact-maker is described which has been found by Dr. Fleming to be satisfactory.

The chapter on dynamo, motor and transformer testing is less adequate and complete. The generally accepted methods of determining transformer regulation and efficiencies without load are not given; nor the short circuit tests of alternators. The testing of alternating current motors is scarcely more than touched upon; rotary converters are ignored. The testing of electrical machinery does not constitute therefore a strong part of the book.

The book as a whole contains much valuable and interesting material and may be read with profit. In many ways it is a treatise rather than a manual or handbook. Particularly valuable are the copious references to original sources.

F. B.

Ready Reference Tables. By Carl Hering. New York, John Wiley & Sons, 1904. Pp. xviii + 196.

The first volume of Mr. Hering's manual contains conversion factors of numerous units used in measurements of every kind together with a wealth of data bearing upon the subject. In the selection and arrangement of his materials Mr. Hering has shown excellent judgment. His tables of units of each class are, for example, divided into two parts. The first contain the *usual* units with the accepted precise relation and the logarithm of the ratio to seven places; together with an approximate value. Unusual and obsolete units, and those used in special trades are classified by themselves under the heading of the countries to which they belong. Great pains has been taken to secure authentic values for the important fundamental units of length, mass, etc., and the list of conversion factors given under each is sufficiently large to serve the purposes of almost every form of computation. The tables of conversion factors are preceded by a table of symbols and abbreviations, by an introduction dealing with the relation of units and physical quantities and by a table of the physical quantities in common use, with dimensional formulæ, name of the c.g.s. and practical unit, etc. A very full index of ten pages greatly adds to the convenience and usefulness of this little volume. The book is printed on thin paper with flexible covers.

E. L. N.

The Mechanical Engineer's Reference Book. By H. H. Suplee. Philadelphia, J. B. Lippencott Company, 1904. Pp. l + 834.

The arrangement of this book is particularly convenient, and the material on the whole is well selected. It is divided into sections, each provided with a thumb index, as follows: mathematics, mechanics, materials of engineering, strength of materials, machine design, heat, air, water, fuel, steam, steam boilers, steam engines, internal combustion motors, electric power, the cost of power, works management.

The work is intended to be a successor to Nystrom's pocket book, now out of print. The material is selected with reference to its usefulness to the mechanical engineer, subjects pertaining to mechanical engineering being treated fully and adequately, allied subjects more briefly. Some tables, as those relating to steam, are given in metric as well as in British units. The section on electric power is unimportant, consisting chiefly of wiring data, underwriters' rules and the standardization report of the American Institute of Electrical Engineers. The sections on mathematics (233 pp.), mechanics, air, heat and water give much material useful to the physicist. The book should take high rank among hand-books.

F. B.

Volume XIX. *September, 1904.* *Number 3*

THE
PHYSICAL REVIEW.

TEMPERATURE MEASUREMENTS TO 1600° C.

BY ARTHUR L. DAY AND E. T. ALLEN.

IN a recent study of the isomorphism and thermal properties of the feldspars[1] in the laboratory of the U. S. Geological Survey, the authors prepared certain apparatus for generating, regulating and measuring high temperatures, which has proved easy of manipulation and uncommonly exact and which may have a wider interest. Through the courtesy of President Kohlrausch and Professor Holborn, of the Reichsanstalt, we were also enabled to obtain unusually direct comparisons with the gas thermometer scale of that institution, and to establish this scale in a conveniently accessible form for use in this country.

It is not our purpose at this time to enter upon an extended discussion of methods or principles of temperature measurement in the upper ranges. This has been done elsewhere,[2] and the difficulties to be dealt with have also received considerable attention. We desire merely to describe the application of the Reichsanstalt high temperature scale with such detail as may be necessary to make it serviceable and with sufficient data to enable measurements of the order of accuracy of the scale itself to be made without the trouble of the inconvenient official calibration and incidental loss of time.

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[1] The entire investigation will appear in a few months as a professional paper of the U. S. Geological Survey.

[2] See Le Chatelier and Boudouard, "High Temperature Measurement." (Translated by Burgess.) Ludwig Holborn and Arthur L. Day, "Ueber das Luftthermometer bei hohen Temperaturen." Ann. d. Phys. u. Chem., 68, p. 817, 1899. (Translated, Am. Jour. Sci., 4, 8, p. 165, 1899.) Ann. d. Phys., 4, 2, p. 505, 1900. (Translated, Am. Jour. Sci., 4, 10, p. 171, 1900.)

By the same authors: "Uber die Thermoelectricität einiger Metalle." Sitzungsber. Berl. Akad., p. 691, 1899. (Translated, Am. Jour. Sci., 4, 8, p. 303, 1899.)

And many others.

In the ordinary routine of temperature measurement in the laboratory it is rarely necessary to refer back to the fundamental temperature scale — the errors at ordinary temperatures are for most purposes insignificant — but in pyrometry, temperature is almost without significance until some specific reference to fundamental measurements is made.

The general plan of the apparatus used in these determinations may be assumed to be fairly well known. It is the same in all essential particulars as that used by Holborn and Day[1] in establishing the high temperature scale with the gas thermometer at the Reichsanstalt. And yet it is plain that such a scale requires some care in the transplanting, particularly as the authors were without a gas thermometer and were therefore not in position to make direct comparisons with the gas scale.

The temperatures were measured with thermo-elements exclusively. We obtained from Dr. Heraeus (Hanau, Germany) a set of four elements cut successively from the same roll of wire, which, when joined together, proved to be identically alike in their readings over the range of temperature covered by the gas scale of the Reichsanstalt (250° to 1150° C.), within the limits of observation error. Through the courtesy of Professor Holborn these were taken to the Reichsanstalt and measured in the original melting-point furnace with the same elements in terms of which the gas thermometer scale had been expressed, and five careful comparisons made. These were the melting points of the pure metals, cadmium (in air), zinc (in air), antimony (reducing atmosphere), silver (reducing atmosphere) and copper (in air). A fortunate circumstance made it possible to send these carefully calibrated elements to Washington by messenger, which made it certain that they suffered nothing in transit.

The elements were then further compared in an electric furnace, which will be described below, and the melting points of the same group of metals again determined in our laboratory. The metals used, however, were from other sources than those which had served for the calibration at the Reichsanstalt. When this test was finished, we were able to assure ourselves that, although all the constants in the measuring apparatus — thermoelements, resistances, standard cells, metals, etc. — had been changed in the transfer from the Reichsanstalt to the Geological Survey at Washington, the aggregate error nowhere exceeded 1° over the entire range from 250° to 1150°. It will be remembered that 1° was about the accuracy which the standard gas thermometer showed at 1000°. Our thermoelectrical system is therefore now doubly established — (1) by direct comparison and (2) through an independent series of metal melting points — upon the gas thermometer scale of the Reichsanstalt within the limits of error of the latter, and can be verified at any time with the help of two of the elements which have been laid aside for this purpose, or by means of the melting points of the metals. The scale is therefore permanent.

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[1] Ludwig Holborn and Arthur L. Day, Am. Jour. Sci., 4, 8, 165, 1899.

As the introduction of the standard high temperature scale of the Reichsanstalt into this country and its establishment by proper fixed points may be a matter of considerable interest to other investigators, some further details regarding the metals chosen for these fixed points are added. We tried to find metals which should not only be of the purity necessary for such standards, but which should be easily obtainable in uniform quality. With four of the five metals of the Reichsanstalt series — cadmium, zinc, silver and copper — no difficulty was experienced, but we were not able to find satisfactory antimony in this country. This need not prove an obstacle, for the four points mentioned will serve most purposes without a fifth, while if the needs of an experiment are so exacting as to require an intermediate melting point, antimony can be imported from Kahlbaum of Berlin without great delay or excessive cost, in the same purity as that originally used at the Reichsanstalt.

The cadmium and zinc in our series were taken from the regular listed chemicals of Eimer and Amend (zinc, "C.P. in sticks"; "cadmium, metal sticks"), the silver was the well-known test silver of the Philadelphia Mint laboratory, while the copper was also from Eimer & Amend ("C.P. copper drops, cooled in hydrogen").

Careful analyses of samples of the cadmium, zinc and copper follow:

Zinc.		Cadmium.		Copper.	
Eimer & Amend's "c. p. in Sticks."		Eimer & Amend's in Sticks.		Eimer & Amend's "c. p. Drops, Cooled in Hydrogen."	
As	none.	As	none.	Te and Se	none.
Cu	trace.	Cu	trace.	Sb	trace?
Pb	.0412%	Pb	.0860%	As	none.
Cd	.0021	Zn	trace.	Bi	none.
Fe	.0053	Fe	.0025	Ag	.018%
Co and Ni	none.	Co and Ni	none.	Pb	.001
S	.005	S	.0005	Co and Ni	none.
	.0005			Zn	.010
				Fe	.011[1]
Total Impurities, .0491%		Total Impurities, .0890%		Total Impurities, .040%	

It was not deemed necessary to make an analysis of the silver as we were assured that it contained no impurity which could be quantitatively determined.

The melting temperatures of cadmium and zinc are relatively low and those of silver and copper comparatively high on the gas scale, with a long interval between, so that it sometimes becomes very desirable to have an intermediate point. The two melting points which are most conveniently located are aluminium and antimony. Aluminium, on account of its low density, and perhaps because it has been less successfully purified than the other metals, does not give a sharp and satisfactory melting point. The melting point of Kahlbaum's antimony, of which a recently published analysis is reproduced here, serves this purpose excellently. It rarely solidifies without considerable undercooling, but the point to which the temperature rises after crystallization begins is sensibly identical with the melting point.

Antimony (Kahlbaum, Berlin).[2]

Fe	.012%
Cu	.004
Pb	.003
	.019%

The C. P. antimony obtained from Eimer & Amend and from Merck & Co., in a careful analysis for which we are indebted to

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[1] This figure is doubtless somewhat too high.

[2] Fritz Henz, Inaugural Dissertation, Zürich. Published Leipzig, 1903.

Dr. W. F. Hillebrand, of the Geological Survey, each showed about one per cent. of the sulphide still present and traces of other impurities. The melting temperatures of these varied under different conditions as much as fifteen degrees and were totally unsuited to this work.

Inasmuch as the melting points of these metals were determined with thermoelements which Professor Holborn had just calibrated with the metals in use at the Reichsanstalt for this purpose, a comparison of the values obtained will show the accuracy with which one may reproduce the Reichsanstalt scale entirely from local sources :

	Reichsanstalt.	Day & Allen.	Difference.
Cadmium.	321.7° (Kahlbaum)	321.7 (Eimer & Amend).	0.0°
Zinc.	419.0 (Kahlbaum)	420.0 (Eimer & Amend.)	1.0
Silver.	961.5 (Gold u. Silber Scheideanstalt).	962.2 (Philadelphia Mint).	0.7
Copper (in air).	1064.9 (Haddernheim Kupferwerk).	1065.3 (Eimer & Amend).	0.4
Copper (reducing atmosphere).	1084.1 (Haddernheim Kupferwerk).	1083.6 (1) (Eimer & Amend).	0.5

For the method of extrapolation of the scale and further information regarding the use and accuracy of thermo-elements at these temperatures, reference is made to the papers of Holborn and Day already cited.

For everyday use, four more elements were prepared and calibrated in the same way. Of these, two are of the usual form (Fig. 1) and two are of a new design (Fig. 2), which has proved very effective in the determination of the melting points of non-metallic substances. It will be seen from the diagram of the insulated element that the hot junction is protected from the melting charge by a casing of platiniridium (0.1 mm. thick) and by a protecting tube of refractory Berlin (Marquardt) porcelain (1.5 mm. thick). Very early in our experiments upon the mineral silicates we became

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[1] A single determination with one element, all others are mean values with two or more elements.

aware that the conductivity of these materials for heat would be much poorer than in similar charges of metal. Furthermore, the charge of mineral which the furnaces could carry was only one fourth to one third as great as the metal charges used in the calibrations because of the great difference in specific gravity and the limited space which could be heated to a fairly uniform temperature. For these reasons the changes of state would be less sharply marked upon the heating and cooling curves than metal melting points, and it was feared that the readings of the protected element might prove too high or too low through inability to take on the temperature of the surrounding mass promptly. It was to discover and obviate this possible source of error that the form of thermo-element indicated in the adjoining diagram was devised. It really amounts to nothing more than the ordinary form of platinum — platin-rhodium element with the platinum wire insulated from the other by a very slender porcelain (Marquardt) tube and the platin-rhodium wire broadened out and wrapped around this tube like a cap over the portion which dips into the charge. The hot junction (*a*) is then the lower extremity of the cap where the platinum wire emerges from its insulating tube and is welded inside the platin-rhodium cap.

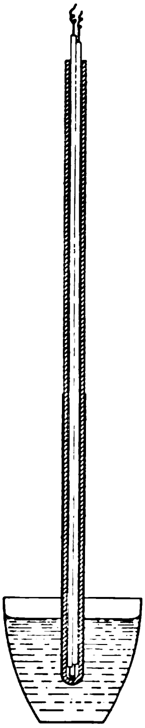

Fig. 1.

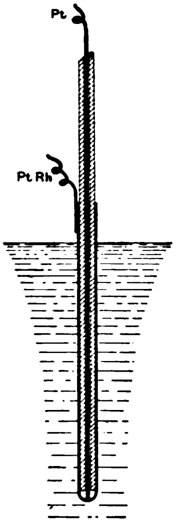

Fig. 2.

This form was dictated entirely by experience to meet conditions where an exposed element might be necessary or desirable. The wires of an ordinary element, if imbedded without protection, are rather frail for the wear and tear of breaking or drilling mineral charges out of the crucibles after the measurements, and they can not be strengthened without introducing a greater error through the amount of heat conducted away from the junction than the one which it is desired to obviate.

It has furthermore been the almost invariable experience of one of the authors[1] that when an element, through exposure to combustion products or otherwise, no longer gives normal readings, the seat of the trouble lies in the five or six centimeters of the *platinum* wire immediately adjacent to the hot junction, and not in the alloy. The pure platinum sometimes seems to absorb enough volatile or other contact products, when unprotected in a furnace at very high temperatures, to alter both its resistance and its thermo-electric potential.[2] Changes of this kind are not serious when a number of control elements are constantly available, and they are usually permanently corrected by half an hour's glowing in the open air at full white heat. The glowing must be done by passing a suitable current through from end to end, and not with a Bunsen burner or gas blast.

In event of a serious accident involving an exposure of the element which can not be corrected by glowing, cutting out the exposed portion of the *platinum* wire and reconnecting will almost always restore the normal readings.

The new form of element seeks to avoid both these difficulties, it offers the advantage of an exposed junction without exposing the platinum wire, and by making the platin-rhodium cap project but little above the surface of the melting charge, it avoids excessive loss of heat by conduction away from the hot junction. In fact in this latter particular the new form enjoys a distinct advantage over the usual form of heavily protected element. It has the disadvantage of being more frail to handle, but there is little danger of anything more serious happening than the breaking of the porcelain tube, which is readily replaced.

These elements are calibrated in metal baths like the others by enclosing in a porcelain protecting tube.

The furnace, in plan, differed but little from that in use for melting-point determinations at the Reichsanstalt. In the working out, two important changes were introduced, in order to enable it to reach the higher temperatures of the mineral melting-points. A more refractory and better insulating material was substituted for fire clay in the hotter parts and the coil was wound on the inside of the oven tube instead of outside. The latter involved some little mechanical ingenuity and skill in winding, but the gain in economy and in the rapidity with which changes could be effected or constant conditions established more than repaid any additional labor in preparation.

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[1] Day.

[2] Holborn & Day, Am. Journ. Sci., 4, 10, 171, 1900.

A diagram of the furnace in section is shown in Fig. 3. It could be used for any temperature up to 1600° C. without any difficulty or especial precautions, and could be regulated to maintain a constant temperature at any particular point for long periods of time.

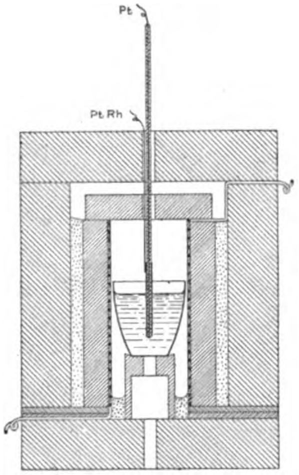

Fig. 3.

The coil, which was obtained from Dr. Heraeus, was of platin-iridium wire (90 parts Pt, 10 parts Ir), 1.5 mm. in diameter, and required about 3,000 watts to maintain a constant temperature of 1600° C. The furnace was carried at times on a 110-volt direct current street main, but accurately constant temperatures could not be depended upon without a storage battery.

The insulation in these furnaces was so perfect that shutting off or reversing the heating current at the highest temperatures did not produce a quiver in the galvanometer to which the thermo-element was connected, although the sensitiveness of the system was such that a leakage from the heating current into the thermo-element, amounting to a single microvolt (corresponding to less than 0.1°) at 1600° would have caused a displacement of about twenty divisions on the scale.

The thermo-electrical potential was measured upon a potentiometer (Wolff, Berlin, Reichsanstalt calibration) in terms of a standard comparison with the earlier ones and were found to agree with them within 0.0001. One of these later cells (the readings of the four were identical to the fifth significant figure) was verified by Dr. Wolff of the Bureau of Standards, by comparison with the standard Clark cells of that institution, and found to be 1.0195 V. at 20° C., assuming the legal value (United States) of the Clark cell, 1.434 V. at 15° C. Substituting the Reichsanstalt value, $\text{Clark}_{15} = 1.4328$,[1] our cells would give the normal potential difference of 1.0186 at 20°.

With the apparatus here described, the authors were enabled to command any furnace temperature up to 1600° conveniently, to regulate it quickly and with great exactness or to hold it constant for long intervals. It could also be easily arranged to introduce an oxidizing or reducing atmosphere whenever desired.

With the help of the metals mentioned, which are readily obtainable and can be used repeatedly, thermoelements or resistance pyrometers can be calibrated in any laboratory, and used for all measurements up to the limit of the Reichsanstalt scale (1150° C.)

Anorthite. First Preparation.

Date.	Element.	Electromotive Force in MV.	Temperature.	Remarks.
Oct. 7, 1903.	A	15939	1534°	Solid charge, open crucible.
" " "	A	15914	1532	" " " "
" 10, "	A	15878	1530	Covered crucible.
" " "	No. 3	16074	1533	" "
" " "	" 3	16058	1532	" "
" " "	" 3	16068	1532	" "
" " "	" 2	16095	1532	" "
			Mean, 1532°	

Second Preparation.

Jan. 16, 1904.	A	15860	1532°	Covered crucible.
" " "	A	15864	1532	
" 20 "	No. 3	15960	1533	
" " "	" 2	16102	1532	
" " "	" 2	16092	1532	
Mar. 31 "	" 3	15932	1531	1st and 2d preparations together.
			Mean, 1532°	

with no greater error than that inherent in the scale itself. Above this temperature to 1600° the continuation of the thermoelectric scale probably still furnishes the most convenient and trustworthy extrapolation which has yet been perfected.

The uniformity and certainty of this extrapolation will best be illustrated by a table taken from our measurements upon anorthite (the highest melting point we measured). The melting temperature of a mineral of very poor conductivity for heat and relatively low specific gravity is much more difficult to measure than that of a metal, but the agreement of the results tabulated above is sufficiently good to demonstrate the accuracy of the extrapolation.

These measurements were made with three different thermoelements, the individual constants of each being determined independently by separate calibration with the cadmium (321.7°), zinc (419°), silver (961.5°) and copper (1065°) melting points as described. The thermoelectric potential therefore appears to deserve entire confidence for consistent extrapolation through the 450° immediately above the present Reichsanstalt scale.

U. S. GEOLOGICAL SURVEY,
WASHINGTON, D. C., June, 1904.

THE CONDUCTIVITY OF A SPARK-GAP.

By W. P. Boynton.

SOME years ago the writer made a study of the high-frequency (Tesla) induction coil,[1] from which he concluded that the resistance of the spark-gap in the primary circuit might be of the order of from 100 ohms down to 10 or possibly even 5 ohms. Results of the same order were obtained a little earlier by Trowbridge and Richards,[2] by a method different in detail, but depending upon the same fundamental principle, namely the damping effect upon the oscillations.

Some work recently done by the writer and Mr. Ralph S. Shelley, Assistant in the Department, leads to the conclusion that the resistance of the primary spark may be, for a part of the time at least, very much less than one ohm.

This work was undertaken primarily with the purpose of verifying experimentally the theoretical conditions of resonance of such a double circuit. For such quantitative work, it was necessary to select the quantity for which resonance was to be sought. Three naturally suggest themselves; the potential difference of the terminals of the secondary condenser, the current in the secondary circuit, and the heating effect, or "mean square" current in the secondary circuit. The previous paper furnishes the basis for the mathematical discussion.

It appears the maximum attainable secondary potential is[3]

$$(1) \qquad \frac{2C_2}{K_2} = \frac{2V_0MK_1}{\sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}$$

$$K_2 = \frac{L_1L_2 - 2M^2}{L_2^2}K_1, \tag{2}$$

an expression independent of the resistances of the circuits, while substituting this value gives this maximum as

$$\frac{L_2V_0}{\sqrt{L_1L_2 - M^2}}. \tag{3}$$

The conditions for maximum current can be easily found, but are not given, as no attempt was made to test this point.

The value for the "mean square" of the secondary current previously deduced was[1]

$$I_2^2 = \frac{nV_0^2M^2K_1K_2(R_1K_1 + R_2K_2)}{R_1R_2(L_1K_1 - L_2K_2)^2 + M(R_1K_1 + R_2K_2)^2}. \tag{4}$$

For which a maximum is found when

$$\frac{K_1}{K_2} = -\frac{R_2}{R_1} + \frac{\frac{R_2}{R_1} + \frac{L_2}{L_1}}{\sqrt{1 + \frac{R_1M^2}{R_2L_1^2}}}, \tag{5}$$

APPARATUS.

The primary capacity consisted of plates of window glass 12 x 16 inches coated on both sides with tinfoil, having a capacity of about .003 microfarad per plate. The secondary capacity consisted of two sheets of zinc, 4 x 6 feet, suspended vertically by white silk ribbons from a wooden frame work, forming an air condenser of adjustable capacity. The ratio of the capacities was measured directly by the bridge method, using intermittent current (110 volts) and telephone.

The primary consisted of five turns of heavily insulated No. 4 wire wound in an open spiral about 9 cm. long and of the same diameter; the secondary consisted of 104 turns No. 20 wire wound on a glass cylinder, forming a solenoid 13 cm. long and 4.3 cm. in diameter. Their inductances were determined by comparison with a standard of self-inductance, whose computed value, .00419 henrys, had been checked by comparison with measured capacities. The

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[1] Loc. cit., p. 47, eq. (26).

Anderson-Maxwell method as modified by Fleming[1] was employed for the self-inductances, using a condenser as an intermediary, and the mutual inductance was determined from the self-inductance of the secondary coil by Maxwell's method,[2] adopting Fleming's modification suggested in the paper last referred to. The values were also computed as a check. They were found to be

$$L_1 = 2{,}000 \text{ cm.},$$

$$L_2 = 130{,}000 \text{ cm.},$$

$$M = 5{,}400 \text{ cm.}$$

The spark gaps employed were zinc knobs of about 2 cm. diameter mounted on glass standards.

EXPERIMENTAL.

A spark gap was placed in parallel with the secondary condenser, and readings of the spark length taken for a series of different distances between the plates of the condenser. With the arrangement employed, namely 8 condenser plates in the primary condenser, and a primary spark of about .13 cm., a distinct maximum of about 1.2 cm. was observed when the plates were about 8 cm. apart. A direct comparison showed the secondary capacity to be 1/71.5 of the primary. The ratio computed by substituting the values of the inductances in formula (2) was 1/83. This discrepancy could easily have been made to disappear by moving the plates slightly farther apart, which would have produced no observable change in the spark length. Substituting the values of the inductances in formula (3) shows that the secondary potential cannot be expected to exceed 8.56 times the primary. The ratio of the spark lengths, $1.2/.13 = 9.22$, is entirely consistent with this conclusion as the potentials are very nearly represented by a linear function of the spark length, so that

$$\frac{V_2}{V_0} = \frac{S_2 - a}{S_1 - a}, \tag{6}$$

in which S_1 and S_0 represent the spark lengths and the not impossible value $a = .0115$ would give exactly the computed ratio, 8.56.

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[1] Phil. Mag. (6), 7, p. 536. 1904.

[2] Elec. and Mag., Vol. II., pp. 397-398 (3d. edition).

This preliminary test serves to indicate that the apparatus functions normally.

Measurements of the "mean square" current were made by a hot wire ammeter similar to that described in the previous paper.[1] Readings of this instrument were taken as in the case of the potential measurements for a series of distances between the plates of the secondary condenser, then these were set at the distance seeming to correspond to the maximum current, and the ratio of primary and secondary capacities determined. This was done for different values of the primary capacity, as, shown in the following tabulation :

No. plates in primary	2	3	4	5	6	8
K_1/K_2	22	29	31	34	35.5	38

For comparison the values of K_1/K_2 are plotted from eq. (5) in the accompanying figure as a function of R_2/R_1. A comparison of the observed ratios with the tabulation indicates that the values of R_2/R_1 lie between 1 and 5, or since the measured value of R_2 is 2.07 ohms, the value of R_1 would seem to lie between 2.07 and .415 ohms. As the latter value is not very different from the ohmic resistance of the metallic circuit, the implication is that the spark

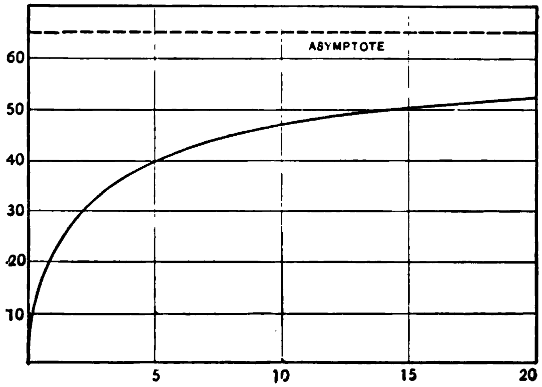

Fig. 1.

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[1] Loc. cit., p. 53.

gap may offer a resistance of much less than an ohm to the passage of a large oscillatory discharge.

DISCUSSION.

The mathematical theory as previously developed has assumed that all resistances are metallic, that is, constant. On this assumption an oscillatory system ought to give a wave train with a considerable number of observable oscillations. Such a system of metallic conductors having a single period of oscillation was investigated by Webster,[1] who records measurements of the period of perhaps 30 oscillations of the system.

But a conducting gas, such as a spark gap, does not have a constant resistance, such an assumption being only a first approximation. A second approximation which might prove useful would be the assumption that the resistance of such a spark has for a short time a definite, finite steady resistance, suddenly changing at the end of that short time to an infinite resistance. Apparently such an assumption would explain the various phenomena, the resonance effect being determined by the instantaneous small value of the resistance, the effect recorded in the previous paper being due to a time-average effect, while the number of spark-records obtained in various photographs would be related both to the instantaneous value of the resistance, and to the time during which this value persisted.

The data taken at this time are not sufficient for a satisfactory quantitative discusssion of this new assumption, hence an attempt will be made to apply it to older data.

In the indefinite integral of the square of an oscillatory function of the form

$$e^{-\alpha t}(A \cos \beta t + B \sin \beta t)$$

the principal term is[2]

$$-\frac{(A^2 + B^2)e^{-2\alpha t}}{4\alpha}.$$

If the integration be taken not from 0 to ∞, but from 0 to a finite limit τ, the definite integral will be

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[1] PHYS. REV., VI., pp. 297–314, 1898.

[2] Loc. cit., p. 44.

$$(1 - e^{-2a\tau})\frac{A^2 + B^2}{4a}, \tag{7}$$

which becomes, if $2a\tau$ is small

$$2a\tau\frac{A^2 + B^2}{4a} = (A^2 + B^2)\tau/2. \tag{8}$$

Applying this result to the secondary potential, and to the secondary and primary currents,

$$\int_0^\tau V_2^2 dt = \frac{(A_2^2 + B_2^2)\tau}{2K_2^2} + \frac{(C_2^2 + D_2^2)\tau}{2K_2^2}, \tag{9}$$

which becomes, neglecting B_2^2 and D_2^2 as small, and noting that $C_2^2 = A_2^2$

$$\int_0^\tau V_2^2 dt = A_2^2\tau/K_2^2. \tag{10}$$

$$\int_0^\tau I^2 dt = (\alpha^2 + \beta^2)(A^2 + B^2)\tau/2 + (\gamma^2 + \delta^2)(C^2 + D^2)\tau/2, \tag{11}$$

which becomes for the secondary current, neglecting α^2, γ^2, B_2^2 and D_2^2 as small and remembering that $C_2^2 = A_2^2$

$$\int_0^\tau I_2^2 dt = (\beta^2 + \delta^2)A_2^2\tau/2, \tag{12}$$

and for the primary current, where C_1 preponderates over the other coefficients.

$$\int_0^\tau I_1^2 dt = \delta^2 C_1^2\tau/2. \tag{13}$$

If, for the sake of definiteness, we assume that τ is the time required for m half-oscillations of the primary current, then evidently

$$\begin{aligned} \delta\tau &= \pi m, \\ \tau &= \pi m, \end{aligned} \tag{14}$$

from which

$$\begin{aligned} \int_0^\tau V_2^2 dt &= \pi m A_2^2/\delta K_2^2, \\ \int_0^\tau I_2^2 dt &= \pi m(\beta^2 + \delta^2)A_2^2/2\delta, \\ \int_0^\tau I_1^2 dt &= \pi m\delta C_1^2/2. \end{aligned} \tag{15}$$

Substituting the values of β, δ, C_1, and A_2, and introducing n, the number of discharges per second of the primary condenser, the following average values are obtained:

$$V_2^2 = \frac{\pi n m M^2 K_1^2 V_0^2 \sqrt{L_1K_1 + L_2K_2 + \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}}{\sqrt{2}[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]},$$

$$(16)\quad \overline{I}_2^2 = \frac{\pi n m M^2 K_1 K_2 V_0^2 (L_1K_1 + L_2K_2)}{\frac{\sqrt{L_1K_1 + L_2K_2 + \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}}{2\sqrt{2}(L_1L_2 - M^2)[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]}},$$

$$\overline{I}_1^2 = \frac{\pi n m K_1 V_0^2 \sqrt{L_1K_1 + L_2K_2 - \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}\,[L_1K_1 - L_2K_2 + \sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K}]}{16K_2(L_1L_2 - M^2)[(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2]}.$$

In these approximate equations the "effective" readings appear to be conditioned not upon the resistance of the spark-gap, but upon m, the number of half-oscillations it permits to pass. The first two quantities also are very simply related, having the ratio

$$(17)\qquad \frac{\overline{V}_2^2}{\overline{I}_2^2} = \frac{2}{(\beta^2 + \delta^2)K_2^2} = \frac{2K_1(L_1L_2 - M^2)}{K_2(L_1K_1 + L_2K_2)}.$$

Substituting numerical values in this equation we have, reducing to practical units and extracting the square root,

$$(18)\qquad \frac{\overline{V}_2}{\overline{I}_2} = 5{,}300.$$

Fig. 2 shows data of the discharge with a 2 mm. spark,[1] plotted as functions of the excitation, I_2 being in hundredths of an ampere, $\overline{V}_2$ in hundreds of volts, while the ratio $\overline{V}_2/\overline{I}_2$, computed from the curves, not the points, shown by the broken line, appears very nearly constant, ranging for the most part between 5,500 and 5,000.

Equations (16) may also be used for the computation of m. Sub-

$$m = \overline{3{,}3}\bar{4}0\,V_2^2/V_0^2,$$

$$(19) \qquad m = 9.42 \times 10^{10}\overline{I_2^2}/V_0^2,$$

$$m = 5.18 \times 10^{8}\overline{I_1^2}/V_0^2.$$

Fig. 3 gives values for *m* from the data just referred to. This indicates a number of half-oscillations varying from 2 to 8, as shown by data from the secondary circuit, or including one point not in the figure, up 16 as shown by data from the primary circuit. As an interesting coincidence, upon reference to the original spark photographs, one can count in many of them 8 striations, rarely 9, but often less. These however were taken under such different conditions that their evidence on this evidence on this point is not conclusive.

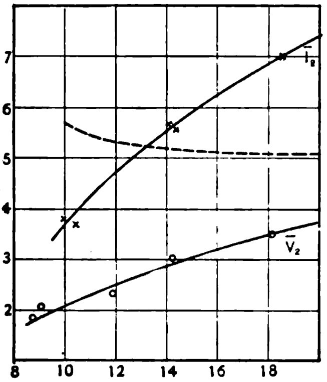

Fig. 2.

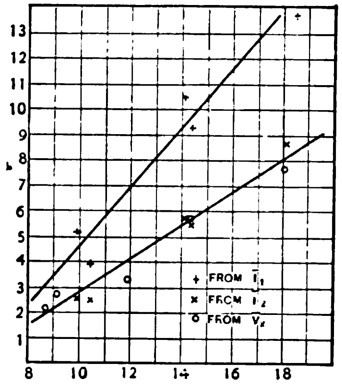

Fig. 3.

CONCLUSION.

Trowbridge and Richards[1] specifically speak of the "spark from a large condenser" as the condition of low resistance of spark gap. The justice of this remark is apparent from the fact that in the earlier experiments with a spark from a condenser of .0016 M.F. capacity only the data taken with the 2 mm. spark were available for the purposes of this discussion, the others showing irregular and abnormally large values of the ratio $\overline{V}_2/\overline{I}_2$, while in the present investigation capacities from 4 to 15 times as large were used, namely, from .006 to .023 M.F., when the low resistance effect became strikingly manifest.

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[1] Loc. cit.

The two sets of equations deduced appear to be adapted to the treatment of the two types of experimental data, the old equations perhaps better expressing the conditions when the condenser is small or the spark long and thin; the newer ones better adapted to the case where the primary condenser is large, and the spark relatively short.

An attempt to find a maximum value of $\overline{I_2^2}$ in equation (16) by differentiation gave forms too complicated for solution; but the method of trial indicates a maximum in the neighborhood of $K_1/K_2 = 90$ for the later apparatus. This by no means accords with the experimental results, which run from 22 to 38; but as the resonance is conditioned upon instantaneous rather than average values, it is not surprising that the newer formulæ, which take account primarily of the time of continuance, should fail to agree with experiment.

The complete mathematical theory would of course treat the resistance of the spark gap as a variable, a function possibly of the time, and thus indirectly a function of the effective current, or the energy of the field.

Physical Laboratory, University of Oregon,
June, 1904.

EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS.

By George W. Pierce.

I. Introduction.

To be enabled to foresee the extent to which confusion may be avoided in the simultaneous operation of a great number of wireless telegraph circuits, it is important to ascertain quantitatively the relative energy communicated between various dissonant circuits compared with the energy communicated between circuits in unison; that is, to ascertain the shape of the curve made by plotting the received energy (or the current in the receiving circuit) as a function of the periods of the two circuits.

Two forms of sending circuit capable of resonance are shown in the accompanying figures. The first of these forms—the "electromagnetically coupled type," Fig. 1—is seen to consist essentially of a Tesla coil with one terminal of the secondary connected to earth, while the other terminal is attached to a more or less nearly vertical wire carried by a mast.

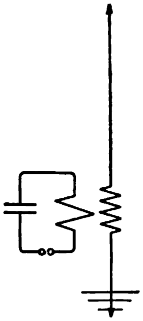
Fig. 1.

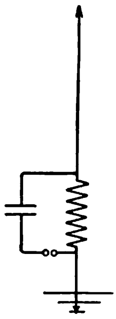
Fig. 2.

The second form — the "directly coupled type"—is shown in Fig. 2. It is seen to contain an autotransformer instead of the Tesla arrangement of Fig. 1.

The corresponding receiving circuits employed by different inventors resemble more or less closely the sending circuit with, however, a coherer and its appurtenances or other form of detector substituted for the spark gap.

The important advantage for resonance of the two types here represented consists in the direct grounding of the antenna, and in the production of the oscillation in the antenna by the inductive action of a condenser discharge in an auxiliary circuit of small damping.

The two forms of *sending* circuit shown in Figs. 1 and 2 seem to have been first clearly described by Professor Ferdinand Braun[1] of Strassburg, and both forms are covered by German Patent No. 111,578, issued Oct. 14, 1898, to the "Gesellschaft für drahtlose Telegraphie," basing its claim on Braun's discovery. The priority of discovery of these two types of sending circuit is attributed to Braun by M. Wien,[2] who says, "Die Bedeutung dieses Fortschrittes wird vielleicht am besten dadurch characterisiert, das so wohl Marconi als Slaby inzwischen zu der Braun'schen Methode der gekoppelten Systeme uebergegangen sind."

While Braun was active in developing the two forms of inductive sending circuits represented in Figs. 1 and 2, Marconi utilized the same principle in the construction of a syntonic *receiving* station. In 1898 he saw the importance of the direct grounding of the receiving antenna, and his English Patent No. 12,326 of June 1, 1898, provides for the location of the coherer in a closed condenser circuit inductively connected with the antenna. His receiving circuit is essentially of the form of Fig. 1 with the coherer in place of the spark gap, and his patent is of slightly earlier date than the corresponding patent of Braun covering the sending circuit.

Later, on the 21st of March and the 26th of April, 1900, Marconi has been granted English patents on a sending circuit closely resembling that of Fig. 1, with, however, the addition of a variable inductance in the antenna for the easy adjustment of the mast circuit to resonance with the condenser primary.

The theory of resonance in wireless telegraphic circuits has been treated by a number of writers. The simple form of circuit, used by Marconi prior to 1898, consisting of vertical wire, spark-gap or coherer, and ground has been discussed theoretically by V. Bjerknes[3] and M. Abraham.[4] The theory of the sending and receiving

[1] Phys. Zeit., 2, 372, 1901; and 3, 143, 1901–2.
[2] Drude's Ann., 8, 686, 1902.
[3] W. A., 55, p. 120, 1895.
[4] Phys. Zeit., 2, p. 329, 1901.

circuits of the type shown in Fig. 1 has been presented by M. Wien,[1] while the theory of the type shown in Fig. 2 has been made the subject of a recent paper by M. Abraham.[2]

Also a number of experiments have been published that treat of certain parts of the problem of resonance in such circuits. P. Drude[3] has made extensive experiments on the period of electrical vibrations of spools of wire with antennæ or metal plates attached. F. Braun,[4] by hanging Geissler tubes in various positions along the antenna of a single circuit has obtained evidence of nodes and loops when the side circuits are properly adjusted. C. A. Chant[5] has made similar investigations with a quantitative instrument in place of the Geissler tubes. James E. Ives,[6] by means of a hot-wire ammeter in the lead to earth has obtained some curves showing resonance of the antenna with the condenser circuit connected to it as in Braun's directly connected system (Fig. 2).

The researches made by the above investigators have all been experiments at the sending station, where the current in the antenna is large. In my own experiments this current, as measured by a hot-wire ammeter, sometimes attains an average value of two or three amperes.

Recently A. H. Taylor[7] has published some results of direct measurement of the current at the *receiving* station. By the use of a bolometer as a receiving instrument, Mr. Taylor has adjusted a pair of circuits to resonance, and with a fixed sending circuit giving out waves of approximately constant intensity and constant period has plotted the current in the receiving circuit when the inductance in the receiving antenna was varied. Mr. Taylor's article contains also interesting lecture-room demonstrations of the phenomena.

In the experiments described in the present paper the relative energy in the receiving circuit has been measured directly for several variations of the capacity and inductance of both the sending and the receiving circuit. The measurements show that the phenomena may be made to give sufficient regularity to permit of quantitative study.

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[1] W. A., 61, p. 151, 1897; and D. A., 8, p. 686, 1902.

[2] Phys. Zeit., 5, p. 179, 1904.

II. Apparatus and Method.

Form of Circuit. — The present experiments are confined to circuits of the electromagnetically connected type of Braun and Marconi. The sending circuit is shown at *S* in Fig. 3.

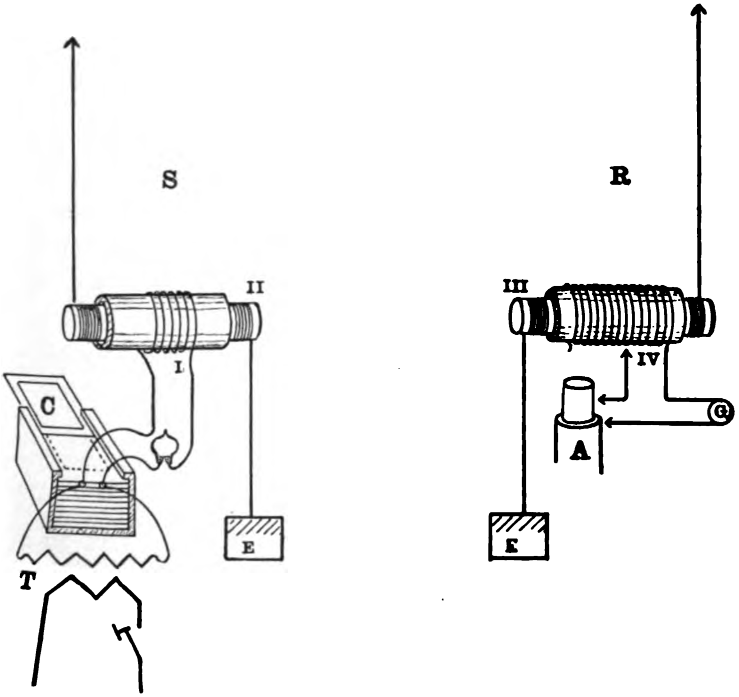

Fig. 3.

C is a variable glass plate condenser, charged by a step-up transformer *T*, operating on the 110-volt alternating light circuit. The transformer is capable of giving a potential of 10,000 volts, but by means of a rheostat in the primary this voltage was usually reduced to about 5,000. The condenser discharges through a Cooper-Hewitt mercury interrupter and through a primary of a few turns of coarse wire (diameter [illegible] cm.) wound on a tube of hard rubber inserted. The secondaries were of insulated wire wound on thin rubber tubes (2 mm. thick) and 10.4 cm. external diameter. Two such secondaries were used with the following dimensions:

No. of Turns.	Diam. of Wire, cm.	Length of Solenoid, cm.	Inductance Henries.
240	.104	46	125×10^{-5}
115	.157	41	31×10^{-5}

The secondary at the sending station, together with its antenna and ground connection, is referred to in the descriptions as Circuit II. At the receiving station *R* the inner coil with one end grounded and the other connected to the antenna (Circuit III.) had the same number of turns and the same dimensions as the corresponding coil at the sending station. The closed circuit electromagnetically connected with it (Circuit IV.) consisted of a variable air condenser in series with the receiving instrument and a bobbin on which the number of turns could be varied by means of a sliding contact.

The sending and receiving circuits were set up at separate small houses 25 meters apart. There were no wires or other metallic connections between the stations. The grounds consisted of pieces of wire netting (1 meter by 3 meters) buried in a horizontal position 40 cm. below the surface of the earth. The antennæ were copper wires 2 mm. in diameter passing in porcelain tubes through the walls of the buildings. These antennæ were usually about 7 meters high.

The Condensers. — The variable capacity at the receiving station consisted of four cylindrical air condensers each made of two concentric drawn tubes of brass 2 mm. thick. These tubes were made by the American Tube Works and are very approximately of uniform bore. They were straightened and freed from the small distortions by turned brass castings slipped around outside the outer and inside the inner tube of each condenser. These tubes are 60.8 cm. long. The outside diameter of the inner tube is 14.62 cm. The distance between the two tubes was obtained by setting a condenser on a block of paraffin and finding the weight of water required to fill the interspace. For the condenser *A* this distance was found to be .147 cm. From these data the capacity of *A* was computed to be

$$1.69 \times 10^{-3} \text{ microfarads.}$$

The other three air condensers did not differ materially from *A*.

The inner tube was supported on plugs of hard rubber and could be slid in and out of the outer tube without much motion sidewise — certainly without a displacement of the axis by as much as fifteen per cent. of the distance between the cylinders, and hence with a change of less than one per cent. in capacity.

The connections to the inner cylinder was made by means of a stiff copper brush. The capacity at the receiving station was varied by throwing in and out these air condensers as wholes or by varying any one of them by withdrawing the inner cylinder and reading on a paper scale the number of centimeters of length left overlapping — 1 cm. being equal to 2.77×10^{-5} microfarads.

The capacity *C* at the sending station consisted of two condensers made with plates of glass as dielectric. The number of such plates could be varied, and final small variations could be made by sliding back the top plate of one of them. The capacity of one of these glass condensers was measured for approximately the frequency at which it was to be used by comparing it with the known air condensers. The comparison was made by a resonance method as follows: Two parallel rectangular loops of wire 20 meters apart were hung up in the laboratory. One of these loops containing a condenser and spark-gap, or better a Hewitt mercury interrupter, was used to produce the waves of the proper period. The other loop, used as a receiving circuit, was adjusted to resonance with the sending circuit, first by the use of the variable air condenser, then by the glass condenser, plus a part of the air condenser. The air condenser being known, the capacity of any number of plates of the glass condenser could be determined. The other of the glass condensers used at the sending station was subsequently measured by comparisons obtained in the experiments that are described later in this account.

The Receiving Instrument. — Fig. 4 shows the instrument used in measuring the energy in the condenser circuit (Circuit IV.) at the receiving station. It is a slightly modified form of the oscillating current galvanometer devised by Fleming. A thick-walled hard rubber tube with an internal bore 1 cm. in diameter is mounted on a metallic base provided with leveling screws. In this tube is hung the suspension, which consists simply of a plane glass galvanometer mirror, 3 mm. in diameter, backed up by a thin disc of silver of approximately the same diameter. The mirror and disc are hung by a fine quartz fiber. Near the disc *M* is a small coil *C* making an angle of about 45° with the plane of the mirror at rest. This coil (diameter 9 mm.) consists of 30 turns of insulated wire (.11 mm. in diameter) and is mounted on the end of a thin hard rubber tube capable of sliding back and forth in the supporting case so that the distance between the mirror and coil can be varied, thus giving the instrument a very wide range of sensitiveness. The coil is connected by binding posts directly in series with the condenser circuit at the receiving station. Oscillations in this circuit conducted through the coil induced oscillations in the suspended disc, which is thus repelled, and tends to set itself at right angles to the plane of the coil.

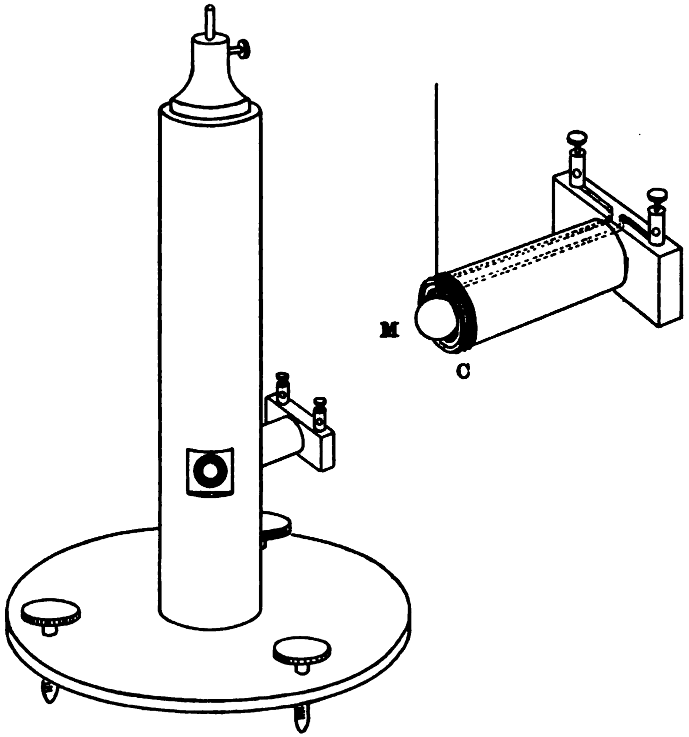

Fig. 4.

The period of swing one way is 3.4 seconds; and the mirror comes to rest after four or five oscillations. The resistance of the coil is 1.33 ohms. Its self inductance measured by Rayleigh's bridge with 1,200 alternations per second is 1.05×10^{-5} henries. This instrument is satisfactorily sensitive for the experiment. In fact, to keep the deflections on the scale when the circuits were in the neighborhood of resonance it was necessary to have the plane of the coil of the instrument at a distance of over 1 cm. from the center of the suspended disc. In order that the distance between the coil and the disc might not be changed by accidental jarring the instrument was screwed rigidly to a heavy shelf attached to a brick wall.

To avoid electrostatic disturbances of the instrument a small quantity of radioactive substance (radium bromide) was put inside the case to dissipate the charge acquired by the mirror in the process of hanging the suspension. This proved a great convenience, and after its use the instrument remained quiet within .1 or .2 mm., and kept the same zero for weeks.

For deflections so small that the change in the mutual inductance between the coil and the disc is negligible simple theoretical considerations show that the deflection is proportional to the square of the current in the coil. It is not presumed that such an instrument could be used as a receiver for wireless signals at any great distance, but without any very great precaution in its use it proves to be applicable to the rapid and consistent measurement of feeble currents of high frequency.

The Hewitt Mercury Interrupter.—In a previous paper[1] I have pointed out the advantages of the mercury interrupter both for uniformity and intensity in comparison with the spark-gap when used to interrupt a condenser discharge. The action of the mercury interrupter is, however, influenced by the temperature outside the bulb. The amount of this influence is shown in Section IV. of this paper.

In the present experiments, where the interrupter was operated consisted of an electrically heated bath of oil with the temperature automatically regulated. The following sets of readings (Table I.) taken at various times for different sensitivenesses of the instrument give an idea of the uniformity that may be attained in such observations. The readings are in centimeters with a scale-distance of 90 cm. Each column gives successive readings under the same conditions.

TABLE I.

To Show Uniformity.

10.8	37.5	28.7	11.8	37.7	11.4
10.9	37.2	28.0	11.2	37.2	11.4
11.4	37.0	28.1	11.8	39.0	11.2
11.4	36.1	28.3	11.9	36.7	11.2
11.2	38.9	28.5	11.7	36.2	11.1
11.1	38.0	27.8	11.6	36.8	10.8
10.8	38.3	29.4	11.4	37.8	11.0
11.0	38.1	28.5	11.2	39.8	11.0
11.0	38.1	29.5	11.4	38.8	11.2
11.2	38.1	29.0	11.4	37.8	11.0

These deflections for various mercury interrupters were from five to seven times as large as could be obtained by the spark-gap operating at the same potential. For low discharge potentials (spark 1 to 2 mm.) by increasing the sensitiveness of the receiving instrument it was possible also to get uniform readings with the spark-gap, but for a long series of observations the mercury interrupter is much more satisfactory. This advantage is, however, partially offset by the annoyance caused by the breaking of the bulb when exposed too long to a heavy current—a difficulty that may possibly be overcome by correct annealing of the glass.

III. RESONANCE CURVES.

Experiment I. Resonance Curves with the Secondaries with 240 Turns.—The two mast circuits were made similar, each containing one of the secondary coils of 240 turns, described on page 200. On the outside bobbins, Fig. 3, there were 9 turns at the sending station and 20 turns at the receiving station. The mast height was 7 meters of bare wire 2 mm. in diameter. The family of curves plotted in Fig. 5 were obtained by varying the capacities at the sending and receiving stations. Each curve corresponds to a fixed sending capacity; the individual points on any one curve correspond to various capacities of the receiving condenser.

The process of taking readings was as follows: The glass condenser at the sending station was set at a given small value (2 plates), the air-condenser *A* at the receiving station was set at 6 cm. (length of cylinder overlapping), the primary circuit of the transformer was closed by a switch operated by a string. This switch was left closed during the outward swing of the mirror, and the throw was read. The value of the throw is plotted as the first point on Curve 1. The sending circuit remaining constant, successive deflections were taken with the receiving condenser set at 8, 10 and 12 cm. respectively. These four values determine Curve 1, in which deflections are plotted against receiving capacity. From Curve 1 it is apparent that the random choice of the sending capacity is not correct for resonance with the mast circuits.

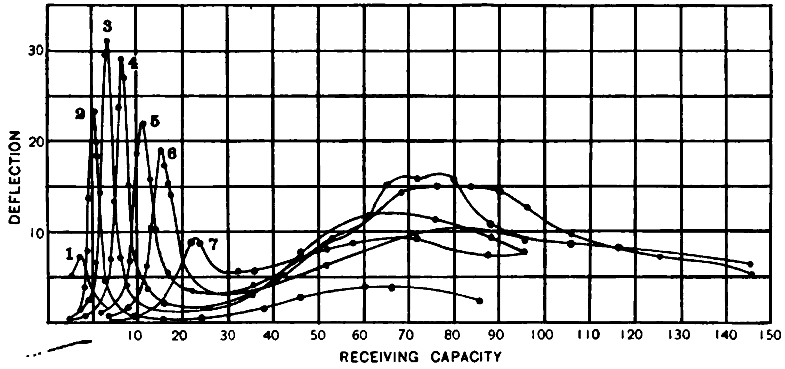

Fig. 5.

For Curve 2, the condenser at the sending station was set at 3 plates. Keeping this constant, deflections were taken for various values of the receiving capacity ranging from 6 to 96 cm. of the cylindrical condensers. Curve 2 thus obtained shows a larger maximum than Curve 1; which indicates that the second adjustment of the sending circuit gave a nearer approach to resonance between the sending condenser circuit and the mast circuits. Subsequent increases of the sending capacity by small stages gave the Curves 3, 4, 5, 6 and 7. Of these Curve 3 is the best. It was obtained with a sending capacity of 3.3 plates of the glass condenser. (The various capacities here designated are given in microfarads in Table II.)

An examination of Fig. 5 shows, however, that the deflections,

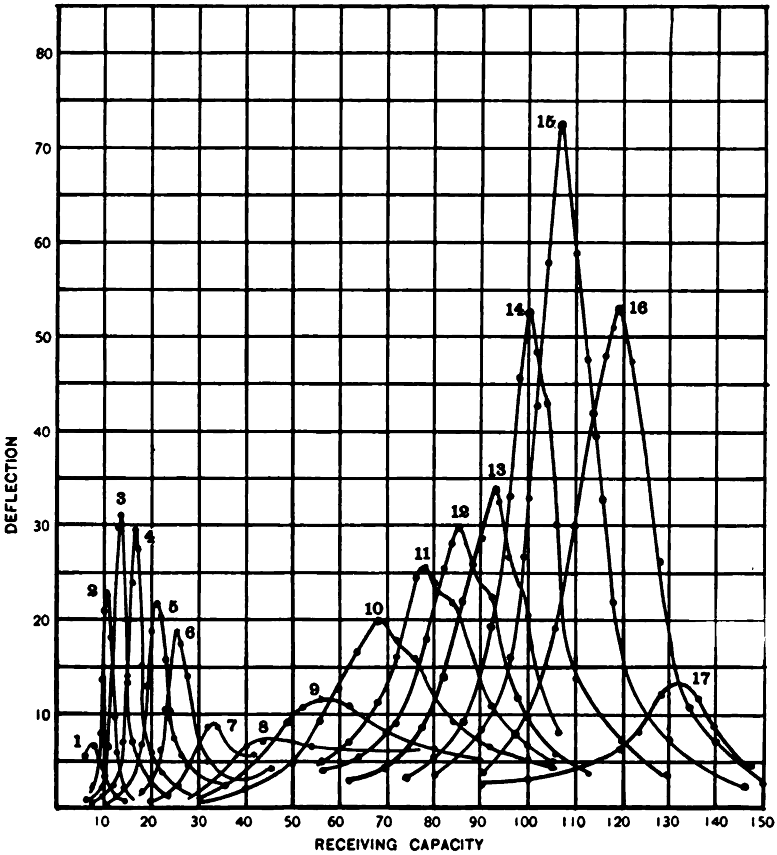

Fig. 6.

after falling for receiving capacities between 30 and 40, rise again

To test this view, the sending capacity was increased further by small stages, and the Curves 8 to 16, Fig. 6, were taken. In this figure Curves 1 to 7 are also plotted, as in Fig. 5, except that their second maxima are omitted in order to prevent confusion with the subsequent curves.

Curve 15 of Fig. 6 examined for percentage decrease of deflection with percentage dissonance, is seen to give resonance more strongly marked than Curve 3.

The 17 curves of Fig. 6 were all taken in a single series of observations in a single day. Their relative values were then checked up by taking the average of three observations at each of the maxima, so as to be sure that no progressive changes occurred in the action of the interrupter during the observations.

One might ask whether Curve 15 is correct for resonance or whether we have again merely resonance of the sending capacity circuit with a harmonic of the mast circuit. The answer is obtained by noticing that Curves 3 and 15 have their maxima at 13 and 107 cm. of the receiving condensers. These numbers are approximately

TABLE II.

Curve No.	Sending Capacity in Microfarads.	Optimal Rec. Capacity in Microfarads.	Ratio, S/R.
1	12.3×10^{-4}	2.21×10^{-4}	5.6
2	17.5	2.90	6.0
3	20.5	3.75	5.5
4	25.6	4.70	5.5
5	30.7	5.80	5.3
6	37.0	6.90	5.4
7	54.0	9.12	5.9
8	73.5	12.2	5.9
9	103.5	15.5	6.6
10	111	18.8	5.9
11	124	21.3	5.8
12	130	23.5	5.5
13	137	25.7	5.3
14	158	27.7	5.7
15	164	29.7	5.5
16	178	33.0	5.4
17	200	36.5	5.5
			Mean 5.7

in the ratio of 1 : 9 (*e. g.*, 12 : 108). The *periods* in these two cases are, therefore, in the ratio of 1 : 3, which is the ratio of the period of the first odd harmonic to that of the fundamental. The same ratio could not obtain for any other two odd harmonics, hence Curve 15 is the case of resonance with the fundamental period of the mast circuits.

In the descriptions given above of the curves of Fig. 6 the capacities are stated in arbitrary units. Table II. contains these values in microfarads. The second and third columns give the sending capacities and the corresponding optimal receiving capacities for the several curves of Fig. 6. The fourth column contains the ratio of these two capacities.

The inductances were kept constant throughout the series and had the following values in henries :

	Henries.	Method of Measuring.
L_2 and L_3	125×10^{-5}	— Maxwell's Bridge.
L_4	6.0×10^{-5}	Resonance, cf. p. 210.
	5.95×10^{-5}	Rayleigh's bridge.
L_1	1.05×10^{-5}	Assuming it to be $L_{4/5.7}$.
M_{12}	4.85×10^{-5}	Comparison with Earth Inductor.
M_{34}	10.9×10^{-5}	" " " "

The approximate constancy of the ratios in the fourth column of Table II. shows that for the particular circuits employed in Experiment I. the several curves of Fig. 6 are the curves of resonance between the two condenser circuits I. and IV. This means that the predominating oscillation in the mast circuits II. and III. is a forced vibration with approximately the period determined by the closed circuit I. The characteristic oscillation of the mast circuits themselves, on account of high damping, does not persist strongly enough to have much effect in determining the *period* of the train of waves that carry the energy. The *intensity*, however, is considerably influenced by the resonant action of the masts. The great intensity obtained in Curve 15 is the effect of tuning the sending condenser circuit to resonance with the masts. For the maximum point on this curve the receiving condenser circuit is also in resonance with the masts. The *purity* of the wave is also enhanced by unison of the discharge circuit with the masts. For the sending circuit in the condition to give Curve 15 there is no evidence of more than one maximum, even when examined over a range of several octaves. Thus with equal mast circuits harmonics appear ouly when the condenser circuit I. is not in resonance with the mast circuits II. and III.

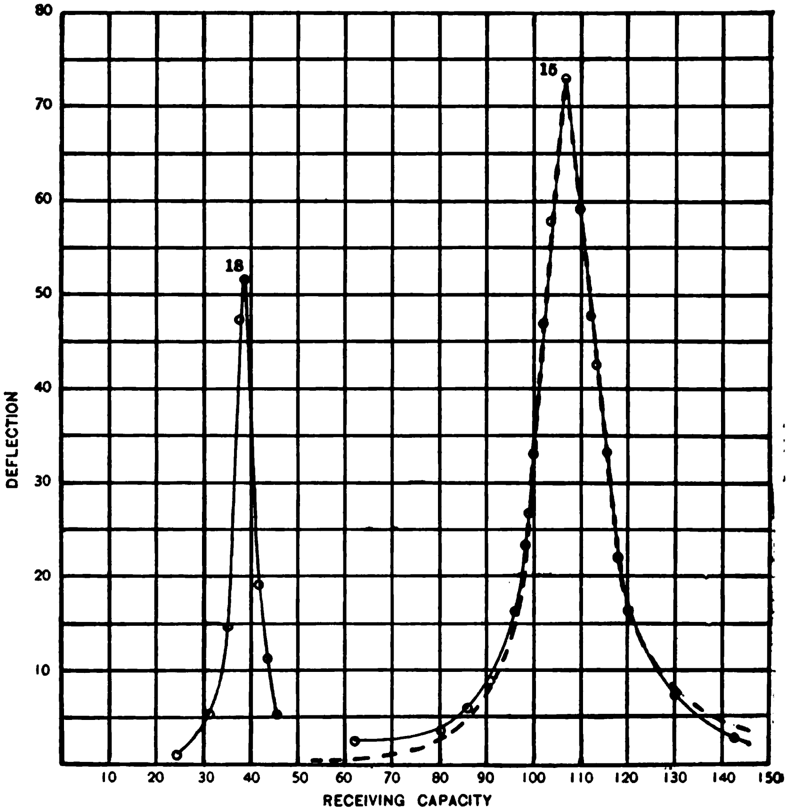

Fig. 7.

Experiment II. Determination of the Fundamental Wave Length for Curve 15.— To determine the wave-length of the apparatus in the condition of best resonance, a known inductance was put in the circuit with the air condenser and instrument at the receiving station, and readings were taken for several successive values of the receiving condenser. The results are plotted as Curve 18, Fig. 7, with a

maximum at $C = 38.5 = 38.5 \times 2.77 \times 10^{-11}$ microfarads. This
7 curve compared with 15, which is also plotted in Fig. 7 gives the data for calculating the fundamental wave-length. The oscillations producing curves 17 and 14 have the same period, since the sending circuit is the same in the two cases; therefore,

$$\begin{aligned} T &= 2\pi\sqrt{L \times 107 \times 2.77 \times 10^{-11}} \\ &= 2\pi\sqrt{(L + L') \times 38.5 \times 2.77 + 10^{-11}}. \end{aligned} \qquad (1)$$

X

In this equation L is the unknown inductance of the leads and instrument, L' the known inductance of the added coil. The coil L' was accurately wound, and its inductance was computed from its geometrical dimensions by the aid of Maxwell's formula. This coil was also measured in comparison with a calculated air condenser, by photographing with a revolving mirror the period of the discharge of the condenser through the coil. These two methods agreeing within one per cent., gave

$$L' = .000106 \text{ henries.}$$

This value substituted in equation (1) gives for the inductance of the leads

$$\begin{aligned} L &= \frac{38.5 \times .000106}{107 - 38.5}, \\ &= 6.00 \times 10^{-5} \text{ henries.} \end{aligned}$$

A subsequent measurement of this quantity on Rayleigh's bridge with 1,200 alternations per second gave

$$L = 5.95 \times 10^{-5}.$$

The agreement is satisfactory. The substitution of this value of L in equation (1) gives for the period

$$T = 2.66 \times 10^{-6} \text{ seconds.}$$

Whence the wave-length

$$\lambda = vT = 798 \text{ meters.}$$

the current) falls to one half its maximum value when the receiving capacity departs by 7 per cent. from the capacity of resonance. Simple theoretical discussion of the case may serve to discover the conditions that have prevented sharper resonance and may serve to suggest methods for improving these conditions. As a rough approximation let us suppose that the condenser circuit containing the instrument at the receiving station has a resistance R, a self inductance L and a localized capacity C, and that there is impressed upon it a sinusoidal electromotive force $E \sin \omega t$, and let us take no account of the neighboring mast circuit, then the square root of the mean square current through the instrument is

$$I = \frac{E/\sqrt{2}}{\sqrt{R^2 + \left(\frac{1}{c\omega} - L\omega\right)^2}}. \tag{1}$$

Second, let us suppose that the deflections of the receiving instrument are proportional to I^2;

$$D = \frac{kE^2}{R^2 + \left\{\frac{1}{c\omega} - L\omega\right\}^2}. \tag{2}$$

In Experiment I., L was kept constant, and by Experiment II. it was found to have the value

$$L = 6.00 \times 10^{-5} \text{ henries.}$$

Also in obtaining Curve ~~14~~ the incident waves were kept constant in intensity and frequency, so that E is a constant, and 15

$$\omega = \frac{1}{\sqrt{LC_m}},$$

$$L\omega = \frac{\sqrt{L}}{\sqrt{C_m}} = \sqrt{\frac{6.0 \times 10^{-5}}{29.7 \times 10^{-10}}} = 144.$$

Then from (2)

$$D = \frac{kE^2}{R^2 + 144^2\left\{\frac{C_m}{C} - 1\right\}^2}. \tag{3}$$

Equation (3) contains two unknown constants kE^2 and R^2. A trial shows that with proper values of these constants equation (3) gives a fair approximation of the values of Curve 15. To this end the constants were determined from two points on the experimental curve — the maximum point $D = 75$, $C = 107$ cm. of cylinder, and the point $D = 16$, $C = 96$ cm. of cylinder. Whence,

$$75 = kE^2/R^2,$$

$$16 = \frac{75R^2}{R^2 + (144 \times .12)^2},$$

and

$$R = 9 \text{ ohms}.$$

With this value of R, Table III. contains the computed values of D compared with the observed values. The formula for the computation is

$$D = \frac{75}{1 + 256\left(\frac{cm}{c} - 1\right)^2}.$$

TABLE III.

C	D calc.	D obs.	C	D calc.	D obs.
53.5	.3	—	107	75	75
80.5	2.6	3.5	113	47	45
86	4.4	6.0	120	16	16.5
91	8.0	9.0	130	8	7.5
96	16	16	143	4.4	3.0
102	47	46			

The calculated values of D are plotted as the dotted curve in Fig. 7.

It is seen that the observed and the calculated values of the deflection agree well for the interval of dissonance between $C = 91$ and $C = 130$. For greater dissonance the agreement is not so good. The apparent resistance used in the calculation is 9 ohms. The actual resistance of the circuit containing the instrument was 1.40 ohms when measured with a steady current. Practically the whole of this resistance is in the coil of the instrument itself. On the supposition that Rayleigh's formula for resistance of a straight conductor as a function of the frequency applies also to such a coil, a computation shows that the resistance of the circuit under consideration is not appreciably greater than its resistance for a steady current.

Thus the greater part of the apparent resistance of 9 ohms is to be sought as effect of the proximity of the other circuits. It is well known that the presence of a closed secondary has the effect of increasing the apparent resistance of a circuit in an amount depending on the self inductances, and resistances of the two circuits and the mutual inductance between them. When the square of the mutual inductance is negligible in comparison with the product of the two self inductances this influence of the neighboring circuit in increasing the apparent resistance is negligible. Thus for sharp resonance, as pointed out by Wien, it is necessary to make the electromagnetic connection between the circuits "loose," that is to make the square of the mutual inductance between the two circuits at each station negligible in comparison with the product of the two self inductances. The circuits employed in experiment I. did not completely fulfill this requirement, as M^2 at each station was equal to about one tenth of the product of the mutual inductances.

Experiment III. Resonance Curves with the Secondaries of 115 Turns. — Another set of resonance curves obtained with another pair of secondary coils is shown in Fig. 8. Here the secondaries had 115 turns of wire 1.57 mm. in diameter. Table IV. contains the values of the capacities corresponding to the maxima of the several curves.

TABLE IV.

Curve No.	Sending Capacity in Microfarads, *S*.	Optimal Rec. Cap. in Microfarads, *R*.	Ratio, *S*/*R*.
19	17.5×10^{-4}	4.40×10^{-4}	4.00
20	25.6	6.25	4.10
21	26.1	6.67	3.90
22	26.6	6.80	3.90
23	27.1	6.85	3.95
24	27.6	6.95	3.97
25	28.2	6.95	4.05
26	30.7	7.20	4.30
27	37.0	8.50	4.35
			Mean, 4.17

In this experiment the inductances had the following values :

	Henries.	**Method of Measuring.**
L_1	1.63×10^{-5}	Raleigh's bridge.
L_2 and L_3	31.0×10^{-6}	" "
L_4	6.00×10^{-6}	" " and resonance.
M_{12}	2.60×10^{-6}	Ballistic galvanometer.
M_{34}	$5.77 \times 10^{1-6}$	" "
L_1	1.44×10^{-5}	L_4 / 4.17, compare Table IV.

The wave-length in this case is 381 meters.

The curves of Fig. 8 were taken in the same manner as those of Fig. 6. Curve 22 when examined for percentage decrease in deflection with percentage dissonance is not quite so sharp as Curve 17 of Fig. 6. The family of curves in Fig. 8 present a different appearance from those of Fig. 6, in that the successive approximations to resonance are grouped closer around the maximum curve than they were in the former experiment. In fact in experiment III., when the dissonance between the condenser circuit and the mast at the sending station is not too great, the curves in which the sending capacity is too small (20 and 21) go up tangential to 22 on its left slope, while those in which the sending capacity is too large (23, 24, 25, and 26) come down tangential to 22 on its right slope. The maximum curve is thus a kind of envelope of the curves in its neighborhood. At first I took this to indicate persistence of the period of the condenser circuit at the receiving station, so that the progression of the maxima at the receiving station did not follow closely the progression in period at the sending station. The ratio of $S : R$ in Table IV., however, does not seem to vary in a manner to make this view tenable. So that, except for somewhat greater damping, the curves of Fig. 8 do not appear to differ essentially in character from those of Fig. 6.

I have taken other families of resonance curves with different values of the inductance and it is my intention to examine the results obtained with various relations between M^2 and the product of the self-inductances in the hope of testing some of the theoretical deductions of Wien and Abraham. Complete data for these tests are not yet at hand.

The curves here published show strikingly the necessity of tuning the condenser circuits to resonance with the masts — a necessity which has been repeatedly emphasized in the theoretical discussions of the subject, and which has also been previously shown experimentally by Mr. A. H. Taylor.

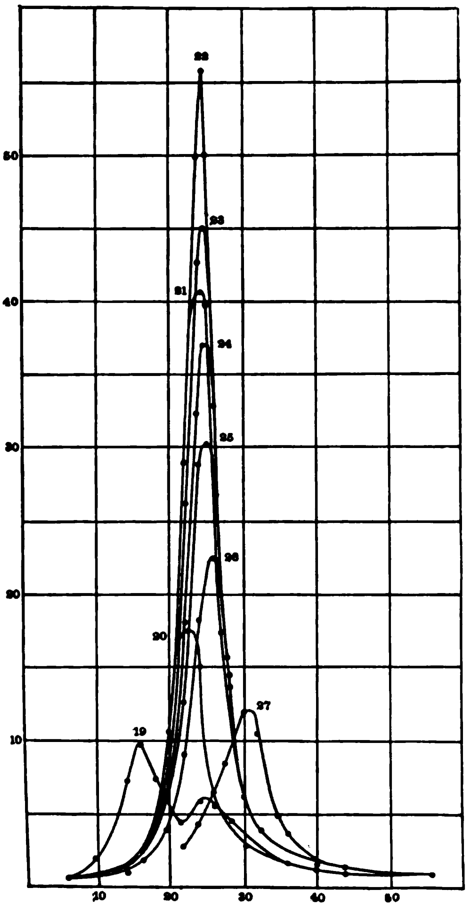

Fig. 8.

IV. The Effect of Temperature on The Hewitt Mercury Interrupter.

In the course of these experiments it became early apparent that the temperature outside the bulb had considerable effect on the action of the mercury interrupter. So the interrupter was put in a thermostat consisting of a bath of oil electrically heated and with an automatic regulation of temperature. In addition to the amount of the effect, question arose whether the ohmic resistance of the interrupter depended on the temperature to such an extent as to affect the shape of the resonance curves taken at various temperatures. Fig. 9 shows the results obtained for a number of resonance

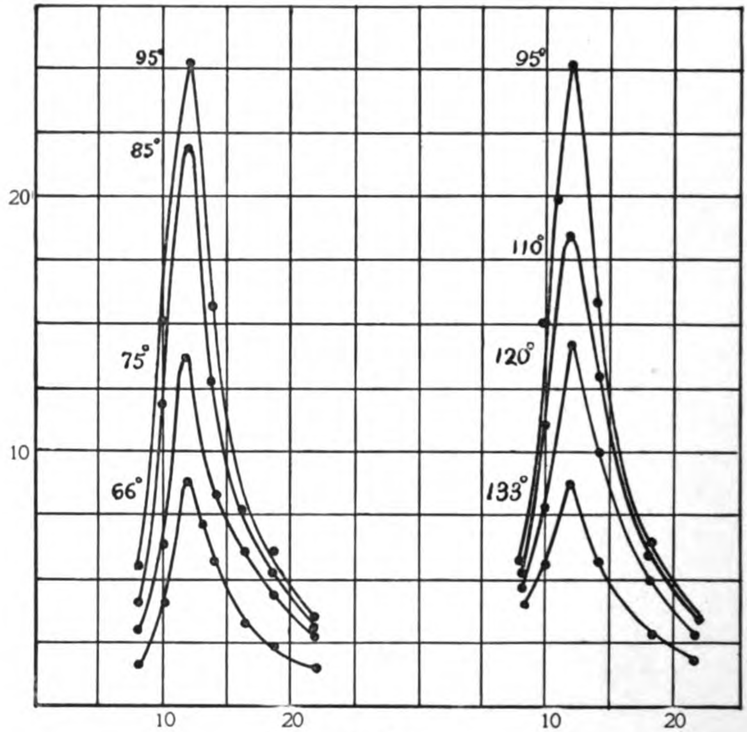

Fig. 9.

ranging from 66° to 95° C. in which the deflections increase with increasing temperature; the other for temperatures from 95° to 133° which show a decrease of deflection with increasing temperature. The best temperature for operating this particular interrupter is 95°. At 66° and again at 133° the deflections are only one third as great as at 95°. The shape of the curves, however, examined for percentage decrease of deflection with percentage dissonance, is not materially different for the different temperatures. Of course it is not presumed that a regulation of the temperature outside the bulb results in an exact regulation of the temperature inside when the discharge is passing, but the ease with which the various observations recorded in this paper could be reproduced shows that the Hewitt interrupter, when operated in a thermostat, gives considerable uniformity in the interruption of a condenser discharge.

HARVARD UNIVERSITY,
CAMBRIDGE, MASS., June 29, 1904.

NOTES ON THE ERRORS IN THE HALF-SHADE ELLIPTICAL POLARIZER.

By D. B. Brace.

IN the measurements of a change of phase and of a rotation of the plane of polarization, the methods should be such that the presence of the one effect should not be confounded with that of the other. In the majority of cases the two occur separately, or can be isolated. In some instances, however, this is not possible. Thus in the differential absorption of the two circular components into which light is broken up in its passage through natural and magnetic rotary substances, plane polarized light, after its passage, is rotated and also becomes elliptically polarized. Similarly, in certain differentially absorbing double-refracting substances, the one linear component is retarded and absorbed more than the other, giving elliptical for plane polarized light, and also a rotation of the major axis. This may occur in a similar manner for plane polarized reflected and refracted light.

In the half-shade elliptical polarizer if the major axes of the vibration in the two halves are parallel and equal there will be no relative change in intensity on rotating the analyzer; if this condition is not fulfilled, there will in general be such a relative change. In the half-shade plane polarizer, if an element is interposed which is double-refracting, a change in the two halves will occur which may be appreciable, particularly if the principal plane lies between those of the polarizing elements, or if the element is not uniform in the amount of double refraction.

In the elliptical polarizer previously described[1] the discussion is based on the assumption of a crossed system of nicols with inter-

and hence, on rotation of the analyzer, the match may be destroyed. However, by lowering the order of the elements, this difference in the azimuths may be reduced so that the resulting change in intensity may not become an observable quantity. Within these limits, which are very small, a rotating section may be used; beyond this a compensating system must be used such that it may be kept crossed with that of the interposed element.

If ξ, η, A and B are the rectangular component vibrations and amplitudes respectively and ε their difference in phase, the resulting elliptical vibration is

$$\frac{\xi^2}{A^2}+\frac{\eta^2}{B^2}-\frac{2\xi\eta}{AB}\cos\varepsilon=\sin^2\varepsilon. \tag{27}$$

The angle δ between the axes of the ellipse and the coördinate axes is given by

$$\tan 2\delta = 2\frac{AB}{A^2-B^2}\cos\varepsilon, \tag{28}$$

and if $\tan\gamma=\beta/\alpha$ is the ratio of the axes of the ellipse

$$\sin 2\gamma=\frac{2\alpha\beta}{\alpha^2+\beta^2}=\frac{2AB}{A^2+B^2}\sin\varepsilon. \tag{29}$$

In order to calculate these relations, the final rectangular components must be determined. Consider two such plates and let i be the azimuth of the first plate referred to the initial plane of polarization, a the azimuth of the second with respect to the first plate and s that of the analyzer with respect to the polarizer, o', e', o and e the equivalent air thickness for the ordinary and extraordinary rays in the successive plates respectively. With an initial amplitude of unity the components leaving the first plate are

$$\left.\begin{aligned} x&=\cos i\sin 2\pi\left(\frac{t}{\tau}-\frac{o'}{\lambda}\right),\\ y&=\sin i\sin 2\pi\left(\frac{t}{\tau}-\frac{e'}{\lambda}\right),\end{aligned}\right\} \tag{30}$$

The components after passing the second plate are

$$\xi=\cos a\cos i\sin 2\pi\left(\frac{t}{\tau}-\frac{o'+o}{\lambda}\right)$$

$$+ \sin a \sin i \sin 2\pi\left(\frac{t}{\tau} - \frac{e' + o}{\lambda}\right) = A \sin 2\pi\left(\frac{t}{\tau} - \frac{C}{\lambda}\right)$$

$$\eta = -\sin a \cos i \sin 2\pi\left(\frac{t}{\tau} - \frac{o' + e}{\lambda}\right) \tag{31}$$

$$+ \cos a \sin i \sin 2\pi\left(\frac{t}{\tau} - \frac{e' + e}{\lambda}\right) = B \sin 2\pi\left(\frac{t}{\tau} - \frac{D}{\lambda}\right),$$

where A and B are the component amplitudes and C and D the equivalent air thicknesses of a single plate which would give the same resultant vibration.

We have to determine $2AB \cos 2\pi\left(\frac{D - C}{\lambda}\right)$ and

$$2AB \sin 2\pi\left(\frac{D - C}{\lambda}\right) \quad \text{where} \quad 2\pi\left(\frac{D - C}{\lambda}\right) = \varepsilon.$$

Expanding and equating coefficients of the time function and applying the formulæ

$$AB\left(\cos 2\pi\frac{D}{\lambda} \cos 2\pi\frac{C}{\lambda} + \sin 2\pi\frac{D}{\lambda} \sin 2\pi\frac{C}{\lambda}\right) = AB \cos 2\pi\left(\frac{D - C}{\lambda}\right)$$

$$AB\left(\sin 2\pi\frac{D}{\lambda} \cos 2\pi\frac{C}{\lambda} - \cos 2\pi\frac{D}{\lambda} \sin 2\pi\frac{C}{\lambda}\right) = AB \sin 2\pi\left(\frac{D - C}{\lambda}\right)$$

we obtain a series of terms containing products of cosines and sines of the different sums of the equivalent thicknesses of the two plates taken two and two. Combining terms with like coefficients and applying the formulæ for the cosine and sine of the difference again, we have

$$\begin{aligned} 2AB \cos 2\pi\left(\frac{D - C}{\lambda}\right) = &- 2 \sin a \cos a \cos^2 i \cos 2\pi\frac{o - e}{\lambda} \\ &- 2 \sin^2 a \sin i \cos i \cos 2\pi\frac{(o - e) - (o' - e')}{\lambda} \\ &+ 2 \cos^2 a \sin i \cos i \cos 2\pi\frac{(o' - e') + (o - e)}{\lambda} \\ &+ 2 \sin a \cos a \sin^2 i \cos 2\pi\frac{o - e}{\lambda} \end{aligned} \tag{32}$$

$$= -\sin 2a\cos 2\pi\frac{o-e}{\lambda} - \sin^2 a\sin 2i\cos 2\pi\frac{(o-e)-(o'-e')}{\lambda}$$

$$+\cos^2 a\sin 2i\cos 2\pi\frac{(o'-e')+(o-e)}{\lambda}$$

and

$$2AB\sin 2\pi\left(\frac{D-C}{\lambda}\right) = 2\sin a\cos a\cos^2 i\sin 2\pi\frac{o-e}{\pi}$$

$$+ 2\sin^2 a\sin i\cos i\sin 2\pi\frac{(o-e)+(o'-e')}{\lambda}$$

$$- 2\cos^2 a\sin i\cos i\sin 2\pi\frac{(o'-e')+(o-e)}{\lambda} \qquad (33)$$

$$- 2\sin a\cos a\sin^2 i\sin 2\pi\frac{o-e}{\lambda}$$

$$= \sin 2a\cos 2i\sin 2\pi\frac{o-e}{\lambda} + \sin^2 a\sin 2i\sin 2\pi\frac{(o-e)-(o'-e')}{\lambda}$$

$$- \cos^2 a\sin 2i\sin 2\pi\frac{(o'-e')+(o-e)}{\lambda}$$

As $A^2 + B^2 = 1$ we have only to determine $A^2 - B^2$. Squaring and adding terms in A and in B respectively and combining cosines and sines of the paths as before, we have

$$A^2 = \cos^2 a\cos^2 i + \sin^2 a\sin^2 i + 2\sin a\cos a\sin i\cos i\cos 2\pi\frac{o'-e'}{\lambda}$$

$$B^2 = \sin^2 a\cos^2 i + \cos^2 a\sin^2 i - 2\sin a\cos a\sin i\cos i\cos 2\pi\frac{o'-e'}{\lambda}.$$

Thus

$$A^2 - B^2 = \cos 2a\cos 2i + \sin 2a\sin 2i\cos 2\pi\frac{o'-e'}{\lambda}.$$

Using N', etc., the differential order for $(o'-e')/\lambda$, etc., we have

$$\tan 2\delta' = \frac{\sin 2a\cos 2i\cos 2\pi N - \sin 2i[\sin^2 a\cos 2\pi(N-N') - \cos^2 a\cos 2\pi(N+N')]}{\cos 2a\cos 2i + \sin 2a\sin 2i\cos 2\pi A'} \qquad (34)$$

and

For $i = 45^\circ$

$$\tan 2\delta' = -\frac{\cos 2a \cos 2\pi A \cos 2\pi A' + \sin 2\pi A \sin 2A'}{\sin 2a \cos 2\pi A},$$

$$= -\frac{\cos 2\pi A}{\tan 2a} + \frac{\sin 2\pi A \tan 2\pi A'}{\sin 2a}, \tag{36}$$

and

$$\sin 2\gamma' = - \cos 2a \sin 2\pi A \cos 2\pi A' - \sin 2\pi A' \cos 2\pi A. \tag{37}$$

If we take $\theta = i + a$ or refer the azimuth of the second plane to the initial plane of polarization $a = \theta - 45^\circ$ and

$$\tan 2\delta' = \tan 2\theta \cos 2\pi N - \frac{\sin 2\pi N \tan 2\pi N'}{\cos 2\theta}, \tag{38}$$

$$\sin 2\gamma' = - \sin 2\theta \sin 2\pi N \cos 2\pi N' - \sin 2\pi A' \cos 2\pi A. \tag{39}$$

If $N' = 0$, *i. e.*, if the first plate be removed,

$$\tan 2\delta = \tan 2\theta \cos 2\pi N, \tag{40}$$

$$\sin 2\gamma = - \sin 2\theta \sin 2\pi N. \tag{41}$$

If the first plate is a double plate of orders N' and N'' where $N' > N''$, we have

$$\tan 2\delta' - \tan 2\delta'' = -\frac{\sin 2\pi N}{\cos 2\theta}(\tan 2\pi N' - \tan 2\pi N''), \tag{42}$$

or

$$\frac{\sin 2(\delta' - \delta'')}{\cos 2\delta' \cos 2\delta''} = \frac{\sin 2\pi N}{\cos 2\theta} \frac{\sin 2\pi(N' - N'')}{\cos 2\pi N' \cos 2\pi N''}, \tag{43}$$

which becomes

$$d\delta' = \frac{\cos^2 2\delta'}{\cos 2\theta} \frac{\sin 2\pi N}{2 \cos^2 2\pi N'} dN', \tag{44}$$

when $\delta' - \delta''$ and $N' - N''$ are small; and if $N'' = 0$ also, we have

$$d\delta' = -\frac{\cos^2 2\delta}{\cos 2\theta} \frac{\sin 2\pi N \tan 2\pi N'}{2}. \tag{45}$$

$$\sin 2\theta = -\frac{\tan 2\pi\left(\frac{N' + N''}{2}\right)}{\tan 2\pi N}. \tag{46}$$

These equations will aid us in determining by inspection the angle θ for an approximate match and the inclination of the major axes to one another, together with the ratio of the axes in each vibration, from which we may determine approximately the intensity of light passing the analyzer.

Then if $N'' = 0$ and $N = N' = \frac{1}{90}$ and $\frac{1}{180}$ respectively, we find from (46) that we will have a match when the principal axis of the compensator N makes angles of $\theta = -14° 58' 45''$ and $-14° 59' 52''$, approximately, with the plane of polarization.

To the same degree of approximation we may determine $d\delta'$, the inclination to one another of the major axes of the elliptical vibrations. The exact determination of these parameters for a match and for a definite difference in intensity will require the use of the original equations (1) and (8). Thus θ determined in this way gives $-15° 0' 36''$ and $-15° 0' 11''$ instead of the preceding values, — differences in settings sufficient to become evident to the eye. The error is due to the unequal angles which the axes of each ellipse makes, on each side of it, with the plane of polarization as determined by (38) and (40), and, consequently, to a slight inequality of the minor axes for giving a match, a condition contrary to the equality $\gamma = -\gamma'$. The smaller the orders, however, the less this error becomes.

If in (1) $i = 45$ and $N'' = 0$ we shall have

$$\begin{aligned} I' = \cos^2 s - \sin 2a \sin 2(a-s) \sin^2 \pi N' - \cos^2 a \cos 2(a-s) \sin^2 \pi(N' + N) \\ + \sin^2 a \cos 2(a-s) \sin^2 \pi(N' - N), \end{aligned}$$

and

$$I = \cos^2 s - \cos 2a \cos 2(a-s) \sin^2 \pi N. \tag{47}$$

If further $N = N'$,

$$\begin{aligned} I' = \cos^2 s - \sin 2a \sin 2(a-s) \sin^2 \pi N \\ - \cos^2 a \cos 2(a-s) \sin^2 2\pi N, \end{aligned}$$

and

$$I = \cos^2 s - \cos 2a \cos 2(a-s) \sin^2 \pi N. \tag{48}$$

If now the nicols are crossed, $s = 90$ and we shall have

$$I' = \sin^2 2a \sin^2 \pi N + \cos^2 a \cos 2a \sin^2 2\pi N,$$

and (49)

$$I = \cos^2 2a \sin^2 \pi N.$$

The value of a is obtained from

$$\tan^2 a = \frac{B \pm \sqrt{B^2 + 4AC}}{2A} \tag{8}$$

where, if $N'' = 0$,

$$\begin{aligned} A &= \sin \pi\,(2N - N'), \\ B &= 4 \sin \pi N' - \sin \pi\,(2N + N') + \sin \pi\,(2N - N'), \\ C &= \sin \pi\,(2N + N'), \end{aligned} \tag{8a}$$

and if further $N = N'$,

$$\begin{aligned} A &= \sin \pi N, \\ B &= 5 \sin \pi N - \sin 3\pi N, \\ C &= \sin 3\pi N. \end{aligned} \tag{8b}$$

For the more intense radiants the equivalent orders in the two portions of the field may be as low as 1/180 and 1/360. This would make N' 2/180 and 2/360 respectively, and N any value which would give for a definite angle θ the former equivalent orders.

Substituting in (8) the values of A, B and C obtained from (8*b*) on making $N = 1/90$ and $1/180$ we find

$$a = \pm 60^\circ\ 0'\ 36''$$

$$\theta = \begin{cases} -\ \ 15^\circ\ 0'\ 36'' \\ +\ 105^\circ\ 0'\ 36'' \end{cases}$$

and

$$a = \pm 60^\circ\ 0'\ 11''$$

$$\theta = \begin{cases} -\ 15^\circ\ 0'\ 11'' \\ \ \ \ 105^\circ\ 0'\ 11'' \end{cases}$$

respectively, since $\theta = a + 45^\circ$.

Using angles with the minus signs and substituting in (49) we shall find in the first case

$$I = .00030486 \quad \text{and} \quad I' = .00030487,$$

and with the second order

$$I = .000076175, \quad I' = .000076173.$$

Substituting again in (48) and making $s = 89°$ we find

$$I = .0005908, \quad I' = .0006644,$$

and hence

$$\frac{I - I'}{I} = .125 \text{ approximately},$$

and for $N = 1/180$,

$$I = .00037611, \quad I' = .00039451$$

and

$$\frac{I - I'}{I} = .049.$$

These latter are the relative differences in intensities between the two fields when we rotate the nicol in the region most sensitive to such a change.

If now we make $N = 1/180$, $N' = 1\frac{5}{8}/180$ we find similarly

$$a = \pm 78° \ 13' \ 36'',$$
$$\theta = -33° \ 13' \ 36'',$$

and

$$I = .000255955,$$
$$I' = .000255954 \quad \text{for} \quad s = 90°,$$

and

$$I = .0005565,$$
$$I' = .0005720,$$

and

$$\frac{I - I'}{I} = .028 \quad \text{for} \quad s = 89°.$$

the latter. In the second example the intensity is one fourth that in the first while the change is about one third. If $N = N'/2$ the axes of the vibrations would be exactly parallel and there would be no change on rotating the nicol. The calculations show that the conditions are best met by setting the thicker section in an azimuth of 45° and so selecting the orders that the two portions of the field approach as nearly as possible a "crossed" system.

We may now calculate from (36), (37), (40), and (41) δ', δ, γ' and γ.

For $N = N' = 1/90$, $a = -60^\circ\ 0'\ 36''$,

$$\gamma' = -59'\ 47''$$

$$\gamma = \quad 1^\circ$$

and

$$\tan\gamma' = -.017392, \quad \tan^2\gamma' = .00030248,$$

$$\tan\gamma = \quad .017455, \quad \tan^2\gamma = .00030468,$$

$$\delta' = -15^\circ\ 6'\ \ 1'',$$

$$\delta = -14^\circ\ 58'\ 47'',$$

and

$$d\delta = 7'\ 14''.$$

For $N = N' = 1/180$, $a = -60^\circ\ 0'\ 11''$.

$$\gamma' = -29'\ 58'', \quad \tan\gamma' = -.0087267, \quad \tan^2\gamma' = .000076155,$$

$$\gamma = \quad 30' \qquad \tan\gamma = .0087268, \qquad \tan\gamma^2 = .000076158,$$

$$\delta' = -15^\circ\ 1'\ \ 33'',$$

$$\delta = -14^\circ\ 59'\ 43'',$$

and

$$d\delta = 1'\ 50''.$$

For

$$N = \tfrac{1}{180},\ N = 1\tfrac{5}{6}/180,\ a = -78^\circ\ 13'\ 26'',$$

It is evident from the above calculations that the difference in intensity of the two portions of the field will depend upon both the inclination of the major axes of the two vibrations with one another, as well as upon the ratio of their axes in a single vibration. In the first example the major axis from the two plates makes an angle of

$$-15^\circ\, 0'\, 36'' - 15^\circ\, 6'\, 1'' = 5'\, 25'',$$

and that from the single plate an angle of

$$-15^\circ\, 0'\, 36'' - 14^\circ\, 58'\, 47'' = -1'\, 49'',$$

thus showing that the two ellipses are not symmetrical with respect to the plane of polarization. In the same way we find the corresponding angles in the second and third examples to be 1′ 22″, − 28″ and 1′ 9″, − 23″ respectively. Since now the ratios of the axes of each of the pairs of vibrations and consequently their squares differ by a much smaller quantity than the intensities calculated directly from (47) and (48) we conclude that this change in the match on rotating the nicol is due mainly to the inclination of the axes of the two vibrations with each other. This inclination we may examine by means of (43) and (44). Supposing it were possible to obtain a vanishing line by placing in juxtaposition and crossed two strips from the same section. If now the element to be examined is placed in the same azimuth of 45° and if $N' - N'' = 2dN = dN'$, where dN is the order of each strip, N'' will represent the order of the interposed element and N the order of the compensating plate.

Since $\delta' = \delta'' = \theta$ nearly, within the limits of a practicable system, since further from (8) $\theta_1 = \theta_2$ approximately; for two different systems in which N and N' are not too large and are in constant ratio, we see that $\delta' = \delta''$ increases approximately in proportion to N. As it has already been shown that the orders which may be used in the half-shade system itself which will show no relative change in intensity on rotating the analyzer, are so low, even with the more intense radiants, as to allow only sufficient intensity for observation, we see that a rotating compensating system N cannot be used.[1] Hence, even if the axes of the vibrations in the two portions of the field are parallel, a rotating compensator is not practicable in the half-shade type of differential refractometers. If we use a single strip dN, the same thing will hold since $2dN$ becomes dN the order of the strip. After once obtaining the sections of a half-shade which will satisfy the necessary conditions, in order to use the same, we must apply some mode of compensation in which the axes do not rotate. For small effects the strained glass strip offers the most satisfactory means. Indeed we may dispense with the second plate N and set for a match by means of the glass strip and then insert the element to be examined and reset for a match. However, for constancy in the two portions of the field under temperature and mechanical changes, a strip N' and second section N are much to be preferred.

From the third example above we see that the nearer $N = N'/2$, providing $N \nless N'/2$ the more nearly we approximate a true elliptical half-shade system, *i. e.*, one which will not show a differential effect when the element to be examined possesses rotary power. For very thin mica films this may be accomplished by the method of overlapping, but this will be at the expense of sharpness of the vanishing line in the field of view. Thus, suppose from the same section we overlap one portion so that the principal axes in each part are as nearly parallel to each other as possible. This double section will now serve as our strip and the other portion of the same section will, when crossed, give us the required half-shade system. These may be mounted together in this position where this is possible, or each may be cemented to its own mount. As it will be impossible to overlap such a strip with the axes exactly parallel, we may readily calculate the change in intensity for the small angle between them.

If, instead of using rectangular components, as is done in deducing Fresnel's expression (1) for the intensity of light, we use conjugate components we may reduce the number of terms and simplify the calculation. Suppose now we have such a double strip followed by the second section crossed or in subtractive order, then muth with the plane of polarization, N_1, N_2 those of the succeeding elements, and a_1, a_2, a' the aximuths of each element with the preceding one. Then if α and β are the conjugate components forming the resultant transmitted by the analyzer, $\alpha^2 + \beta^2 = I'$ will be the intensity of this portion of the field. The conjugate components α and β are given by the following relations:

$$\begin{aligned}\alpha = {} & \cos \pi N_1 [\cos \pi N_0 \cos \pi N_2 \cos (a_0 + a_1 + a_2 + a') \\ & \qquad - \sin \pi N_0 \sin \pi N_2 \cos (a_0 - a_1 - a_2 + a')] \\ & - \sin \pi N_1 [\cos \pi N_0 \sin \pi N_2 \cos (a_0 + a_1 - a_2 + a') \\ & \qquad + \sin \pi N_0 \cos \pi N_2 \cos (a_0 - a_1 + a_2 + a')] \\ \beta = {} & \cos \pi N_1 [\cos \pi N_0 \sin \pi N_2 \cos (a_0 + a_1 + a_2 - a') \\ & \qquad + \sin \pi N_0 \cos \pi N_2 \cos (a_0 - a_1 - a_2 - a')] \\ & + \sin \pi N_1 [\cos \pi N_0 \cos \pi N_2 \cos (a_0 + a_1 - a_2 - a') \\ & \qquad - \sin \pi N_0 \sin \pi N_2 \cos (a_0 - a_1 + a_2 - a')].\end{aligned} \tag{50}$$

Make

$$a_0 = 45°, \ a_1 = 0°, \ a_2 = -75°, \ a' = 120°,$$

and

$$N_0 = \tfrac{1}{90}, \ N_1 = \tfrac{1}{6}, \ N_2 = \tfrac{1}{4}, \text{ or, } \pi N_0 = 2°, \ \pi N_1 = 30°, \ \pi N_2 = 45°.$$

This will make the polarizer and analyzer crossed.

Substituting these values in the above equations we find

$$I' = .056016.$$

If we now rotate the overlapped portion so that its principal axis makes 3° with that of the other we shall have

$$a_0 = 42°, \ a_1 = 3°, \ a_2 = -75°, \ a_1 = 120°.$$

Substituting again we have $I' = .056098$.

The change in intensity due to the rotation through 3° of this strip is thus about four times smaller than could be detected. This example is not that of a perfectly compensated or crossed system. However, the rotation of the strip destroys the compensation in part since it subtracts from the element N_1 and adds slightly to the compensator N_2 and hence the intensity should increase. Such a system will, therefore, meet all the requirements of a half-shade, leaving out of consideration the comparative sharpness of the vanishing line.

If in (50) we interchange a_0 with a' and a_1 with a_2 and N_0 with N_2 we find that α and β remain unchanged and therefore that the system is a reversible one; and the intensity will not be changed if we reverse the direction of the light, or if we reverse the order of the elements themselves.

It should be noted that in the mica rotating compensator its differential dispersion is of course the same as that of the strip and once a match is obtained for any one color it will remain a match for all colors. If the differential dispersion (or its equivalent in change of phase in any case of relative retardation) of the element to be examined is not that of mica, it will not remain a match for all colors. The same will be true in using the glass-strip compensator, whose differential dispersion is probably different from that of mica as well as from that resulting from a simple change in wavelength although the latter has not yet been detected.

Physical Laboratory,
University of Nebraska,
July 18, 1904.

Errata to Previous Papers.

P. 72, line 10, read — $\dfrac{\sin\pi[2N + (N' + N'')]}{\sin\pi[2N - (N' + N'')]}$.

P. 87, line 29, read "*a gain of an order*" instead of "*a gain of half an order.*"

TESTS OF SOME OILS SUITABLE FOR USE IN AIR PUMPS.

By P. G. Nutting.

THE large and rapidly increasing use of oil flush air pumps (Fluess patent) makes it highly desirable to know what standard grades of oil available in American markets are suitable for use in these pumps. A good pump oil must have first of all a very low vapor pressure and great fluidity. The vapor pressure of the oil used of course limits the vacuum attainable. With a well cared for pump, a good Röntgen ray vacuum should be easily and quickly attainable. Heavy, viscous oil of course makes the pump run less easily and rapidly.

From among a large number of stock samples of mineral oils kept by the Standard Oil Company, five were selected for the vapor pressure test. These were: Ruby Capital Cylinder, Dark Neutral, Standard Gas Engine, Atlantic Red, and Renown Engine. Beside these, Transformer oil supplied by the General Electric Company, and the pump oil supplied by the Pulsometer Company, were tested.

A specially constructed McLeod gauge was used in making the tests. The oil was placed in a small bulb connected with the gauge by a narrow tube, and the whole arranged to permit of a hot water bath being placed about the oil bulb and an easy and thorough cleaning between tests. The evacuating was done with a motor-driven Barr & Stroud rotary mercury pump in series with Geryk oil pump.

All the oils tested give off considerable dissolved air or some light petroleum product as soon as the pressure is reduced to about 3 mm. It seems most probable that it is dissolved air that comes off. The boiling caused by it soon ceases entirely at lower pressure. When using an oil pump for very low vacua it is advisable to place the oil to be used in a bottle and exhaust it before introducing it into the pump, but for ordinary work this previous exhaustion is unnecessary, as the pump will exhaust its own oil quite thoroughly. In fact, the vapor of volatile fluids like benzine may be pumped off with an oil pump without seriously affecting its working capacity. Chemically active gases and vapors like chlorine and bromine may also be pumped without injury to the exposed parts of the pump. Many of these gases are at once taken up by the oil, and the contaminated oil may be easily and quickly replaced by fresh.

None of these oils appears to possess a definite vapor pressure, just as they possess no definite boiling point. The pressures quoted in the table are to be regarded as superior limits to the vapor pressures rather than mean or minimum values. They are a good indication of the relative merits of the oils.

Owing to the extreme slowness with which vaporization takes place and the rapidity of action of the oil pump, a pressure much lower than the vapor pressure of the oil used may be obtained provided the leading in tubes are clean. The fact that a considerably better vacuum may be obtained when the pump is cool than when it is warm, appears to be due to the effect of temperature on the rate of vaporization rather than on the vapor pressure. With a pump using oil No. 5, whose static vapor pressure was found to be 0.015, a pressure as low as 0.004 mm. has been recorded. The vapor pressures were tested at various temperatures from zero to 60° C., but were found not to decrease more than from two to five per cent. for the whole interval.

The Transformer Oil appears to be identical with the Standard Gas Engine; while the oil originally supplied with the pump, in color and general appearance, as well as in boiling point, is very similar to the Dark Neutral.

No.	Oil.	Vapor Pressure in Millimeters of Mercury.	Name of Firm Supplying Oil.
1	Ruby capital cylinder.	0.03	Standard Oil Co.
2	Dark neutral.	0.04	" " "
3	Pump oil (No. 2).	0.02	Pulsometer Co., London.
4	Transformer oil (No. 5).	0.015	General Electric Co.
5	Standard gas engine.	0.015	Standard Oil Co.
6	Atlantic red.	0.02	" " "
7	Renown engine.	0.03	" " "

Both the Transformer and Standard Gas Engine oils showed a lower vapor pressure than the oil supplied with the pump. All three varieties are very fluid. The Standard Gas Engine oil is now being used exclusively in the pumps belonging to the bureau. It is a limpid, amber-colored petroleum product intended primarily for high temperature lubrication.

National Bureau of Standards.

NEW BOOKS.

The Metric Fallacy, by A. Halsey, and *The Metric Failure in the Textile Industry*, by Samuel S. Dale. New York, D. Van Nostrand Company, 1904. 8vo. Pp. 231.

In the midst of the admiration universally expressed for the metric system a few voices have been raised, more noisy than numerous, to maintain that its exclusive adoption would be, if not a step backward, at least an advance of little importance, and one purchased at too great expense in many countries because of the difficult and costly period of transition. Among the authors who have undertaken to combat the metric system none has been more harsh, and, we are compelled to say, less fair than Mr. Halsey. For several years he has employed in the combat against this admirable system an amount of effort, perseverance, and vehemence worthy of a better cause, bringing forward arguments whether good or bad, and finding even in the most beautiful relations of the metric system reasons for opposing its adoption. His work is voluminous; it abounds in assertions, in citations, and in tables, whose appearance of having been duly and seriously verified might easily deceive. To criticise his argument in detail would necessitate an extended review, for there is hardly a line which does not contain a questionable statement and where it would not be easy as well as interesting to demonstrate the contrary. But we must restrict ourselves to the most typical cases, which will suffice to show in what a spirit of prejudice Mr. Halsey's work has been written.

After a short introduction the author presents the pro-metric and the anti-metric arguments. The readers of this review doubtless know them, either from having heard them many times or from their own experience. Nevertheless none of my readers would expect to see the latter argument summed up in this sentence, which will be sufficient for our purposes without quoting the rest: "We have the simplest and the most uniform system of weights and measures of any country in the world." To sign one's name to such a statement indicates at once that the writer must give up the idea of being taken seriously. Nevertheless Mr. Halsey finds difficulty in suppressing a certain admiration, even though a wholly theoretical one, for the metric system. He patriotically cites the American authors who have spoken in its favor, but he relieves himself of the need of combatting their approval by a very simple step, which consists in denying that a system which is theoretically perfect can offer any practical advantage over a system which is more complicated and of less complete coördination. This antagonism is placed clearly in evidence in sentences such as the following: "Arguments based on 'the beautiful interrelation and correlation of the units' have little more application than a philosophical speculation regarding the appearance of the back side of the moon."

Citing also the opinion of Dr. Giding "it is simple, elastic, scientific, and on the whole a beautiful structure," Mr. Halsey replies "Was there ever such a case of rainbow chasing? Are we a nation of dreaming idealists and transcendentalists that we should be swayed by such considerations?"

Nevertheless we must not be unjust. Even while writing his sonorous sentences Mr. Halsey feels that they are purely declamatory and will not convince anybody. They are a flourish of trumpets in preparation for the attack by solid arguments, which constitute, as it were, the heavy artillery of his army. Our system, he says in substance, possesses over the metric system the inestimable advantage of being accepted without exception by all English-speaking people, while the so-called metric countries still swarm with old units, retained by custom as an imperishable evidence of the impossibility that the metric system should anywhere obtain firm foothold. The evidence for this argument is furnished by a table of about ten pages, in fine print, giving all the various nonmetric units that are still in use in countries where the law requires the use of the metric system. This would certainly be an argument of great weight if the table were not for the most part pure fantasy. Mr. Halsey however seems to treat the metric system well, for he has not cited any non-metric units employed in Denmark or in Russia; for this we should be glad to give due praise, if it were not explained by the fact that the Danish law (the only exception in Europe) does not accept the metric system, and that it has been in use in Russia for only three years. Nevertheless he cites units employed at Riga and at Copenhagen, which, if we are not mistaken, are situated in countries which he excludes from the table. The hospitality of this table is great, for it gives asylum to units of China and of Persia!

But the summit of absurdity is reached in speaking of Malta. Here, Mr. Halsey says, the barrel, containing eleven gallons, is employed as a unit. Why should the metric system be responsible for this, since Malta is an English possession? But can it be that eleven American gallons have suddenly became a British unit? It would seem difficult to believe this; and even if it were true the uniformity claimed by the author for the Anglo-Saxon units would not gain thereby, since (and Mr. Halsey cannot deny it) the English gallon is 21 per cent. larger than the American gallon. We do not reproach the Maltese for using a unit which is not contained in the English table of measures. That is of no consequence to us, but it must be admitted that it is not fair to make of the fact a reproach to the metric system.

Among the units which Mr. Halsey cites there are some which no one in the country indicated by him has ever heard of. For example, one would greatly surprise the inhabitants of Neuchâtel and Geneva, two cities using the French language, by telling them that they make use of the "Stütz" or of the "Ruthe." The latter is a measure of length formerly employed in Germany, but I do not believe that the "Stütz," in spite of its German form, was ever known there. At all events as far as Neuchâtel is concerned this unit is pure invention. I have much difficulty also in believing that the "Pfundeschwer" can be a unit of length at Cairo and at Constantinople. Here, as elsewhere, Mr. Halsey has drawn his information from a bad source. He gives as reference the Special Consular Reports, Vol. XVI. Apparently this document contains typographical errors; or possibly it requires complete revision. Since Mr. Halsey makes use of official publications, how much better he might have done if, in place of reproducing a document which gives us somewhat the effect of a catalogue for a collection of antiquities, he had read and meditated upon the "Report from Her Majesties Representatives on the Metric System." But he takes care not to do this, for this document, written with an impartiality which cannot be gainsaid, is fatal for the thesis which he maintains. I doubt, however, whether he is ignorant of these "Reports," for they are mentioned among others in the replies made by a Commission of the Society of the Engineers of France to a series of questions proposed in consequence of the earlier writings of Mr. Halsey, and showing how little founded were the objections. It would have been at least polite to make reference to this latter document, which had been translated and published by the American Chamber of Commerce at Paris. Mr. Halsey does not do so, and with reason.

But Mr. Halsey's table, referred to above, does not contain merely errors; it also contains numerous truths. Unfortunately these are hard for his contention, as we shall see. Mr. Halsey tells us very truly that there is employed in Greece under the name of "Libra," and in Holland under the name of "Kan," a unit equivalent to 0.2645 gallons, and another equivalent to 2.2046 lbs.; at Milan a "Mina" of 2.6418 gallons, in Belgium a length of 3.937 inches, in Holland an "Ell" of 1.094 yards, in Germany a ton of 2204.6212 lbs. Now if he had taken the trouble to calculate the metric equivalents of these various units he would have found one liter, one kilogram, 10 liters, 10 cm., 1 meter, 1000 kilograms! Since he has given eight figures for this last unit, we may verify the fact that it is equal to the metric ton to within a gram. Mr. Halsey had decidedly bad luck in the choice of his proofs.

Relieved of its references to countries where the metric system is not legally adopted, of units that are either fantastic and unknown, or which, in spite of their local names, correspond exactly with the metric system, Mr. Halsey's table is reduced to an insignificant affair, made up of a few units which have remained in use among the common people, often only in the spoken language. Often, too, it is not a question of units at all, but rather the names of receptacles, whose capacity is specified in terms of the legal units. For example the Fass, the Eimer, and the Sac are the names of vessels for holding liquids, or substances in small pieces; their capacity has a more or less definite value determined by usage; but their contents must be measured or weighed, and are recorded in liters or kilograms. Would it not be contrary to the spirit of good law to require that wine should be sold only by the hectoliter and flour by the hundred weight?

The second part of the work written in collaboration with Mr. Dale, is of more value. It shows that in the textile industries the metric system has not been generally used and that the conditions have been met by using a number of different arbitrary units, more or less local, but with a certain preponderance of British units. We admit to Mr. Dale, as we have to Mr. Halsey, that up to the present time no convention, national or international, has fixed standard values for the pitch and diameter of screws in terms of the millimeter. These are faults which can be only gradually removed, because reforms of this nature require, in addition to national action, international agreements, which are brought about slowly. But the progress is already great. The reproaches of Messrs. Halsey and Dale would have been better justified ten years ago than now. For the textile industries international agreements, have either been made or are about to be made in several European countries. Unification on the basis of such agreements is proceeding at an increasing rate; but so radical a change in customs and habits cannot occur except at considerable expense, nor without some slight temporary loss in manufactured products. This is an objection that can be urged against any change, and it is quite comprehensible that an industrial country like the United States should have hesitated to assume such expense as long as it could be avoided. But the progressive extension of the metric system throughout the whole world leaves no doubt that the Anglo-Saxon countries will some day be forced to its adoption. Mr. Halsey claims that in countries that have adopted the metric system *two* systems are in use at the same time. If we neglect a few cases of local usage in agricultural communities, this is not true. On the other hand in the Anglo-saxon countries two systems are certainly in use, for it has been impossibe to close the door to the metric system, which the laws already recognize, and which is used exclusively by all men of science. It is this fact that has made the old system tolerable, for one is set free from the restrictions of the old system as soon as a scientific or technical problem is approached.

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Le Point Critique des Corps Purs. By E. MATHIAS. 14 × 22 cm. Pp. viii + 256 Paris, C. Naud, 1904. Paper, 7 francs.

This book is an outcome of the report made in 1900 by the author before the International Congress of Physics. The opening chapter deals with the theory of Andrews, with density and saturation curves and with Natterer's tubes. Chapter II. contains a discussion of the Cagniard-Latour phenomenon and points out the insufficiency of Andrews' theory. The next three chapters are devoted to the discussion of experimental methods giving respectively three, two or one of the critical constants. In Chapter VI. the author supplements the preceding discussion with a full account of the methods for calculating the critical constants, and in the chapter following tabulates the critical data for 165 substances covering a wide range of elements and compounds.

The remainder of the book — about 60 pages — is devoted to a discussion of various theories regarding the condition of matter in the critical state. In opposition to the opinion of de Heen and others the author believes that the two-phase system liquid, vapor is univariant after equilibrium has been reached. Two classes of theories are discussed: (1) theories conforming to Avogadro's Law, and assuming the polymerization of gasogenic molecules to form liquidogenic molecules; (2) theories not conforming to Avogadro's Law, but assuming that the volume of gasogenic and liquidogenic molecules is different, or that the density of the molecule may vary. The theories of de Heen and of the author appear under the first of these heads; the theory of Traube under the second. Both de Heen and Traube ignore the pressure of the saturated vapor.

The author calls attention to the importance of time as a factor in the determination of critical phenomena, for the reason that a certain amount of time must always elapse before equilibrium between the two phases can be established. In the concluding chapter he discusses from the standpoint of the liquidogenic theory the properties of matter at temperatures above that at which the meniscus disappears, and explains the alleged existence of liquid above the critical temperature by assuming the presence of partially decomposed liquidogenic molecules.

The book is comprehensive in scope, lucid in style. That its author has himself contributed largely to our knowledge of critical phenomena is a still further reason for predicting that the book will be of interest to all students of physical science. It is only to be regretted that in so broad a treatment of the subject the author ignores those pseudo-critical phenomena observed when superimposed liquids are heated in a closed tube. A. W. Rowe.

Festschrift, Ludwig Boltzmann gewidmet zum Sechzigsten Geburtstag 20 Februar 1904. Leipzig, Barth, 1904. Pp. xii + 930.

The invitation issued to physicists last May to contribute papers to a Jubilee Volume in celebration of the sixtieth birthday of Professor Boltzmann has resulted in the production of a book of 930 pages and containing 117 papers. On the list of authors appear eight names from the British Empire, nine from the United States, and one from Japan. The remainder of the list includes the names of 100 physicists from the continent of Europe. The book, in spite of the restriction as to space made necessary by the large number of contributors, constitutes what is probably the most representative collection of papers on physics ever gathered into a single volume. It is a worthy tribute to Ludwig Boltzmann and bears evidence of the widespread recognition which he has gained through his work. The task of editing the volume fell to Dr. Stefan Meyer to whom the hearty thanks of the contributors and physicists generally are due for the admirable promptness with which the material has been brought together and published. E. L. N.

Electrical Engineering Measurements. By G. D. Aspinall Parr. New York, D. Van Nostrand Company, 1903. Pp. l + 322.

With scores of books dealing with the various special subjects related to electrical engineering, it seems strange that the subject of measuring instruments should not have had its share. Parr's book may be looked upon as a contribution, although it by no means covers this field. The subject is discussed under the following heads: general considerations, moving-needle electromagnetic instruments, moving-coil electromagnetic instruments, hot wire and electrostatic instruments, electromagnetic wattmeters, recording ammeters, voltmeters and wattmeters, miscellaneous standard and other instrumens, electricity supply meters. This last chapter constitutes over one third of the book.

Theory is not discussed. Many instruments are described which are of little interest to an American reader. The descriptions are general and copiously illustrated, trade cuts supplied by the manufacturer being freely used. F. B.

in addition to the wealth of data concerning prices and quantities produced, a short history of each metal and an account of its occurrence and of the methods employed in preparing it for market. The volume contains information not only concerning the commoner metals such as iron, copper, lead and zinc, but likewise a brief account of the rarer metals such as vanadium, tantalium, niobium, thallium, etc. While chiefly of economic interests it will be found a valuable and interesting book of reference for the physicist and the chemist.

E. L. N.

American Meter Practice. By Lyman C. Reed. New York, McGraw Publishing Company, 1903. Pp. l + 196.

The scope of this book is exactly indicated by its title. The author treats the subject from the commercial as well as the technical standpoint, discussing such matters as administrative policy, the treatment of customers, methods of charging, etc. Descriptions of many forms of meters are given, together with the commercial methods of using and testing; but no discussion of theory or special refinements. The book is commended to one seeking information as to the commercial meter and its use

F. B.

Experimental Researches on Reinforced Concrete. By Armand Considere; translated and arranged by Leon S. Moisseiff. New York, McGraw Publishing Co., 1903. Pp. l + 188. (*Received.*)

A First Course in Infinitesimal Calculus. By Daniel A. Murray. New York, Longmans, Green & Co., 1903. Pp. l + 432. (*Received.*)

Theoretische Grundlagen der Starkstrom-Technik. By Charles Proteus Steinmetz. Braunschweig, Germany, Friedr. Vieweg & Sohn, 1903. Pp. l + 323. (*Received.*)

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THE

PHYSICAL REVIEW.

THE TEMPERATURE OF THE ARC.[1]

By C. W. Waidner and G. K. Burgess.

THE comparison of the methods of extrapolation that must be resorted to in the estimation of extremely high temperatures is of growing importance in establishing a satisfactory tentative scale of temperature which is already required in many scientific and industrial operations.

In a study of the possibilities of the application of optical and radiation methods of pyrometry to the estimation of extremely high temperatures we have been led to compare a number of carefully calibrated optical pyrometers at the "temperature of the arc."

The early attemps to estimate the so-called temperature of the arc, or more precisely the temperature of the hottest portion of the positive crater were based on the extrapolation of empirical relations connecting radiation and temperature (Newton, Dulong and Petit, Rosetti, etc.), that were only applicable through very narrow ranges of temperature and the results to which they have led are now only of historical interest.

The first important measurement was that of LeChatelier,[2] who determined for a number of bodies the relation between the photometric intensity of the red light emitted and the temperature. The photometric measurements were made with his optical pyrometer and the temperature measurements with the now well-known LeChatelier thermocouple (platinum, platinum-rhodium 10 per cent.).

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[1] Communication from the National Bureau of Standards.

[2] LeChatelier, C. R., 114, p. 737, 1892; Jl. de Phys. (3), 1, 185, 1892.

This empirical relation is based on experiments extending over the range 700° C. to nearly 1800° C. LeChatelier found by the extrapolation of this relation ($I = 10^{6.7} T^{-3210/T}$) for the temperature of the arc 4400° abs. The red light was obtained by passing the radiation through red glass which probably lets through the shorter wave-lengths at high temperatures so that the measured intensity would increase more rapidly than the formula would indicate. This would act in the direction of making the result come out too high.

Violle[1] made an estimate of the temperature of the arc by a calorimetric method. A small removable button of carbon on the end of the positive crater was dropped into the calorimeter. The specific heat of carbon was measured and found to obey a linear relation for a considerable range above 1,000°, and the relation thus established was assumed to hold as far as the temperature of the arc. The final value found by Violle was given as 3875° abs. Among the great experimental difficulties of this method are those due to loss of heat by the button in falling to the calorimeter and to non-uniformity of heating. Violle investigated these by varying the height of fall and size of the button. In general criticism of this method it should be noted that the specific heat of most substances increases more rapidly as the temperature corresponding to change of state is approached than at low temperatures. Although the difficulties of this method are very great, as admitted by Violle, the result is nevertheless interesting as a determination by a method widely different from those usually employed based on the extrapolation of relations connecting radiation and temperature.

Wilson and Gray[2] made an estimate of the temperature of the arc by a method in which the radiation from the positive crater falling on one junction of a differential radiomicrometer was balanced by the radiation on the other junction from a known area of incandescent polished platinum strip, whose temperature was measured by its expansion after the principle of the Joly meldometer. The relation between the radiation of polished platinum and of assumption made that the radiation from the carbon obeyed the same law as from the copper oxide, which is very nearly the case. Knowing then the apparent areas of the two sources of radiation, and knowing the relation connecting radiation and temperature, they could at once find the temperature to which the platinum strip would have to be raised to balance the arc if the apparent areas were equal. The result to which these experimenters were led was 3600° absolute. In discussing this experiment which was admirably carried out it must be remembered that it was done at a time when the laws of radiation were not so well understood as they were a few years later. The method of measuring the temperature made use of in these experiments is capable of considerable precision. Their temperature scale is probably about 20° low at 1000°, inasmuch as the melting point of gold was taken as 1041° in the calibration of the platinum strip. The radiation of the bare platinum was found to increase approximately as the 4th power of the absolute temperature and of the blackened platinum about as the 3.4th power. This is not in agreement with subsequent researches of Paschen and of Lummer, Pringsheim and Kurlbaum. The bare platinum was enclosed in a gilt case, which made it approximate more or less to a black body. The strongest criticism of this method is undoubtedly that it is based on the extrapolation of an empirical relation which was studied only through a comparatively narrow range of temperature (650° to 1150°), and the results cannot, therefore, be given so much weight as those obtained by the extrapolation of Planck's law, which has been found to satisfy the results of experiment throughout the range from — 200° C. to + 1500° C.

Wanner[1] studied experimentally the relation connecting the temperature and the photometric intensity of monochromatic light of different wave-lengths emitted by several black bodies of different construction, up to about 1400° absolute, and found that the results were in agreement with Wien's relation for the spectral distribution of energy for a black body, $J = c_1\lambda^{-5}e^{-c_2/\lambda T}$, as shown by the linear relation between $\log J$ and $1/T$. Wanner then applied this method to an estimation of the temperature of the arc, by comparing with a spectrophotometer the intensity of the red ($\lambda = 0.6563\mu$) and green ($\lambda = 0.5461\ \mu$) radiation with the intensity of the same radiation emitted by a black body (indirectly for convenience through the intermediary of an amylacetate flame whose radiation for these wave-lengths had been compared with black body radiation). Using red light Wanner finds for the temperature of the hottest portion of the positive crater 3720°, and using green light 3700° abs. when cored carbons (Dochtkohle) are used; using retort carbons he finds for measurements with the red and green light 3875° and 3895° abs. respectively. If it can be assumed that the Wien equation continues to hold for such extremely high temperatures, which will be referred to again, this determination of Wanner must be given considerable weight, as the method is capable of precision. It must be noted that inasmuch as this determination is based on the extrapolation of the Wien equation which applies to a black body, the temperature thus found is the "black body temperature," *i. e.*, the temperature that a black body would have to emit light of the same intensity. Barring the presence of luminescence, this method therefore gives the lower limit of temperature, so that the true temperature of the positive crater must at least be higher, by an amount depending on how much its radiation differs from black body radiation.

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[1] Wanner, Ann. d. Physik, 2, 141, 1900.

Lummer and Pringsheim[1] made use of the relation $\lambda_m T =$ const. connecting the wave-length λ_m having maximum energy and the absolute temperature T of the radiating body, to estimate the limiting temperatures of a number of incandescent bodies, such as the Nernst filament, the Welsbach mantle, Argand gas flame, and the electric arc. The value of the constant for a black body is 2940 according to their experiments and 2921 by those of Paschen,[2] a most satisfactory agreement. For the radiation from polished platinum Lummer and Pringsheim have found for the constant 2630. Assuming then that the "displacement law" continues to hold, and that the radiation from the crater of the arc is intermediate between these, which they have shown is very probable, these investigators concluded that the temperature of the positive crater must be between

$$T_{\text{min}} = \frac{2630}{0.7} = 3750^\circ \text{ and } T_{\text{max}} = \frac{2940}{0.7} = 4200^\circ \text{ abs.}$$

From the flattened form of the energy curve and the effect of atmospheric absorption, it is difficult to locate the position of the maximum with precision. It must be said that the "displacement law" on which this method is based is one of the best established laws of radiation, both from the theoretical and experimental side.

Petavel[1] has found that his observations on platinum radiation satisfy the formula

$$(t - 400) = 889.6 \sqrt[6.9]{b}$$

where t is degrees centigrade and b the intrinsic brilliancy per square centimeter measured photometrically. For the crater of the arc he found $b = 11{,}000$ candles per cm.[2], which would give $t = 3830^\circ$ C. (4100° abs.) assuming that carbon and platinum obey the same radiation law. This assumption gives too high a value for t. With the constants proper to carbon in the above formula, the method might give good results.

By an examination of a spectral energy curve of the positive pole of an electric arc in a paper by Abney and Festing,[2] F. W. Very[3] was led to fix the maximum at about 0.73 μ. This gives for the upper and lower limits between which the temperature of the positive carbon must lie

$$T_{\text{max}} = \frac{2940}{0.73} = 4025^\circ \text{ abs.}$$

$$T_{\text{min}} = \frac{2630}{0.73} = 3600^\circ \text{ abs.}$$

More recently, Féry[4] has made estimates of this temperature by two diff a minute iron-constantan thermocouple which was joined in circuit with a galvanometer. A preliminary calibration of this thermoelectric telescope with the radiation from an electrically heated black body as far as 1500° C. showed that the observed deflections of the galvanometer were in most satisfactory agreement with those calculated from the Stefan-Boltzmann law. By this method Féry was led to the value 3763° abs. as the "black body temperature" of the arc. In this connection it is of interest to note that Lummer and Kurlbaum[1] have found that while the energy of total radiation from iron oxide, which is very approximately the same as from carbon, is only 30 per cent. of that from a black body at 654° abs.; at 1481° abs. it has already grown to 60 per cent. This would indicate that the "black body temperature" of the arc as found by the energy of total radiation from carbon would not differ very much from its true temperature, probably by less than 200°.

Using the photometric method and a modified form of the LeChatelier optical pyrometer and assuming that Wien's law continues to hold, Féry finds 4140° abs. with red light and 4170° abs. using green light. As an explanation of this high value, want of monochromatism of the glass used at once suggests itself. The explanation certainly applies for the usual red glasses that are sent out by Pellin with the LeChatelier optical pyrometer. Moreover, the "center of light" transmission of this glass is nearer to 0.631 μ than 0.659 μ, the value usually assumed from LeChatelier's early determinations, probably made with a different kind of glass.

EXPERIMENTAL RESULTS.

The methods we have used are all dependent on the extrapolation of Wien's equation for the distribution of energy in the spectrum of a black body,

$$J = c\lambda^{-5}e^{-c_2/\lambda T}.$$

and Kurlbaum,[1] and Beckmann[2] have shown that this law does not hold for long wave-lengths and high temperatures. The results of experiment are better represented by the equation deduced by Plank,[3]

The experiments of Rubens and Kurlbaum have shown that this equation is in most satisfactory agreement with the results of experiment throughout the widest range of measurable temperatures, − 200° C. to + 1500° C., and for the longest waves of the infra-red portion of the spectrum obtained by multiple reflection from fluorite ($\lambda = 24.0\mu$ and 31.6μ) and rock salt ($\lambda = 51.2\mu$) surfaces.

However, for the wave-lengths of the visible spectrum, and even for the region of the infra-red in which the largest part of the total energy of the spectrum is found, Wien's law applies with sufficient precision. Lummer and Pringsheim have shown that for values of λT less than 3000, Wien's equation represents the results of experiment to an order of accuracy of 1 per cent. For the photometric measurements in the visible spectrum where the wave-length does not exceed 0.65 μ, the use of Wien's equation is justified for extrapolation to temperatures exceeding 4000° C.

We have determined the temperature of the brightest portion of the positive crater of the electric arc by means of the three pyrometers devised respectively by Le Chatelier, Wanner and Holborn-Kurlbaum. The three instruments are photometers. In the Wanner a narrow spectral band in the red is used as monochromatic source, while in the other two dependence has to be placed upon colored glasses for monochromatism. In the Holborn-Kurlbaum[4] pyrometer, the tip of a very fine incandescent lamp filament is made to disappear against the bright background observed, by varying the current through the lamp. The current is then a function of the temperature which may be found by calibration against a thermocouple.

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[1] Rubens and Kurlbaum, Sitzungsber. der k. Akad. Berlin, 41, 929, 1900; Ann. d. Phys., 4, 649, 1900.

[2] H. Beckmann, Inaug. Dissert. Tübingen, 1898.

[3] Planck, Verh. d. Deutsch. Phys. Ges., 2, 202, 237, 1900; Ann. d. Phys., 4, 553, 1901.

$$J = c_1 \frac{\lambda^{-5}}{e^{/} - 1}.$$

[4] Holborn and Kurlbaum, Ann. der Phys., 10, 225, 1902.

In the Wanner[1] instrument a polarizing device permits balancing photometrically the intensities of the same spectral hues coming from a standard light and the source observed. The angle of the analyzer may be converted into degrees of temperature by calibration. In the LeChatelier[2] instrument the light from the source is cut down by a cats-eye diaphragm until photometric equality is had with light from a comparison source, when from the opening of the cats-eye the temperature may be calculated after the instrument has been calibrated.

Between such a bright source as the arc and any of these instruments it is necessary in order to render photometric comparison possible, to interpose absorbing glasses whose absorption factors (K) have to be determined for the different wave-lengths used.

Wien's law gives:

$$(1) \qquad \log_{10} K = \log \frac{J_1}{J_2} = \frac{c_2}{\lambda} \log_{10} e \left(\frac{1}{T_2} - \frac{1}{T_1} \right),$$

where $c_2 = 14,500$ for a black body, where λ is expressed in μ, e is the base of Naperian logarithms, J_1 and J_2 are the light intensities expressed in any units corresponding to the temperatures T_1 and T_2, T_1 being the absolute black body temperature of the source and T_2 its apparent temperature when the absorbing glasses are interposed. This formula may be used with any of the above pyrometers to determine the arc temperature when K is known and the reading of each instrument has been found for some source which can be viewed without the absorption glasses. As a convenient reference temperature, or source of constant intensity, that of the central area of a definite acetylene flame viewed normally was used. The apparent black body temperature of this source was 1625° abs., the different instruments agreeing to within 3° in this determination. Each of the instruments was also calibrated over a considerable temperature range in terms of a thermocouple using a Lummer-Kurlbaum black lengths. Thus a given absorbing glass gave $K = 1500$ for $\lambda = 0.651\,\mu$ and only 360 for $\lambda = 0.550\,\mu$. Also with these glasses the value of K for n glasses is not K^n, as is generally assumed, but something less than this quantity. Four glasses of the same quality and thickness gave, for $\lambda = 0.651\,\mu$, the following results as determined from a large number of observations using all three pyrometers in this determination :

No. of Glasses.	1	2	3	4
Value of K.	11.9	137	1500	15500
Increment of K.	11.9	11.5	10.9	10.3

The results of our determination of the arc temperature for a current of fifteen amperes are given in the three following tables. The carbons were exceptionally homogeneous, prepared especially for optical projections and arcing very readily ; the positive carbon (13 mm. diameter) being mounted horizontally and the negative (11 mm. diameter) being vertical. The potential across the arc was about 65 volts. In order to prevent the formation of a deep crater on the under side of the positive carbon the latter was occasionally trimmed down so that the hottest part of the crater could be readily viewed in a horizontal line.

Although the three pyrometers give practically an identical value (3700° abs.) for the arc temperature, yet we do not consider all the observations of equal weight. The red glass $\lambda = 0.630\,\mu$ is not sufficiently homogeneous for good photometric measurements ; with the Holborn-Kurlbaum instrument, for instance, the filament of the lamp cannot be made to disappear when using this glass. Glass $\lambda = 0.651\,\mu$, on the other hand, leaves practically nothing to be desired as to its homogeneity and the results obtained with it have the greatest certainty. The values of the equivalent wave-lengths for the red glasses were verified by Dr. Nutting to an accuracy well within the limits of error from other sources.

The relative temperatures obtained with the various instruments are independent of the absorption factors, the latter being dependent upon the absorption glasses, which were the same for all the pyrometers, and upon the wave-length used. The absolute values we have obtained for the arc temperatures would evidently be in

Determinations with Holborn-Kurlbaum Pyrometer.

Date, 1904.	Number of Lamp used in *H-K* Pyrometer	Wave-length trans. by Glass.	K = Absorption Factor.	Absolute Temperature of Arc.
June 14.	No. 132	0.651 μ	1500	3670
" "	156	"	"	3680
" "	145	"	"	3680
" 15.	132	"	"	3630
" "	156	"	"	3690
" "	145	"	"	3670
" 20.	132	0.630	1350	3750
" 15.	156	"	"	3790
" 16.	132	0.550	360	3640
				Mean, 3690

Determinations with LeChatelier Optical Pyrometer.

Date.	Wave-length Used.	K = Absorption Factor.	Absolute Temperature of Arc.
June 17.	0.651 μ	15500	3740
" "	"	"	3750
" "	"	"	3750
" "	"	"	3690
" 16.	"	1500	3730
" 18.	0.630	14850	3680
" "	"	1350	3730
			Mean, 3720

Determinations with Wanner Pyrometer.

Date, 1904.	Wave-length Used.	K = Absorption Factor.	Absolute Temperature of Arc.
June 20.	0.656 μ	1500	3660
".	"	"	3700
			Mean, 3680

error if the absorption coefficients are incorrect. The values chosen are the best that could be deduced from several hundred observations made with all the pyrometers. The absorption factor for green light ($\lambda = 0.550\,\mu$) is however less well determined.

As to the pyrometers themselves and the reliance to be placed upon the results obtained with the different instruments, the greatest uncertainty seems to be with the Wanner because it is more difficult with this instrument than with the others to be sure that one is sighting upon the desired spot, as the instrument is not a telescope; also the extrapolation of the scale of this particular instrument is somewhat uncertain. As between the LeChatelier and Holborn-Kurlbaum determinations, the preference is in favor of the latter, as a smaller area is used in the photometry and there are more independent checks upon its calibration; thus using three comparison lamps, identical results are obtained.

On the whole, a better agreement among different instruments employing such varied methods of measurement could hardly be expected at such a high temperature. Our experiments with pure graphite have shown that the value, 3700° abs., would not be increased by more than 50°. The effect of material of carbons and of current density will also be considered.

VARIATIONS IN TEMPERATURE OF THE ARC.

(*a*) *With Current.* — It is generally accepted since Violle[1] so stated, that the temperature of the arc is independent of the current and this temperature is assumed to be the boiling point of carbon. There are, however, at least, two reasons for which we should expect this temperature to vary with the current. As the current is increased there will be a tendency to superheat the viscous carbon layer from which the vapor boils, even though this vapor does not have a higher temperature than the normal boiling point, and as it is this viscous layer which is observed, variations of current above a certain limit should be accompanied by changes in temperature. Again, with low current a smaller area is heated so that it will be more cooled by conduction.

An examination of the observations from which it has been inferred that the arc temperature is independent of current seems to indicate that this conclusion is unwarranted, for no observer has published a series of results sufficiently concordant in themselves or of sufficient number to enable him to state with certainty whether or not the effect exists. This has been due largely to the tediousness or inadequacy of the methods employed.

The constancy of brightness was first announced as probable by

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[1] Violle, Jour. de Phys. (3), 2, 545, 1893.

Rossetti in 1878, and Violle was the first to state as a result of his own experiments that "the brightness of the positive carbon is rigorously independent of the electric power expended to produce the arc," changing from within the limits 10 amperes at 500 watts to 400 amperes at 34,000 watts. Carbons of 3.5 cm. diameter were used for the high currents, but no details of observations are published nor the precision of his spectrophotometric method, which is certainly sensitive though it would be expected that any effect for very high currents would be neutralized in part at least by the larger carbons used. Furthermore, results with carbons of varying size and quality are not comparable. Although Violle showed undoubtedly that with carbons of different sizes adapted to carrying currents of from 10 to 400 amperes, there is no considerable change in temperature, yet it would seem that he did not show conclusively that for a given sized carbon there is no variation of temperature with current for the brightest portion of the positive crater.

Wilson & Gray found that changing from 14 to 25 amperes "the temperature then appeared to be a little higher than with the smaller current. . . . Later experiments . . . indicate an exact equality of temperature." They do not give any data, but their published observations on the arc temperature indicate that it would be difficult to detect variations by their method.

Wanner states that "changing the current 1½ times (15 to 22 amperes) remains without influence." Wanner's published results for a given kind of carbon are few in number (four) and vary over ninety degrees under the same conditions, so that any difference due to current changes might be masked.

Our experiments on the variation of arc temperature with current were first made with a Holborn-Kurlbaum pryometer which is peculiarly well adapted for these measurements, as it is readily sighted and thus the wandering of the brightest spot on the crater can be easily followed; a very small photometric area is employed; and observations may be taken rapidly, within intervals of a few seconds.

In order to secure a good sized image of the crater so as to facilitate the photometric measurements, the instrument, provided with auxiliary lenses and suitable absorption glasses, was placed as near as possible (12–25 cm.) to the arc, which was mounted as previously described. The current from a 120-volt storage battery was varied without the observer being aware of the actual changes, and to render the settings of the pyrometer as unbiased as possible an additional rheostat in its lamp circuit was also worked independently of the observer. No observations were taken on a hissing or humming arc, nor until the arc burned normally after changing current, and the conditions of constant length of arc and constant P.D. across the arc were very closely maintained, the latter being measured with a Weston voltmeter; the value of the P.D. was about 65 volts. Two observers obtained practically identical results which are here summarized:

Date.	Number of Observations.	Current in Arc (Amperes).	Temperature of Arc (Absolute).
June 23, 1904.	51	15	3690
	48	22	3727
	49	30	3782
June 28, 1904.	23	15	3690
	37	22	3741
	42	30	3762

A few observations were also taken with a current of 40 amperes which indicated that the effect persists with increasing current density to the limit that the auxiliary apparatus would stand.

The precision with which this determination can be made on a single carbon with the Holborn-Kurlbaum instrument is shown by the observations of June 23, 1904, namely, that the probable error of the mean in any one of the three series on that date is less than 3° and no photometric observation with a current of 15 amperes was as high as the mean value for 30 amperes. Whatever may be the best value for the arc temperature, this value is apparently a function of the current when the P.D. across the arc is a constant and the arc is burning normally.

If the current is kept constant and the P.D. varied it is necessary to vary simultaneously the length of the arc, therefore temperature results obtained by this method of varying the power are not directly comparable as the length of the arc is a variable not easily eliminated quantitatively.

It does not seem that our finding a small temperature change with current can be explained on physiological grounds, due to contrast in light intensities entering the eye, since with the smallest current used the image of the hottest part of the crater was many times larger than the image of the filament of the comparison lamp. To further test this point we determined the difference in temperature with a LeChatelier optical pyrometer provided with auxiliary lens and a very homogeneous red glass before the eyepiece. A number of consistent observations indicate that changing the current from 15 to 30 amperes causes a rise in temperature of about 70°. With this instrument the eye does not receive light except through a 1 mm^2. opening, so that the effects of the surrounding field cannot influence the eye.

(*b*) *With the Material of Carbons.* — Wanner finds a difference of about 170° in changing from cored to retort carbon, the former giving a low value, and our experiments also show that the kind of carbon used will influence the apparent temperature of the arc. The homogeneous carbons described above gave us results 40° lower than with a very pure graphite furnished by the Acheson Company. This result was obtained both with the Holborn-Kurlbaum and LeChatelier optical pyrometers.

In commercial carbons, the presence of salts seems to facilitate the arcing, and with large currents a steadiness can be had which cannot be approached with pure graphite carbons. It appears that the arc temperature as determined with commercial carbons is lower than the point obtained for pure carbon.

(*c*) *Effect on the Arc as a Standard of Light.* — These questions are of interest in connection with the proposed use of the radiation from a definite area of the brightest part of the crater of the + carbon of the electric arc as a standard source of light.. If the variations in temperature are as great as indicated in our experiments a variation of current from 15 to 22 amperes (corresponding to a temperature variation of 40° C. in the temperature of the arc) would correspond to a variation in the photometric intensity of something like 7 per cent. Even with the same current and the same carbons our results appear to show relative time changes of photometric intensity that seem considerably greater than the experimental errors, that may be due to varying shape of the carbons and changes in current density, as well as variations in the composition of the material. While it would be possible to define the current, it would not be as easy to control the form of the crater and the current density in the region from which the light is to be taken.

Another difficulty in the above method of defining a light standard arises from the variations that may result from the use of different kinds of carbon as shown above. The experiments of Wanner indicate an enormous change in the intensity of light due to this cause.

Blondel[1] recognized the objections to the use of a screened area of the positive crater as a primary standard of light and recommended its use as a secondary standard for arc photometry. He found the nature of the carbon to be important in determining the absolute brightness (of a square millimeter); thus for five different qualities of homogeneous carbon an average value of 158 candles was found, the extremes being 150 and 163. With cored carbons he obtained values lower than 130 candles for the maximum brightness.

CONCLUSION.

A résumé of the estimates of the arc temperature that have been discussed in the preceeding pages is given in the table.

From this table it will be seen that the photometric methods based on the extrapolation of Wien's equation show that the "black body temperature of the arc" (pure graphite) is at least 3750° abs. so that its true temperature must be higher than this; it is not possible to state how much, in the absence of more definite knowledge concerning the departure of carbon from black body radiation. In the light of the best evidence that is at present available it would seem that the true temperature of the hottest part of the positive carbon is between 3900° and 4000° absolute.

If we are justified in assuming that the high values found by LeChatelier and by Féry, using photometric methods, are satisfactorily explained by the reasons we have advanced, a comparison of the results contained in the above table will show, when we consider

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[1] Blondel, Proc. Int. Elec. Congress, Chicago, 1893.

Observer.	Absolute Temperature of the Arc.	Method.
LeChatelier.	4370	Photometric: Intensity of red light.
Violle.	3870	Calorimetric: Specific heat of carbon.
Wilson & Gray.	3600	Total radiation of copper oxide, empirical relation for.
Wanner.	3700-3900[1]	(Varying with carbons used.) Photometric, in terms of Wien's law.
Very.	Between 3600 and 4000	Wave-length of maximum energy Wien's displacement law: $\lambda_m T=C$.
Lummer and Pringsheim.	Between 3750 and 4200	Wave-length of maximum energy Wien's displacement law: $\lambda\ T=C$.
Féry.	3760[1]	Total radiation: Stefan-Boltzmann law.
	4150[1]	Photometric: Wien's law.
Waidner & Burgess.	(2) 3690[1]	Holborn-Kurlbaum Pyrometer, Wien's law.
	3680[1]	Wanner Pyrometer, Wien's law.
	3720[1]	LeChatelier Pyrometer, Wien's law.

the experimental difficulties and uncertainties that may enter due to varying conditions of the arc (such as current strength, length of arc, quality of carbon, etc.), that the results obtained, dependent as they are on the extreme extrapolation of different laws, are in quite satisfactory agreement, and diminish somewhat the doubt which is always present when we must venture beyond the domain of experiment.

The value obtained by Féry for the "black body temperature" of the arc, by making use of the Stefan-Boltmann law for the total radiation of a black body, is in most satisfactory agreement with the results obtained by Wanner, dependent on the extrapolation of the Wien-Planck relation, and is nearly identical with our own photometric determinations, based on the same relation. It is interesting to note in this connection that Lummer and Pringsheim[3] have found these laws in agreement at 2300° abs.

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[1] Black body temperature.

[2] Pure graphite gives a temperature not over 50° higher.

[3] Lummer and Pringsheim, Verh. Deutsch Phys. Ges. (5), 1, 3, 1903.

If we can assume that the Wien-Planck and the Stefan-Boltzmann relations continue to hold good for thermodynamic radiation at these high temperatures, it would seem, in view of the above facts, that the phenomena of luminescence plays no very important part in the radiation from the positive pole of the electric arc, for, if so, it might be expected to lead to higher values in the estimates based on the photometric methods than would be found by methods dependent on total radiation.

An examination of the available estimates of the temperature of the sun made by methods based on the photometric intensity seems to indicate that these methods lead to higher values than those based on the energy of total radiation. Thus, using the former method LeChatelier[1] finds 7900°[2] abs. The estimates[3] based on total radiation and on the position of the wave-length of maximum energy in the solor spectrum lead to values in the neighborhood of 5500° to 6000° absolute. If this difference in the methods really exists, it would at least suggest the presence of luminescent radiation. It would also be of interest in this connection to use the photometric method with green and blue in addition to red radiation, to see if the former led to higher values as might be expected if luminescence played an important rôle.

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[1] LeChatelier, C. R., 114, 737, 1892.

[2] As LaChatelier's determination of the arc temperature is probably several hundred degrees high, his value of the effective temperature of the sun should be lowered.

[3] Day and Van Ostrand, Astrophys. Jl., 19, p. 39, 1904.

(Contribution from the Cryogenic Laboratory of Wesleyan University.)

EFFECT OF MECHANICAL VIBRATION UPON CARBON DIOXIDE NEAR THE CRITICAL TEMPERATURE.[1] I.

BY W. P. BRADLEY, A. W. BROWNE AND C. F. HALE.

NEARLY three years ago, two of the writers undertook a review of some of the ordinary phenomena of the critical state. The apparatus used was of the Cailletet type, and carbon dioxide was chosen as the subject of the experiments. Since no considerable accuracy was required for the object in view, the apparatus was not mounted upon a pier, but upon an ordinary wooden laboratory desk. This fact led to a peculiar observation.

It happened one day that a small wrench in use about the apparatus fell and struck rather sharply upon the desk. At the same instant a bright flash of fog appeared for a moment in the pressure tube. The tube contained at the time both liquid and gaseous carbon dioxide, and the fog was noticed in both phases. The temperature of the bath was about 30°.

Further investigation showed that almost any form of tapping, as with the knuckles, either upon the desk near the apparatus or upon various portions of the apparatus itself, could be employed to produce the effect, and that the density of the fog was clearly dependent upon the violence of the shock thus imparted.

An investigation of the phenomenon was immediately undertaken, which continued with many interruptions and delays for two years or more. The results thus obtained are far from being exhaustive, but are published at this time for the reason that the pressure of other duties prevents a continuation of the inquiry, for the present at least.

viously the first problem calling for attention was the connection between the blow delivered outside the apparatus and this change of physical state.

To set the problem in better perspective it may be remarked that the appearance of fog in substances which are near their critical state is by no means new or uncommon. So far as we can ascertain, however, it has never been observed under such conditions as have suggested mechanical shock as the cause of it. It usually results from thermal changes in the bath.

Quite independently of the critical state fog can be obtained by expansion in almost any gas or vapor, provided the expansion be of suitable proportions and rapidity. This fact was demonstrated in 1877 by Cailletet in his classical researches upon the liquefaction of the so-called permanent gases. The production of fog in this manner is due to the sudden fall of temperature resulting from the expansion. This leads to the condensation of a multitude of minute liquid droplets, which remain temporarily suspended in the vapor. As the fall of temperature is most pronounced in the center of the tube where the expansion is most nearly adiabatic, the fog is densest in that region. It is always transient, owing to the inflow of heat through the walls of the tube. It may be made to disappear instantly by suddenly restoring the original volume of the system.

Fog may be produced in the liquid phase also by suddenly increasing the volume of the system. In this case, however, it is not due to any change of temperature caused by the expansion, but to the temporary decrease in the pressure to which it was subjected. Multitudes of minute gas bubbles are formed throughout the liquid, rendering it more or less opaque. If the volume of the system undergoes no further change, these bubbles do not disappear quickly like the liquid droplets just mentioned, but remain and rise more or less rapidly and break at the meniscus. They may be made to disappear as suddenly as they came by restoring the original volume. A connection will presently appear between these facts and those which were noticed in our experiments.

To return now to the fog produced by mechanical shock, it was quickly discovered that the direction of the blow was a factor of prime importance. Delivered vertically it produced fog. If horizontally, it did not. A slanting blow gave fog of less density than a vertical one of the same force. Clearly the slight vertical displacement caused by the blow was the cause of the fog, but the manner of its working was not at once apparent.

A little consideration, however, led us to recognize that one portion of the apparatus was capable of a slight motion, or rather vibration, independently of the rest. Moreover, this motion was exclusively vertical in direction and was calculated to produce small but rapid changes in the volume of carbon dioxide. The part referred to is the mercury column in the glass pressure tube.

It is obvious that this column is held in place vertically by the carbon dioxide above and the steel walls of the pressure jacket below. As both of these are elastic, the column is suspended as if between two springs of very high tension, and vertical vibration is thus theoretically possible. It seemed almost incredible, it must be confessed, that so minute a motion as might be expected on these grounds could have produced the fog, for the walls of the steel jacket were more than half an inch thick.

To test the point, a metallic strap was adjusted around the center of the jacket. The strap was provided with thumb screws by means of which pressure could be applied to the jacket. Tightening the screws would diminish slightly the volume of the jacket, and so, indirectly, the volume of the carbon dioxide.

The result was surprising. It is true we could not obtain any visible effect by tightening the screws, but with each release, even though slight, there appeared a fog of much greater density than had ever been secured by rapping. In fact the circumstances were such as to illustrate in most extraordinary fashion the sensitiveness of matter near its critical state to slight increase of volume. *This* fog was due beyond any question to a sudden increase in the volume of the carbon dioxide.

It was now possible to understand how a blow from without could produce the same effect. The immediate effect of a downward blow is to depress for an instant the entire apparatus with the exception of the mercury column. This column, by virtue of its suspension between springs, does not partake at once of the motion, but lags behind the apparatus as a whole. The situation, so far as the carbon dioxide is concerned, is the same as if the apparatus remained stationary and the mercury column alone moved *upward.* Extra pressure is temporarily exerted in the tube, and the volume of the carbon dioxide is correspondingly diminished.

By the time the mercury begins to follow the apparatus downward, the latter has begun to rise. This condition produces just the opposite effect — a momentary decrease of pressure and increase of volume. The decrease of pressure is responsible for the gas bubbles in the liquid phase, and the fall of temperature which accompanies the increase of volume produces the condensation of droplets in the vapor. Both bubbles and droplets disappear again in the next instant when the column returns to its normal position.

Strictly speaking, the fog so produced is not a single flash, but a series of flashes of decreasing density, which cease when the motion of the column becomes sufficiently feeble. But the individual flashes follow each other so rapidly that they cannot be distinguished by the eye, and the series itself is only of momentary duration.

The explanation of the production of fog which has just been given assumes that the effect in both phases is due only to the fall of the mercury column and the consequent increase of volume in the carbon dioxide. It may be urged that the rise of the column also should produce condensation in the saturated vapor. It is quite true that condensation is effected in saturated vapor by decrease of volume, but under these circumstances it never assumes the form of droplets. Decrease in volume results in a rise of temperature in the vapor, which in so far tends to restrict condensation. This rise of temperature is most pronounced near the axis of the tube, where the compression is most nearly adiabatic, and is least felt at the walls which have the temperature of the bath. For this reason the condensation occurs on the walls, which are the coolest part of the system, and the liquid so formed adheres to the walls and flows downward upon them. We shall return again to this form of condensation in a later paper, in presenting some data as to the condition of matter above the critical temperature.

It is quite unnecessary to say that vapor bubbles could never be produced in the liquid phase by decrease of volume.

The next advance in our study will doubtless have been already anticipated. If a single blow causes a transient fog, a rapid succession of blows should make the phenomenon practically continuous.

To test this, a set of tuning forks of rather large size was brought into requisition, not without some misgivings it must be confessed, as to the adequacy of the force of their vibration. Each of the forks was tried in turn, while the carbon dioxide was held at a constant temperature. Each of them produced a transient fog at the moment when its stem was brought, not too carefully, into vertical contact with some part of the apparatus. This was clearly the result of the impact only, as in previous experiments. Only one of the forks gave rise *by its vibrations* to a fog of longer duration. The rest failed entirely to act as desired, even though they were caused to vibrate with much greater force than proved necessary in the case of the one referred to. The temperature of the bath at the time was still about 30°.

It may be noted in passing that the superiority of the single fork would not have been nearly as pronounced, as we afterwards learned, and that all of the forks might in fact have produced fog, though of varying densities, if the temperatures had been a little nearer the critical temperature — at 31° for instance instead of 30°. This later discovery militates in no way against the obvious conclusion to be drawn from the experiment as described, namely, that none of the forks yielded vibrations of sufficient force in themselves to cause fog, and that one of them proved successful only because that one happened to be in resonance with the vibrating column.

If instead of applying the stems of the vibrating forks to the apparatus, the vibrating tines themselves were allowed to beat against it, all of the forks gave fog, but the effect in this case was transient again because the vibrations ceased almost immediately.

The extreme sensitiveness of the carbon dioxide to such minute changes of volume as were produced in these experiments by the vibrating mercury column is dependent upon a variety of factors, the discussion of which will be reserved for the present.

The essential features of the apparatus employed are illustrated in Fig. 1. *A* is the pressure jacket, the internal diameter of which is 3.8 cm., and the external diameter 6.7 cm. Its internal length, measured from the gasket downward, is 19.4 cm. The valve *B* admits water from a hydraulic pump which is not shown, while the similar valve, *C*, leads to the pressure gauge. The pressure tube, *D*, has a capacity of 40 cc., and is mounted in a steel cap, *E*. The manner of the mounting has been described in a previous article.[1] The exposed end of the pressure tube has an internal diameter of 6 mm. and a thickness of wall of 2.5 mm. Both ends of the tube carry arbitrary graduations which were carefully calibrated with due regard to the curvature of the mercury meniscus. The lower end dips under a mercury seal in the jacket, while the upper end is surrounded by apbath which contains two liters of water, the temperature of which may be regulated by means of a very sensitive thermostat to one thousandth of a degree. The thermostat has already been described in detail.[2] For the present purpose it was found quite superfluous to read temperatures to more than two decimal places.

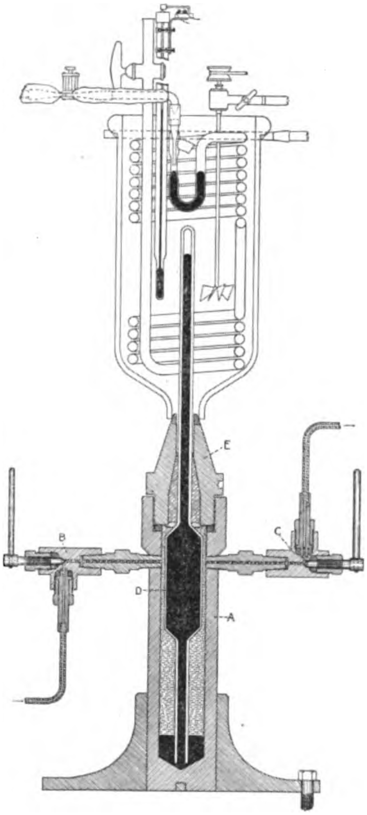

Fig. 1.

The pressure was read when necessary upon the eight-inch dial of a metallic pressure gauge. The gauge was carefully calibrated by comparison with an air manometer, due allowance being made for the deviation of air under pressure from Boyle's law.[3] But it should be said that the chief value of pressure readings in these ex-

and precisely than in any other way (except indeed by the fog itself, as will appear later). The dial was graduated to thirds of an atmosphere (5 lbs.), and when the lag of the needle was corrected by gentle tapping upon the case, readings could be made with considerable accuracy to single pounds. The gauge showed no appreciable change of registration during the period of the experiments.

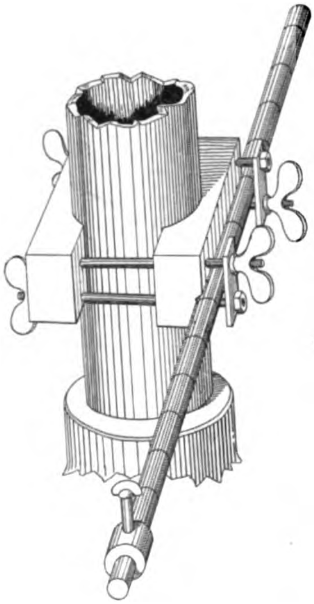
Fig. 2.

Much time was spent in efforts to determine the most practicable form of vibrator with which to induce the vibration of the mercury column. One device after another was tried and abandoned. The best one on the whole proved to be a simple cylindrical steel rod, about 1.1 cm. in diameter, attached horizontally to the center of the pressure jacket by means of an adjustable clamp.

From experiment to experiment the length of the vibrating portion could be changed at the clamp so as to secure coarser changes of pitch, while the finer variations required during the progress of an experiment were provided for by means of sliding weights. These could be fixed by means of suitable setscrews at any point upon the rod. The vibrations were produced by drawing the bow of a bass violin across a suitable point near the end of the rod. This method of inducing vibrations is superior in one respect at least to the electrical one. It permits of instantaneous changes of amplitude of a very wide range, which is an important factor. Direct comparison with a monochord gave the pitch of the rod. It is needless to say that the bow had to be drawn vertically to produce the fog.

Fig. 2 illustrates the vibrator, and the way in which it was clamped to the pressure jacket.

The carbon dioxide employed in the experiments was generated by the action of chemically pure boiled sulphuric acid upon sodium carbonate, later upon sodium bicarbonate. As some discussion of pure carbon dioxide and the precautions necessary in its production will be given in another place, it may be sufficient at this point to add that the amount of impurity (unabsorbed by caustic potash solution) found in the various samples employed varied from one part in 5,000 to one in 40,000, and that the purity tests were made upon gas which had passed through the pressure tube during the act of filling it. The sample of least purity showed a difference of less than one fifth of an atmosphere between the pressures of initial and final liquefaction, while with the best sample the difference fell below the error of reading upon the gauge. It seems clear, therefore, that the phenomena which are associated with the production of fog in the carbon dioxide cannot be ascribed to the effect of impurities in the gas, particularly as there was no discernible difference in the conduct of the different samples in these respects.

The observations so far gathered upon the effects of vibration fall naturally into two classes. One of these contains the results of certain attempts at quantitative study, while the other includes a number of qualitative, and to some extent disconnected, phenomena, which seem to us nevertheless from a theoretical standpoint of rather more importance than the former. Only the former are given in this paper.

Such quantitative data as we have were secured during an attempt to determine the relation between the length of the mercury column and the most effective pitch of the vibrator. It seemed of considerable importance to determine whether the theory of resonance already given in explanation of the genesis of the fog might not be confirmed in this way. If the theory is correct, a longer column, under otherwise equal conditions, should correspond to a lower pitch.

The only practical way of changing the length of the column without interfering with other conditions was to alter the quantity of mercury which served as a seal in the lower part of the jacket. In the tables below one of the lengths of column corresponds to the smallest total quantity of mercury which could safely be used when the carbon dioxide was all liquid and therefore at its minimum volume; another, to the greatest quantity which could be used without danger of overflowing into the valves when the carbon dioxide stood at atmospheric pressure, that is, at its greatest volume; and the third was intermediate between these two.

For the object in view it might have sufficed to make but three observations only, one for each length of the mercury column. It had been observed, however, that the vibratory period depends also upon the temperature of the carbon dioxide. Accordingly for each length of the column a number of tests were made at different temperatures. Still another factor demanded recognition, namely, the position of the meniscus in the system, or in other words, the relation between the volumes of liquid and vapor. The influence of this factor was too great to be neglected, particularly at the lower temperatures, and consequently for each length of column and for each temperature the dependence of pitch upon this relation was studied.

Production of fog either in the vapor or in the liquid is practically limited to the saturated condition, so that the tables refer to the two-phase system only.

It will be noted that the thermal range of the observations is only 2.2 degrees. The upper limit for instance is 31.2°. This does not mean that fog ceased at that temperature, but that it became very difficult indeed to determine accurately the position of the meniscus at temperatures higher than that. The lower limit is 29°. This in turn was not the point at which fog ceased to appear. The lower the temperature the greater the amplitude of vibration required to produce fog. Below 29° the necessary vibrations were of such force that the thermostat was often seriously disturbed, in spite of the fact that it was specially provided with a soft-rubber buffer for the sake of avoiding such an effect. It seems unlikely, however, that fog could have been obtained at a temperature much more than half a degree lower, without the substitution of direct blows for the bowing of the vibrator.

In the tables, the length of the mercury column represents the vertical distance between the upper meniscus, the position of which was read directly, and the lower meniscus within the jacket, the height of which was determined by calculation. Both levels are referred to a temperature of 30° and to the minimum (all liquid) volume of the carbon dioxide.

TABLE I.

Mercury column = 37.54 *cm.*

I. Temp.	II. Pitch.	III. Vapor Length.	IV. Total Length.	V. Vapor Volume cmm.	VI. Total Volume cmm.
30.2°	137.0	3.0	10.5	4.5	123
	132.7	5.0	10.75	19.0	128
	127.8	5.75	11.0	28.0	133
	124.3	6.75	11.25	42.0	138
	121.9	7.25	11.5	51.0	143
	115.0	9.0	12.25	83.0	159
31.0	110.4	4.0	11.0	9.0	133
	109.3	6.0	11.5	31.0	143
	106.8	7.0	11.75	46.0	149
	104.2	9.0	12.25	82.5	159
	103.4	10.0	12.5	102.0	164
	102.0	11.0	13.0	123.0	175
31.2	97.9	4.0	11.0	9.0	133
	96.8	6.0	11.5	31.0	143
	95.9	8.5	12.0	74.0	154

TABLE II.

Mercury column = 35.66 *cm.*

I. Temp.	II. Pitch.	III. Vapor Length.	IV. Total Length.	V. Vapor Volume cmm.	VI. Total Volume cmm.
30.0°	138.2	3.75	10.5	7.9	123
	132.7	6.25	11.0	34.5	133
30.2	134.7	3.0	10.5	4.5	123
	132.7	5.0	10.75	19.0	128
	126.6	6.75	11.25	42.0	138
	115.5	9.0	12.25	82.5	159
	106.8	11.0	13.0	123.0	175
30.5	132.7	2.5	10.5	2.5	123
	129.4	4.5	10.75	13.5	128
	122.4	6.75	11.25	42.0	138
	104.6	11.0	12.75	123.0	170
	102.7	12.0	13.25	144.0	180
31.2	110.4	2.75	11.0	3.5	133
	108.3	5.5	11.25	24.5	138
	102.7	9.25	12.0	87.5	154

Table III.

Mercury column = 33.78 *cm.*

I. Temp.	II. Pitch.	III. Vapor Length.	IV. Total Length.	V. Vapor Volume c.mm.	VI. Total Volume c.mm.
29.0°	145.8	2.0	10.0	1.25	112
	142.0	4.5	10.25	13.5	117
	136.8	6.0	10.75	31.0	128
	133.0	7.0	11.0	46.0	133
	129.1	7.5	11.25	55.0	138
	127.3	8.25	11.5	69.0	143
30.0	143.5	1.75	10.5	1.9	123
	137.4	5.25	10.75	21.75	128
	126.6	7.75	11.5	60.0	143
	120.0	9.0	12.0	83.0	154
	105.7	11.25	13.0	128.0	175
30.5	135.1	4.5	10.75	13.5	128
	123.5	6.75	11.25	42.0	138
	116.0	8.5	12.0	74.0	154
	111.7	9.0	12.0	83.0	154
	111.0	9.5	12.25	93.0	159
	108.3	10.5	12.75	113.0	170
	106.8	11.0	12.75	123.0	170
31.0	116.0	2.75	11.0	3.5	133
	112.1	6.0	11.5	31.0	143
	110.4	8.0	12.0	64.0	154
	106.8	10.0	12.5	102.0	164

Column I. contains the temperatures; Column II., the pitch required to produce fog of greatest density with least vibratory amplitude; Column III., the observed length of the vapor phase in divisions of an arbitrary scale; Column IV., the total length of carbon dioxide of both phases (chiefly for comparison)[1]; Columns V, and VI., the calculated gas and total volumes, respectively, in cubic millimeters.

In the diagrams which follow, abscissas represent the volume of

Diagram I. (Table I.).
Diagram II. (Table II.).
Diagram III. (Table III.).

An inspection of these diagrams reveals the following facts:

1. The longest mercury column corresponds to the lowest pitch. The thesis, to test which the readings were taken, is proved. The curve of any given temperature is lowest in Diagram I., and highest in Diagram III.

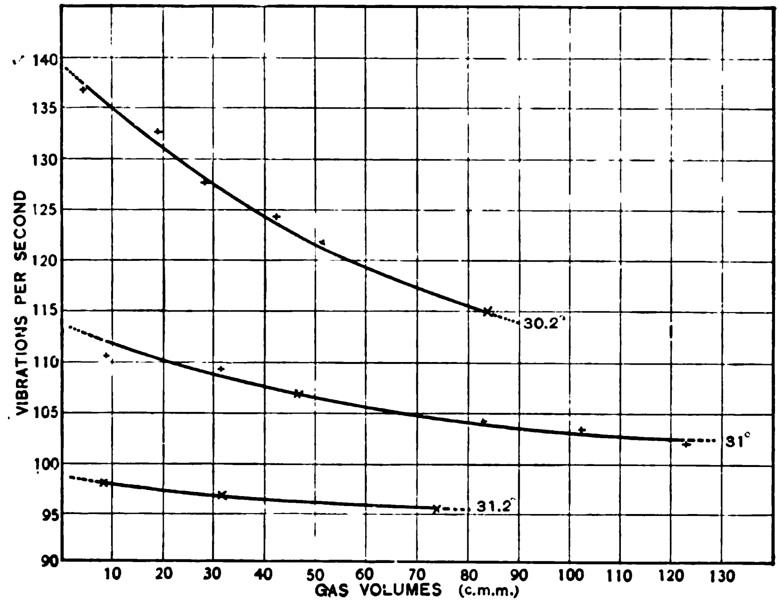

Fig. 3.

It is a curious circumstance, however, that the difference in the height of corresponding curves is greatest for temperatures nearest the critical, and practically disappears for the lowest temperatures in the tables. Thus, the curve of 31.2° in Diagram I. is much lower than that of the same temperature in Diagram II. The curve of 31.0° in Diagram I. is considerably lower, though less so, than that in Diagram III. That of 30.5° in Diagram III. approaches still more nearly the level of the corresponding curve in Diagram II., while there is hardly any difference at all in the height of the curves of 30.2° in Diagrams I. and II. We are quite unable to account for this rather striking peculiarity.

2. For a given length of mercury column, and for a given vapor volume, higher temperature corresponds to a lower pitch. This fact is very noticeable indeed.

If one thinks of change of temperature as affecting only the pressure of the system, this conclusion must seem astonishing. On this basis, rise of temperature, causing as it does an increase in the pressure of the carbon dioxide, should, in the language of our theory, increase the tension of the springs which control the vibrating mercury column. But increased tension means increased promptness of restitution, or in other words, *higher* frequency of vibration.

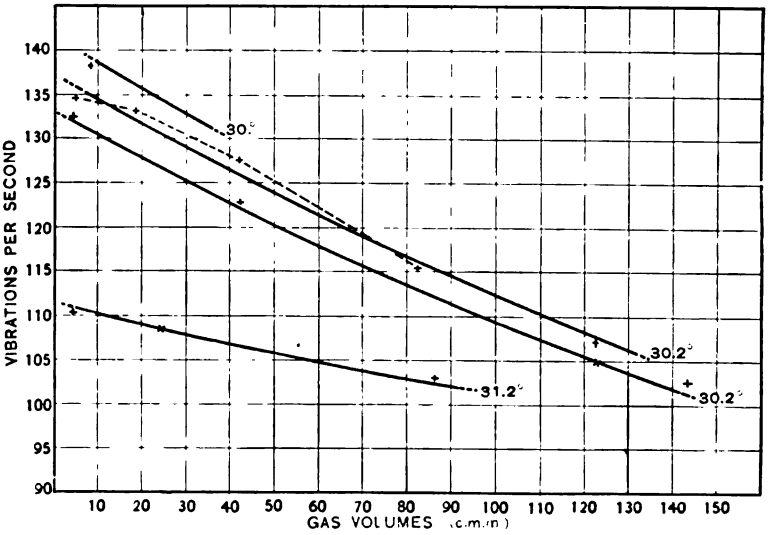

Fig. 4.

Rise of temperature has another result, however, whose effect may perhaps account for the observed reversal of this order. It is well known that the latent heat of a substance rapidly approaches zero in the immediate neighborhood of the critical temperature. Decrease in latent heat means a decrease in the amount of energy required to convert a given mass of saturated vapor into saturated liquid, or *vice versa*. Now copious liquefaction in the vapor phase must interfere with the power of restitution of this "spring," and make it relatively less prompt in reversing the motion of the mercury column. To repeat, rise of temperature tends on the one hand to weaken the effect of the spring by decreasing the latent heat of the vapor, and on the other, to strengthen it by increasing the pressure. If the former effect is relatively greater, as it may be in such close proximity to the critical temperature, the relation between higher temperature and lower pitch would be explained. This matter deserves further study.

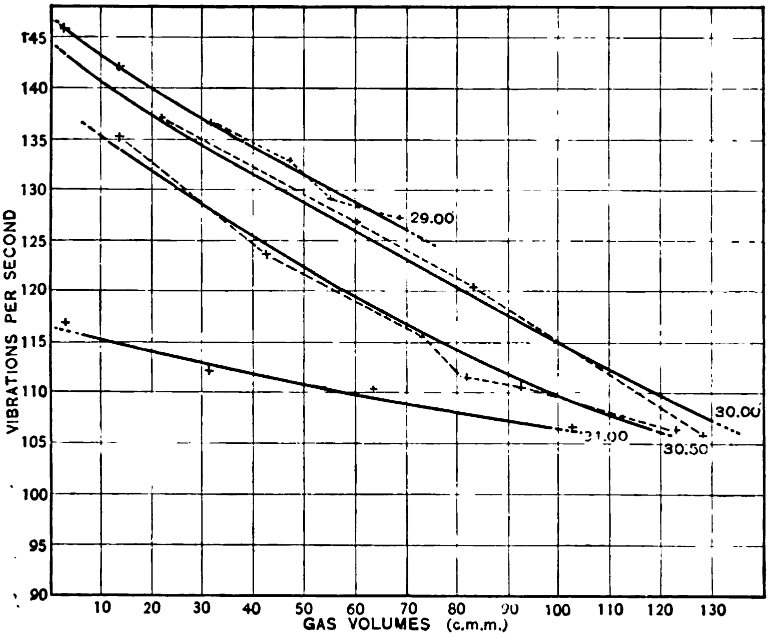

Fig. 5.

There is one other factor which helps, in a small way indeed, to an understanding of the connection of higher temperature and lower pitch. With a given vapor volume, higher temperature means a smaller total volume of carbon dioxide, and consequently a longer mercury column. As we have seen, lengthening the column lowers the pitch. To be sure, the increased length thus gained is very slight as compared with those considered in (1), and cannot play more than a relatively insignificant part in the problem now under discussion.

3. For a given length of mercury column, and for equal increments of temperature, the curves of pitch become more and more

widely separated as the critical temperature is approached. This is probably to be explained by the increasing rapidity with which the latent heat diminishes in this region as the temperature rises. The line of reasoning would be similar to that of (2).

4. (*a*) All else being constant, the pitch depends on the relation between the volume of the vapor and that of the liquid, or, to state it more simply, on the height of the meniscus in the carbon dioxide. The higher the meniscus the higher the pitch.

This result was to have been expected. Of course it is the more compressible vapor phase which constitutes the more effective portion of the upper "spring." The smaller the proportion of the vapor phase the greater the relative displacement produced in it by vibrations of given amplitude, and the greater the force of restitution, that is, the more rapid the vibration.

(*b*) This influence is greatest at lower temperatures. At higher temperatures the two phases resemble each other more and more closely in compressibility as in every other way. Thus, their relative volumes are of less moment, and the curves of pitch are flatter.

WESLEYAN UNIVERSITY,
MIDDLETOWN, CONN.,
May 21, 1904.

THE COMPARISON OF SMALL INDUCTANCES AT HIGH FREQUENCIES.

By A. H. Taylor.

CAPACITIES of the order of 10^{-11} farads have been measured with fair accuracy[1] but the measurement of correspondingly small inductances is more difficult.

By using an optical telephone with a bridge method, Wein[2] succeeded in measuring inductances of the order of 10^{-6} henrys to within less than 1 per cent. of their calculated values, using a frequency of 256. While this method is one of great percision, it is not applicable to the comparison of inductances at very high frequencies.

Seiler[3] measured inductances at high frequencies, his method in volving a measurement of a capacity, and of the period by a modification of the pendulum method of Helmholtz. The smallest inductance he measured was comparatively large, 316×10^{-6} henrys.

M. E. Maltby[4] measured both periods and inductances at high frequencies, in terms of capacity and electrolytic resistance. At a periodicity of 2.56×10^{-7} her results agree within 1.2 per cent. of the theoretical value of the period, but at higher frequencies there is a wide discrepancy.

Small capacities have been compared with small inductances at high frequencies by the use of the Nernst differential exciter. Battelli and Magri[5] using this method measured as small an inductance as 3.6×10^{-6} henrys. Judging from what details of the measurement they give, their limit of error was about 1 per cent.

Martienssen[1] using a modification of the method of "Phasenverschiebung" measured 2.5×10^{-6} henrys with great accuracy at a frequency of 500. His method could probably be extended to moderately high frequencies.

There are two criticisms to be made upon any form of bridge balance of inductance at high frequency. First, the difficulty of avoiding the reactions of neighboring parts of circuits. This objection can be partially removed by widely separating the inductances to be compared by the use of bifilared leads. In the case of very small inductance, the leads are not negligible, while in any case they act like a capacity in parallel with the inductance. Second, the high frequency currents used to operate the bridge must unavoidably have a pronounced group frequency, and the balance attained may not be a true high frequency balance.

It was in order to avoid these two difficulties that I had recourse to a resonance method which affords at the same time a determination of the period with very fair accuracy.

Let C_1 be a capacity in series with a spark-gap and the inductance L_1, while from two points a few centimeters apart on the circuit is run an auxiliary circuit consisting of the capacity C_2 in series with inductances L_c and L, and having an iron-constantin junction at T. The inductance L may be varied by moving the slider S, until the auxiliary circuit is in resonance with the main circuit, which will be indicated by the maximum deflection of the galvanometer connected with the iron-constantin junction at T.

If the point E be kept earthed the auxiliary circuit cannot appreciably affect the period of the main circuit. When in resonance, the auxiliary circuit, of low resistance throughout, needs only a very small potential difference to actuate it. Hence the main circuit may under favorable conditions be tapped by the auxiliary circuit at points distant only two centimeters on a No. 14 copper wire.

L_c is a circle of copper wire of diameter .146 cm. The diameter of the circle is 31.86 cm. The value of L_c according to the formula:[2]

$$L_c = 2l' \left(\log. \frac{l'}{R_1} - 1.5 \right)$$

is 1,154 cm., where $l =$ circumference of circle $R =$ radius of wire.

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[1] Ann. der Phys., p. 95, 1899.

[2] Drude, Physik des Aethers, p. 361.

The mercury cups connected to this inductance occupy 1 cm. of the circumference. Applying an approximate correction,

$$L_c = 1,143.$$

In order to test the accuracy of the method, the auxiliary circuit was brought into resonance with L_c in, and then with L_c cut out, the difference in self-induction being compensated by moving the desirl *S*, whose position was read on a meter scale.

TABLE I.

L_c in.	L_c out.	ΔL.
40.0	95.0	55.0
40.0	95.0	55.0
40.1	94.9	54.8
39.7	94.9	55.2
		Average 55.0

The main circuit was excited by an induction coil as a step-up transformer, the spark-gap being in oil.

The parallel wires are of diameter .146, and are 10.08 cm. distant (d) from each other. The inductance added by shifting the slider a distance $\Delta L = 55$ cm. may be computed from the formula :[1]

$$L = 4\Delta L\left(\log\frac{d}{R} + \frac{1}{4}\right),$$

since the slider itself is a thick brass bar whose inductance may be neglected. The change in L is hence 1,139 cm., which compared with the computed value of L_c (1,143) shows an error of 0.35 per cent.

This is a fair sample of the accuracy with which two inductances of the order of a micro-henry may be compared by this method. The total value of L need not be known; in fact it would be difficult to compute L for a practical case. But the change in L can be computed easily, and according to these observations may be relied upon to within one third of one per cent.

The capacity C_2 should be variable, and kept as large as possible and still allow the attainment of resonance, since greater accuracy will be possible when the change in L is a large part of the whole inductance of the auxiliary circuit.

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[1] Drude, loc. cit.

The mutual action of parts of the circuits should be prevented as much as possible. For the greatest precision, an unknown self-induction (in the place of L_c) should be measured in three pairs of positions 90° apart. The average will nearly eliminate disturbing effects due to mutual induction with neighboring parts of the circuit. Since the leads to L_c need not be known, it can be placed at some distance from the rest of the circuit.

If a variable self-induction is to be calibrated, the slider S can be moved so that L is as small as possible. Then the maximum value of the self-induction must be inserted at L_c and the capacity C_2 varied until resonance is attained. Then L_c can be decreased in steps, the circuit being brought back to resonance on increasing L by moving S. The increase in L can then be calculated, and is equal to the decrease in L_c. If the lowest value at L_c is greater than the range of change in L, this lowest value can be obtained by comparison with other known small inductances connected in series.

The method is not applicable to the comparison of inductances whose resistances differ so widely as to seriously affect the period of the auxiliary circuit, unless a compensating inductionless resistance be inserted in the auxiliary circuit.

The same set-up may be used with advantage to determine the period of the auxiliary, and hence of the main circuit.

Since there is no spark-gap in the auxiliary circuit, its resistance is constant and may be kept as low as a few tenths of an ohm. Hence the period is:

$$T = 2\pi\sqrt{LC_2}.$$

Now let the capacity C_2 take a new value, C_2', and let the change in L (as computed from the distance the slider S is moved) be dL.

Let

$$C_2' = C_2 - dC.$$

Then

$$\frac{T^2}{4\pi^2} = LC_2 = (L + dL)(C_2 - dC), \qquad (1)$$

or

$$\frac{L}{C_2} = \frac{dL}{dC} - \frac{dL}{C_2}, \qquad (2)$$

Dividing (2) by (1):

$$\frac{1}{C_2^2} = \left[\frac{dL}{dC} - \frac{dL}{C_2}\right]\frac{4\pi^2}{T^2}.$$

Hence

$$T = 2\pi\sqrt{\frac{C_2C_2'}{C_2 - C_2'}}\sqrt{dL}. \tag{3}$$

The capacities may be determined with great accuracy; hence that which determines the limit of error is the factor $\sqrt{dL}$. In a specific case,

C_2 was 0.00658×10^{-6} farads;

C_2' was 0.00474×10^{-6}.

Table II. shows the setting of the slider to determine dL.

TABLE II.

R_1	R_2	$R_1 - R_2$
91.8 cm.	37.8 cm.	54.0 cm.
91.4	37.7	53.7
91.6	37.5	54.1
91.5	37.5	54.0
91.5	37.6	53.9
		Average 53.94 cm.

R_1 is the setting for resonance with C_2 and R_2 the setting for C_2'. The value of dL corresponding to 53.94 cm. of the parallel wires is 0.9566×10^{-6} henrys, while the maximum variation in $R_1 - R_2$, hence in dL, is less than 0.8 per cent.

We are therefore sure of $\sqrt{dL}$ within 0.4 per cent., and the period as computed from (3) is 0.7997×10^{-6}. Allowing for a small possible error in the determination of the capacities, we may rely upon this value of T to within one half of one per cent.

An approximate calculation showed that the resistance of the auxiliary circuit would have to amount to 2 ohms in order to affect the period one tenth of one per cent. Since the actual resistance was only a few tenths of an ohm, there is no appreciable error involved in neglecting the resistance in calculating the period from (3).

The same values for C_1 and C_2 may be used to measure periods over a considerable range, the periods being in such case proportional to $\sqrt{dL}$.

Several periods were measured, Table III giving the settings of the slider for the shortest period measured, C_2' was 1.004×10^{-10} farads, C_2 was 1.670×10^{-10}.

TABLE III.

R_1	R_2	R_1-R_2
86.5 cm.	67.0 cm.	19.5 cm.
88.0	66.5	21.5
87.0	68.0	19.0
		Average 20.0 cm.

The parallel wires in this case were of diameter 0.146 as before, but their distance d was 5.85 cm. making:

$$dL = 0.3706 \times 10^{-6}$$

and

$$T = 6.07 \times 10^{-8}.$$

The maximum variation of $R_2 - R_2$ amounts here to 7.5 per cent., and the value of the period can be relied upon to only 3.8 per cent. Part of the difficulty in obtaining consistent values of $R_1 - R_2$, hence dL, was due to the fact that the iron-constantin couple was connected to a high resistance D'Arsonval galvanometer, not suitable for low resistance work and the deflections were small. I think it safe to say that by using a sensitive low resistance galvanometer the method could be applied to the measurement of periods of the order of 2×10^{-8} with a probable error of about 4 per cent

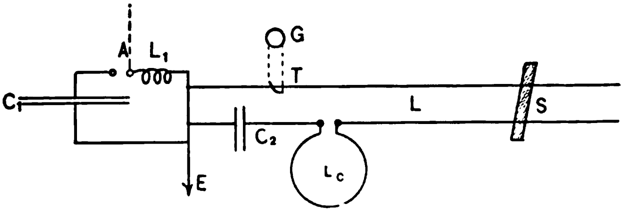

Fig. 1.

the difference being that while he computes both C_2 and L, this method involves the measurement of C_2 and C_2', and the computation of dL, dispensing with the consideration of parts of the circuit awkward to compute.

In case the main circuit with the spark-gap is a radiating circuit, *i. e.*, if a tuned aërial be connected with it, a second aërial several meters distant may be tuned to it, and the auxiliary circuit tapped off from this receiving aërial, hence avoiding any possible direct influence of the auxiliary circuit upon the period of the main circuit.

A series of observations similar to those in Table II. for such a case showed a period of 0.710×10^{-6}, and a possible error of 0.4 per cent.

The double resonance makes it possible to determine dL with somewhat greater accuracy, so that for periods of this order it is better than the simpler method.

Table IV.

R_1	R_2	$R_2=R_1$
Circle.		
12.0	17.8	5.8
11.5	17.5	6.0
11.2	17.2	6.0
11.5	17.5	6.0
Square.		Average 5.95
12.0	16.7	4.7
12.1	17.1	5.0
12.2	17.0	4.8
11.8	16.6	4.8
		Average 4.825

In order to see what could be done in comparing still smaller inductances, the inductance L of the main circuit was made of parallel wires distant 5.85 cm. and of diameter 0.146, and provided with a slider.

A tuned aërial was connected at A, and a tuned receiving aërial with the auxiliary circuit branching off from it was placed 2.5 meters distant from the transmitting aërial. The auxiliary circuit was brought into resonance and left so permanently, the galvanometer showing a maximum deflection. The small inductance to be measured was now inserted in the main circuit in series with the now variable L_1, and throwing the main circuit out of resonance with both transmitting and receiving aërials, also with the auxiliary circuit. L_1 was then varied by moving the slider until resonance was attained again. Table IV. gives the results for:

1. A circle of wire of diameter 0.146, diameter of circle 4.82 cm.
2. A square of the same wire 2.8 cm. on a side.

The computed inductance for the circle is 116×10^{-9}, while by comparison with dL, it is 110×10^{-9} showing a discrepancy of 5.3 per cent.

The inductance of the square considered as a combination of two sets of parallel conductors, and computed from the formula:

$$8S\left(\log\frac{S}{R} + \frac{1}{4}\right) = L,[1]$$

where $S =$ side of square, was 87.29×10^{-9}.

As compared with dL, the inductance of the square was 89.4×10^{-9}, the discrepancy being 2.3 per cent.

The variation in $R_2 - R_1$ indicates, however, that these results cannot be relied upon closer than to within 6 per cent.

Physical Laboratory,
University of Wisconsin, April 27, 1904.

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[1] Drude, loc. cit.

SOME PHYSICAL PROPERTIES OF CURRENT BEARING MATTER. I. TENSILE STRENGTH. II. MELTING POINT.

By Paul R. Heyl.

A DESCRIPTIVE résumé of the work that has been done in determining the physical properties of current bearing matter will be found in the Journal of the Franklin Institute for March 1904. The object of the present article is to present the results of some additional investigations in this matter. It may be stated at once that the results about to be presented are negative. Any change in the tensile strength of iron for a current density of 2,000 amperes per square centimeter cannot exceed one half of one per cent., and for copper, with a current density of 1,000 amperes per square centimeter, the alteration cannot exceed one fifth of one per cent. Any change in the melting point of tin for a current density of about 2,500 amperes per square centimeter can hardly exceed 2°. There is some reason to believe that a more powerful method would definitely show a small rise in the melting point.

Tensile Strength.

The substances tested were iron and copper in the form of wire. To eliminate the heating effect the wire under test was kept surrounded by a stream of cold water. Fig. 1 shows the arrangement of the apparatus. *BC* is the fine wire to be broken, stretched vertically between stronger wires in the axis of a glass tube of about two centimeters diameter. The lower end of this tube is closed by a cork *D* provided with a large hole through which one wire passes loosely on its way to a projecting support below. The upper end *A* of the tube is open, and the other wire passes out and is continued by means of a fish line over two pulleys to a pan with weights. A stream of water from the faucet is introduced at *A*, fills the tube to overflowing and escapes freely through the hole at *D*. The waste water falls into a sink over which the apparatus is set up. The temperature of the water is taken by a thermometer which hangs in the tube. When the wire to be broken carried a current this was provided by a storage cell in connection with a suitable rheostat and an ammeter reading 0.05 ampere, the terminals being loosely hooked to the wires at *A* and *D*.

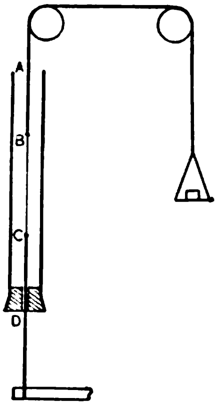

Fig. 1.

To the weights in the pan at the time of breaking is to be added a correction consisting of the weight of the pan itself minus the static friction of the cord and pulleys. The weight of the pan was nearly 100 grams, and the friction was about half a gram. As the weight in the pan was always greater than 1,000 grams, and was never increased by less than 5-gram steps, the friction was neglected and the correction considered to be an even 100 grams.

To insure that the test wire should break between the points *B*, *C*, at which it was fastened (by twisting) its middle part was slightly reduced in diameter by rubbing it with emery paper, any slight ellipticity of cross section thus introduced being allowed for by measuring the diameter in two perpendicular directions. In Tables II., III. and IV., the diameter given was measured after breaking, at the broken ends. In Table I. the diameter is that before breaking. This was obtained by measuring the diameter in three places, the thinnest part and a little on each side of it, and recording the diameter at the point nearest which the break afterward occurred, which in nearly every case was the thinnest point. This proved a somewhat unsatisfactory method, but as the result of Table I. is merely a small correction to the result of Table II., it is amply accurate for the purpose. Of course, the resulting tensile strength in Table I. would be slightly different from the corresponding value in Table II., but as comparisons are made only within each table and not between the tables this is of no moment.

Tables II. and IV. contain the results of the experiments on the tensile strength of the wire with and without a current. These experiments extended over several days, and in consequence the temperature of the tap water varied somewhat. It was therefore thought advisable to determine the effect of the variation of temperature upon the tensile strength. These results are found in Tables I. and III. As the extreme range of temperature in Tables II. and IV. is but 6°, an approximate value of the temperature coefficient is all that is necessary. To determine this the apparatus shown in Fig. 1 was slightly modified. The cork with the large hole *D* was replaced by another in which the wire fitted tightly; the water in the tube remained stationary, and was heated, when desired, by conducting into it a current of steam. It was found that the temperature coefficient for iron was negligible, but that for copper this correction could not be disregarded.

A word of explanation may be necessary concerning the mode of tabulating the results. The tensile strength of a wire is by no means uniform from point to point in the wire. To arrive at a correct idea of the average tensile strength one must break a great many short pieces and average the results. To determine the tensile strength as an end in itself it would be necessary to accumulate observations of this kind until the mean became practically constant; but this method of procedure may be shortened considerably if we desire to know only the ratio of the tensile strengths under two different conditions, such as heat and cold, current or no current. The experiments to be described were performed in this way: a spool of wire was cut into short pieces about 17 cm. long; the first, third, fifth, etc., were broken under the first set of conditions, and the second, fourth, sixth, etc., under the opposite conditions. The cumulative mean was calculated after each observation, as was also the ratio of the corresponding means. It will be seen from the tables that the ratio of the means approaches its limit more rapidly than does the mean value of the tensile strength. It is not claimed that the experiments described in Table I., have been pushed far enough to afford an accurate value of the temperature coefficient of cohesion in iron, but we may conclude from the last value in column 6 that the iron is not weakened more than about five per cent. for a rise in temperature of some 75°. We shall see that this may be neglected in the results of Table II.

By far the greatest error in the second table lies in the determination of the diameter. This was done by a micrometer caliper of 0.5 mm. pitch, graduated to hundredths of millimeters. The error in any diameter may be as much as one per cent., which would lead to an error in the area (and consequently in the tensile strength) of two per cent. The weights in the pan were added by 5-gram steps, making an error of less than one half of one per cent. The extreme range of temperature is 2.5°, by neglecting which we introduce an error of only one sixth of one per cent., according to the result of Table I. The current density is a little more than 2,000 amperes per square centimeter, and the last value in column 6 shows that any alteration of the tensile strength under a current of this intensity does not exceed one half of one per cent. Whether there is any effect less than this is doubtful; these figures can hardly be said to establish it, considering the error of a single observation.

From the last value in column 6 of Table III., we see that copper wire is weakened about fourteen per cent. for a rise of temperature of about 76°. It is worthy of notice that the values in column 6 are practically constant after the second observation while those in column 5 have not reached their limit in eight observations.

In Table IV., the greatest error, as in Table II., is in the diameter, but is somewhat less than in the former case for two reasons: because the quantity to be measured is greater, and because each diameter here recorded is the result of four measurements instead of two, as was the case with the iron wire. The error in the tensile strength arising from this measurement is probably about one per cent. The weights in the pans were added by 10-gram steps, introducing an error of about one third of one per cent., and the tensile strength is calculated to the uniform temperature of 20°, as the greatest error that would be introduced by neglecting this correction would be about one per cent. The current density in this case is about 1,000 amperes per square centimeter.

From the last value in column 6 of this table we see that any alteration in the tensile strength under a current of this intensity does not exceed one fifth of one per cent. and from the way in which the values in column 6 fluctuate above and below unity it is altogether likely that the current produces no effect at all.

Table I.

Effect of Temperature on the Tensile Strength of Iron Wire.

1. Diameter in mm. before breaking.
2. Temperature.
3. Weight in pan at breaking; grams.
4. Tensile strength in kg. per sq. mm.
5. Successive values of the mean tensile strength.
6. Ratio of the means (hot : cold).

1	2	3	4	5	6	5	4	3	2	1
0.200	23.1	1155	39.9	39.9	0.947	37.8	37.8	1135	100.2	0.204
188	24.5	1090	42.9	41.4	0.914	37.9	37.9	1115	100.2	202
201	24.1	1165	39.8	40.9	0.958	39.1	41.7	1265	100.3	204
202	24.7	1305	43.8	41.6	0.955	39.8	41.7	1185	100.3	198
203	25.1	1150	38.6	41.0	0.976	40.0	40.9	1120	100.3	195
201	25.2	1220	41.6	41.1	0.985	40.5	43.0	1120	100.3	190
206	24.9	1285	41.4	41.1	0.973	40.0	37.3	1335	100.4	221
200	24.5	1325	45.3	41.7	0.953	40.2	41.3	1250	100.2	204

Table II.

Effect of Current on the Tensile Strength of Iron Wire.

1. Diameter in mm. after breaking.
2. Temperature.
3. Weight in the pan: grams.
4. Tensile strength in kg. per sq. mm.
5. Successive mean values of tensile strength.
6. Ratio of means (current : no current).

Current Nothing.						Current 0.7 Ampere.				
1	2	3	4	5	6	5	4	3	2	1
0.210	21.9	1253	39.1	39.1	1.11	43.3	43.3	1260	21.8	0.200
.204	21.9	1208	40.0	39.6	1.03	40.6	37.9	1237	21.9	.212
.210	21.9	1160	36.4	38.5	1.02	39.2	36.4	1150	21.9	.209
.212	21.9	1120	34.6	37.5	1.02	38.3	35.4	1150	21.9	.212
.213	21.9	1270	38.4	37.7	1.02	38.3	38.5	1245	21.9	.211
.203	22.0	1220	40.8	38.2	1.02	39.0	42.4	1245	22.0	.201
.205	22.0	1240	40.6	38.6	1.02	39.2	40.5	1275	22.1	.208
.204	22.2	1285	42.4	39.0	1.008	39.3	40.3	1205	22.7	.203
.203	22.1	1200	40.2	39.2	1.007	39.4	40.1	1210	23.0	.204
.203	23.2	1235	41.3	39.4	1.012	39.9	43.7	1300	22.6	.202
.199	22.8	1225	42.6	39.7	1.011	40.1	42.8	1300	23.3	.204
.203	24.3	1205	40.3	39.7	1.012	40.2	41.2	1245	23.4	.204
.200	23.5	1300	44.6	40.10	1.005	40.28	41.1	1230	23.3	.203
.203	23.3	1260	42.0	40.24	1.004	40.40	42.0	1245	23.7	.202

Table III.

Effect of Temperature on the Tensile Strength of Copper Wire.

1. Diameter in mm. after breaking.
2. Temperature.
3. Weight in pan: grams.
4. Tensile strength in kg. per sq. mm.
5. Successive mean values of tensile strength.
6. Ratio of means (hot : cold).

1	2	3	4	5	6	5	4	3	2	1
0.272	23.3	2500	44.7	44.7	0.72	32.0	32.0	2640	100.3	0.330
.365	22.7	2950	29.2	37.0	0.86	31.8	31.6	2600	100.1	.330
.307	22.5	2680	37.6	37.2	0.86	32.2	32.9	2580	100.2	.322
.308	23.7	2870	39.9	37.9	0.86	32.5	33.5	2530	100.1	.316
.324	24.1	3030	38.0	37.9	0.86	32.7	33.4	2500	100.1	.315
.322	24.7	3020	38.3	38.0	0.87	32.9	34.1	2700	100.1	.323
.310	24.5	2960	40.5	38.3	0.86	32.8	32.3	2630	100.1	.328
.315	23.4	2950	39.1	38.4	0.86	33.2	35.7	2540	100.1	.307

MELTING POINT.

The metal selected for this investigation was tin. The first method used was to immerse a thermometer directly in the molten metal. In order to get as great a current density as possible the arrangement shown in Fig. 2 was employed. The tin was melted in a hard glass test tube and a smaller tube of hard glass open at both ends was immersed in it, reaching nearly to the bottom of the melted metal. A thermometer was placed within the inner tube, its bulb being thus surrounded by a thin cylinder of molten metal which was in turn surrounded by a much larger quantity of the same substance. An iron-tipped wire coiled once around the thermometer bulb supplied the current to the inner shell of tin. The current passed down this shell, under the edge of the inner tube and out by another wire from the larger mass of melted tin outside.

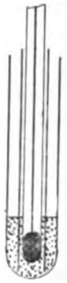

Fig. 2.

The thermometer used was one of short range (200°–250°) graduated to fifths of a degree, and read with a lens to fiftieths. Its bulb was 0.55 cm. in diameter and about the same length. The inside diameter of the inner tube was 0.75 cm., thus leaving a thin cylinder of tin whose sectional area was about 0.21 sq. cm. The current used was 3 amperes, giving a current density of about 14 amperes per sq. cm.

TABLE IV.

Effect of Current on the Tensile Strength of Copper Wire.

1. Diameter in mm. after breaking.
2. Temperature.
3. Weight in pan : grams.
4. Tensile strength in kg. per sq. mm.
5. Successive mean values of tensile strength.
6. Ratio of means (current : no current).

Current Nothing.						Current 0.8 Ampere.				
1	2	3	4	5	6	5	4	3	2	1
0.331	24.4	3100	36.9	36.9	1.008	37.2	37.2	2930	22.8	0.321
.325	22.3	2900	36.1	36.5	0.999	36.5	35.7	3000	21.4	.332
.322	21.2	3000	38.0	37.0	0.978	36.2	35.7	3000	21.6	.332
.319	21.5	2940	37.9	37.2	0.969	36.1	35.7	2850	21.3	.324
.315	21.2	2950	39.0	37.6	0.967	36.3	37.4	2950	21.1	.322
.320	21.0	2940	37.7	37.6	0.975	36.6	38.1	2820	20.8	.312
.325	20.7	2870	35.8	37.4	0.989	36.9	38.6	2910	20.7	.315
.320	20.6	2860	36.8	37.3	0.993	37.0	37.7	2950	20.5	.321
.308	25.0	2700	37.2	37.3	0.990	36.9	36.1	2860	22.3	.322
.317	21.1	2840	37.1	37.3	0.995	37.1	38.3	2930	20.8	.317
.315	20.9	2850	37.8	37.3	0.990	36.9	35.8	2930	21.0	.328
.322	22.4	3020	38.2	37.4	0.994	37.2	39.7	3010	23.1	.315
.325	22.9	2850	35.4	37.2	1.003	37.3	39.5	2900	22.8	.310
.300	21.9	2800	40.9	37.5	1.002	37.6	40.3	2700	21.5	.297
.322	21.4	2880	36.5	37.4	1.007	37.7	39.3	3010	21.2	.317
.306	21.1	2740	38.5	37.5	1.003	37.6	36.5	2680	20.9	.311
.320	19.6	2860	36.8	37.5	1.003	37.5	36.7	2990	19.2	.328
.325	19.0	3100	38.7	37.52	1.001	37.55	37.7	2970	19.3	.322
.323	19.2	3080	38.9	37.59	0.999	37.54	37.2	2700	19.0	.310
.312	19.3	2830	38.3	37.63	0.998	37.56	37.9	2800	19.4	.312

It was found by some preliminary experiments that this current in the iron-tipped wire close to the thermometer stem produced no effect on the latter. To ascertain this the thermometer bulb was immersed in water and the current was led down and up close to its stem. In the actual test the liability to such an effect would be halved, as the return current was at a little distance from the stem with the wall of the glass tube between them.

The experiments on the melting point were conducted as follows: The tin was melted and raised to about 250°, and then allowed to cool. It usually fell a variable fraction of a degree below 230°, then rose to the constant melting point, finally falling slowly as the solid metal cooled. This operation was repeated alternately with and without a current in the tin with the following results:

Melting Point.

Without Current.	With Current.
230.80	230.80
230.80	230.80
230.80	230.80

It is evident from this that the melting point of tin is not altered by as much as one fiftieth of a degree by a current density of 14 amperes per sq. cm. As the absolute melting point is unessential to this conclusion no correction was applied to the thermometer.

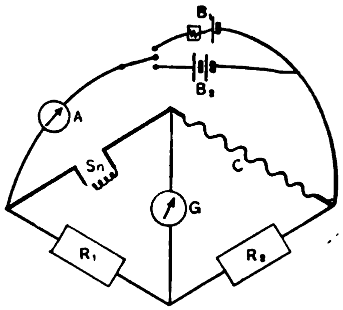

Fig. 3.

It was found impracticable to reach a very high current density by this method, and a second plan was devised. In outline, this was to use a strip of tinfoil of small cross section as one of the arms of a Wheatstones' bridge adapted to carry a current of two or three amperes, and determine the melting point of the tin by its resistance at the instant of melting. The details are shown in Fig. 3.

C, a standard one ohm coil.

A, an ammeter reading 0.05 ampere.

R_1, R_2, ordinary resistance boxes.

G, a D'Arsonval reflecting galvanometer.

B_1, a Leclanché cell with an additional resistance of 50 ohms.

B_2, two storage cells.

Sn, the tinfoil strip. To prepare this a sheet of tinfoil was cut as in Fig. 4, the narrow strip being 1 mm. wide and about 30 cm. long, while the wide parts were about 2.5 cm. wide. The wide parts were rolled up tightly to form handles for the strip, by means of which connection was made to heavy copper wires by screwconnectors. The narrow strip was wound on a small square of glass and tied on by a thread. The handles and connecting wires were vertical so that the strip on its glass plate could be hung in any desired liquid.

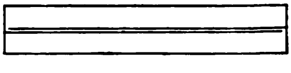

Fig. 4.

The resistance in the box R_2 was kept constantly 1000 ohms, and that in the box R_1 varied to make a balance. The resistance of the tinfoil strip was usually about one ohm when cold, so that the total resistance in the upper half of the bridge was about two ohms, while that of the lower half was a thousand times as great. Consequently, when a cell of no internal resistance, such as a storage cell, was applied to the bridge a heavy current would traverse the upper arms of the bridge and a feeble one the lower; and the current in the tinfoil strip would be very nearly equal to the current shown by the ammeter.

The standard one ohm coil was constructed of bare German silver wire 0.8 mm. diameter, used double and wound on a wooden block. It was kept immersed in kerosene, the temperature of which was noted by a thermometer. The coil was provided with short, heavy copper terminals dipping into mercury cups, which cups formed the terminal points of this arm of the bridge. The total resistance in this arm was measured against a standard correct to one part in a thousand, and was found to be 1.000 ohms at 18°. Its temperature coefficient was assumed to be 0.00044 for the few degrees that its temperature varied from 18°. It was found that the coil thus arranged would carry a current of one ampere for an hour with a rise of temperature of less than one degree.

The experiments were conducted as follows: a tinfoil strip having been put in place its resistance was measured at the room temperature in kerosene, by the feeblest current that would give a definite result. This was about 0.02 ampere, furnished by switching in the Leclanché cell B_1 with its additional resistance of 50 ohms. The temperature of the one ohm coil was noted. The strip was then hung in melted paraffin whose temperature was gradually raised with continual stirring until the tinfoil melted. As the temperature rose the resistance in R_1 was increased by one or two ohm steps to maintain the balance. The melting point of the tinfoil was indicated by a sudden throw of the galvanometer, the resistance in R_1 protecting the instrument from any excessive current. The temperature of the one ohm coil was again noted. The melted paraffin was removed, the handles scraped clean, bent toward each other and fastened by a clip, and the resistance of the connections measured.

From these data the resistance of the tinfoil strip was calculated at its melting point and at 20°. For the latter part of this calculation the temperature coefficient of tin was taken as 0.0037. We then obtain finally the ratio of the resistances of the strip at 20° and at the melting point, for a small current density.

Another strip was put in place, and the operations repeated, except that for the hot resistance the storage cells were used, sometimes in series, sometimes in parallel, according to the resistance of the strip, so as to furnish a current of about one ampere. The ratio of the resistances at the melting point and at 20° was calculated as before. As the current density was now small at 20° and great at the melting point any effect of the current on the melting point would show itself in the value of this ratio as compared with that obtained by using the weak current throughout. It was of course assumed that the resistance of tin is not a function of the current strength. While this has not been proved for the special case of tin it does not seem a dangerous assumption.[1]

The tinfoil strip was always about one millimeter wide and varied in thickness from 0.04 to 0.05 mm. Consequently the two current densities used were about 50 and 2,500 amperes per square centimeter in the thin parts of the strip. The varying thickness plays a curious part in the experiment. Its effect may be neglected for feeble currents, but for heavy currents any irregularity in thickness will reduce the apparent melting point, and the fewer the thin places the greater the apparent reduction. The thin parts, being more strongly heated by the current, will reach the melting point and break while the average temperature of the strip is below the melting point; and the fewer the thin places the lower will be this average temperature.

Experiments were made alternately with the weak and the strong current and the results are given in the following tables. A number of unrecorded experiments were made before the method was well

TABLE V.

1. Temperature of the one ohm coil.
2. Temperature of tinfoil strip.
3. Resistance in box R_1.
4. Resistance of connections to tinfoil strip.
5. Corrected resistance of strip (for low temperatures, at 20°).
6. Ratio of resistances (hot : cold).

Current 0.02 Ampere.					Current 0.02 Ampere. Temperature, Melting.			
1	2	3	4	5	6	5	3	1
21.3	20.9	1136	0.008	1.125	1.537	1.729	1734	23.0
21.3	21.2	977	0.008	0.966	1.532	1.480	1486	22.0
20.0	18.9	1112	0.008	1.109	1.526	1.692	1698	20.1
17.6	16.7	1157	0.008	1.160	1.533	1.778	1786	18.4
20.1	19.0	843	0.008	0.840	1.549	1.301	1308	20.0
18.2	19.5	867	0.008	0.861	1.528	1.316	1324	19.4
20.3	20.0	891	0.008	0.884	1.520	1.344	1351	21.2

Mean value 1.532
Average departure from mean 0.0065

TABLE VI.

Current 0.02 Ampere.					Current 1 Ampere. Temperature, Melting.			
1	2	3	4	5	6	5	3	1
23.8	24.6	1160	0.008	1.138	1.520	1.730	1733	24.6
22.7	22.2	1437	0.008	1.423	1.526	2.171	2173	23.8
20.2	19.0	940	0.008	0.937	1.512	1.417	1424	20.4
19.0	18.7	1050	0.008	1.047	1.521	1.593	1601	19.8
19.8	19.0	831	0.008	0.826	1.553	1.283	1290	20.4
20.0	19.4	731	0.008	0.726	1.560	1.133	1140	20.3
21.9	22.9	947	0.009	0.930	1.565	1.456	1462	23.4
20.1	19.0	965	0.008	0.962	1.530	1.472	1480	19.9

Mean value 1.536
Average departure from mean 0.018

The cause is undoubtedly the higher current density in the latter case which causes the unavoidable irregularities of the strip to have a greater effect upon the result. The rise in temperature of the tinfoil increased its resistance in the ratio of 1 : 1.53, the excess of 0.53 being due to a rise of about 200°. The difference in the final means in the two tables is 4 in the third decimal place, and 4 parts in 530 for a rise of 200° would mean a difference of less than 2° in the melting point. Are we justified therefore in concluding that the melting point of tin is raised by this amount under a great current density? Undoubtedly the results of Table VI. in themselves cannot be taken as proving this, so great is the average departure from the mean; but when we recollect that the effect of great current density upon an irregular strip is always to *decrease* the apparent melting point, and when we notice that several of the values of Table VI. are greater than any of the quite concordant values of Table V., it seems likely that there may be a small rise of the melting point under a great current density which would require closer measurements to prove its existence.

It is hoped to extend this investigation to some metals of much higher melting point in the near future.

Central High School,
Philadelphia.

AN AUTOMATIC COMMUTATOR AND GALVANOMETER KEY FOR MEASURING PERIODICALLY RECURRING PHENOMENA.

By Arthur W. Gray.

In an earlier paper[1] the writer described an apparatus which he had constructed for automatically placing a galvanometer in the circuit of a condenser (a Siemens ozone generator), which was being charged and discharged in rapid succession, just long enough to measure ballistically a *single* charge or discharge. Although this device answered fairly well the purpose for which it was intended, considerable room for improvement remained. In the first place it performed its function as a shunt of negligible resistance in parallel with the galvanometer, which was automatically removed at will when it was desired to obtain a deflection; so that during all but about 40 of the 12,000 to 25,000 passages of electricity that were effective in producing ozone, the resistance and self-induction in the circuit of the generator were negligible, while during the measurement of the quantity of electricity about 200,000 ohms and 27 millihenries were inserted. Whether this difference would alter the conditions within the generater could not at the time be known, but it seemed not at all improbable that it might exert a considerable influence, and that such was actually the case is clear from the results of the experiments to be described in a later paper.

Then again the mechanism was weak and more complicated than necessary, and was not suitable for use with potentials much over 15,000 volts.

To overcome these defects the writer designed the apparatus represented in orthographic projection by Fig. 1. By means of this the resistance and self-induction in the electrical circuit of the generator could be kept the same whether the current passed

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[1] A. W. Gray, Ann. d. Phys., 13: p. 481, 1904.

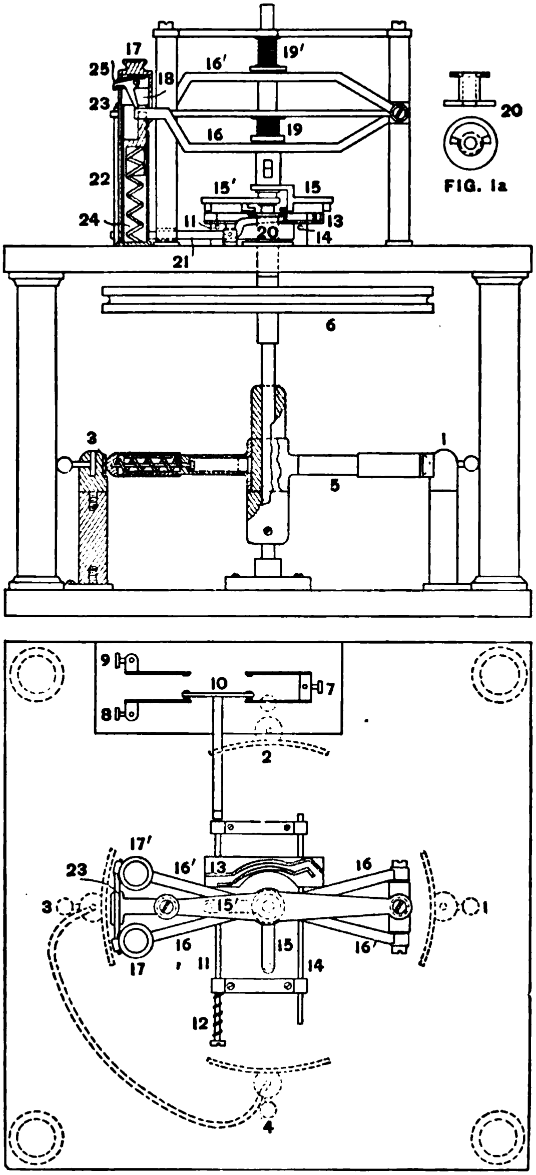

Fig. 1.

rotating ones, thus doing away with troublesome vibrations; the mechanism was simplified and made so much more durable that it would stand long-continued operation without the slightest attention beyond occasional oiling; the resetting after obtaining a galvanometer deflection was performed automatically, and the working was so positive that even many thousands of trials failed to disclose a single false move, even when the speed was made much greater than was necessary for the purpose.

It should be remarked here that, while the machine was designed for the purposes of the work in hand, its application is by no means limited to this. The essential ideas embodied in it can, in fact, be employed to advantage wherever it is desired to single out for measurement a phenomenon which recurs periodically, provided the intervals between recurrences are not too short.

The construction and working of the machine will be made apparent by a study of Fig. 1, which shows some parts in section for the sake of greater clearness.

The four well-insulated metallic terminals, 1, 2, 3, and 4 are uniformly distributed on the circumference of a circle; 1 is connected to the Leyden battery, 2 with the earth, 3 and 4 with each other and with the inner electrode of the generator. The metallic bar 5, insulated from the vertical axis about which it is turned by the pulley 6, has its ends provided with spring contacts which rub against the curved plates attached to the terminals, so that connection is made alternately between 1 and 3 and between 2 and 4, thus charging and discharging the generator in rapid succession.

Upon a shelf above the rotating commutator is located the mechanism that constitutes the galvanometer key. The wire from the generator leads to 7, while 8 leads through a resistance and self-induction equal to that of the galvanometer and 9 through the galvanometer itself to earth. Either 8 or 9 can be connected through spring platinum-tipped contacts with 7 by means of the metal bar 10. This is insulated and carried on the end of a rod 11 which slides in guides and by means of the spring 12 is held ordinarily in the position shown. The plate 13, which carries on its upper surface a grooved way shaped as indicated, is firmly attached to 11 and to the similar rod 14, which serves merely as a support and guide.

Two similar arms, 15 and 15′, are rigidly attached to sleeves surrounding the axle of the commutator 5, so that they turn with this but are free to be moved independently of each other a short distance along the axle. The two arms are fixed at right angles to each other as represented in the horizontal projection, although in the vertical projection 15 is represented as moved back 90° from its true position in order to show its shape. Ordinarily 15 and 15′ are held up in the position indicated by flanges at the upper ends of the attached sleeves resting on supporting collars carried by the levers 16 and 16′; so that the short rods projecting downwards at the extremities of the arms pass just over the grooved way of plate 13. If, however, the cap 17 is depressed, the bent lever 18 within it carries along the lever 16, so that the spring 19 is free to lower 15 about 0.5 cm. The grooved way of 13 is so shaped and situated that just after 5 has ceased making contact between 2 and 4 the projection at the end of 15 will now strike 13, carrying it along until 10 joins 7 with 9 when one end of 5 is just about half way between the contact pieces of 1 and 2, while the other is half way between 3 and 4 — that is, long before there is any danger of sparks jumping from 1 through 5 to 3 and charging the generator. 10 now remains held in this position until after the generator has been charged and the current has passed through the galvanometer; but as soon as 5 ceases to make connection between 1 and 3, 10 is promptly carried back so as to reëstablish the connection between 7 and 8 before the following discharge can occur, and then 15 is automatically raised and will appear presently.

The upper end of the brass bushing 20 through which the axle passes carries a horizontal flange of the shape indicated in Fig. 1*a*; and to the under sides of the arms 15 and 15′ are fastened hooked-shaped projections; so that if one of the arms happens to be over the flange it cannot be dropped, or, if previously dropped, it will be held down by the hook until the flange has been passed. This is to make sure that the arm will be dropped in time to begin properly its movement of the plate 13 and that it will not be lifted out of action until the movement has been completed. To guard against the danger of one of the hooks doing damage by striking against the flange in case it happened to be lowered just before this was reached, the flange and the approaching sides of the hooks are beveled to sharp edges, so that the arm will be carried either below or above so as to fall as soon as the other end of the flange is reached.

To effect the automatic raising of the arm after it has re-established connection between 7 and 8, the lever 21 is provided. One end of this curves around under the flange of 20 in such a way that the hook of 15 presses it down soon after it passes well under the flange. This raises the rods 22 with their attached cross-piece, 23, which presses up one arm of the bent lever 18, pulling the other arm away from the end of the lever 16. The strong spring 24, contained in the tube which is capped by 17, now raises 16, and with it 15, as far as the hook on the under side of the latter will permit; and as soon as the flange is passed, 15 is carried up to the position shown in the figure so that it will merely pass over 13 without striking it, even though the finger continues to hold down the cap 17. When the finger is removed, a spring surrounding the tube containing the spring 24 raises the cap to the position shown, and the small flat spring 25 throws back the bent lever 18 until it is again in position for engaging the end of 16, thus automatically re-setting the instrument so that another deflection will be obtained as soon as the cap 17 is again depressed.

Depressing the cap 17′ causes the current of a single *discharging* to pass through the galvanometer by lowering 15′, which is just 90° in advance of 15.

It is advisable to make the pulley 6 heavy, so as to secure steadiness of rotation. If desirable, the rotating commutator could readily be immersed in an oil-bath.

For determining the number of times the generator is charged and discharged, any kind of rotation counter may be attached to the upper end of the axle, but for convenience one that can be thrown in or out of action at will is preferable.

Berlin, Physical Institute of the University,
April, 1904.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTY-FIFTH MEETING.

The Fall Meeting of the Physical Society was held in the Coliseum Building, St. Louis, Mo., on Friday, September 16, 1904.

During the forenoon the Physical Society met in joint session with Section A of the International Electrical Congress. President Webster of the American Physical Society presided.

Announcement was made by the president of the election of Professor Svante Arrhenius as an Honorary Member of the Physical Society.

The following papers were read during the joint session:

1. The State of Our Knowledge Regarding the Earth's Magnetism and Recent Remarkable Magnetic Storms. L. A. Bauer.

2. The Electric Charge of the Sun. S. Arrhenius.

3. Radioactive Change. E. Rutherford.

4. Report on Recent Developments in Electrical Theory. A. G. Webster.

5. Magneto-optics. (By title.) D. B. Brace.

During the afternoon session the following papers were read:

6. On the Nature of Thermo-electromotive Force. C. B. Thwing.

7. Concerning the Presence of Yttrium and Erbium in Fluorspar. W. J. Humphreys.

8. The Effect of Pressure upon the Light from the Electric Arc. W. J. Humphreys.

9. A Differential Galvanometer of the d'Arsonval Type. (By title.) J. C. Shedd.

10. On the Conditions which Govern the Appearance of Spark Lines in Arc Spectra. Henry Crew.

ON THE NATURE OF THERMO-ELECTROMOTIVE FORCE.[1]

BY CHARLES B. THWING.

ON the hypothesis that an electric current in a metal consists of a displacement of negative corpuscles it might be expected to follow that the direction, as well as the magnitude, of the pressure tending to produce such a flow in a metallic circuit due to unequal temperatures at two points in the circuit would depend upon the rates of expansion of the metal at the two temperatures.

The coefficient of expansion of many metals, and of a number of alloys having high thermoelectric force, was determined both between 20° and 60° and between 60° and 100° with a good degree of accuracy, and the thermo-electric force of the same specimens was determined over about the same range of temperature. Comparing the rate of increase of coefficient of expansion from 40° to 80° with that of lead, the thermoelectric power of which is reckoned at null, it was found:

1. That for nickel and bismuth and for nickel alloys, having like these metals, a negative e.m.f. the rate of increase in expansion is negative and numerically larger for the metals having a high e.m.f.

2. That for metals and alloys like antimony and its alloys having a positive e.m.f. the rate of increase of expansion is positive and numerically larger for the metals having a high e.m.f.

3. That for alloys of Ni-Cu, Sb-Zn and Sb-Cd, all of which have an e.m.f. much higher than either constituent, the coefficient of expansion is abnormally low as compared with the value computed from those of the constituents, the value in the case of one of the Sb-Cd alloys being but one third the calculated value.

These results agree perfectly with the hypothesis referred to above. If the corpuscles behave in some degree like a gas under pressure the low expansion of bismuth at the hot junction would cause a pressure tending to make the corpuscles flow from hot to cold while the large value of the expansion of antimony at the hot junction would result in a diminished pressure at that point causing a flow of negative corpuscles from cold to hot. In the case of the alloys referred to above this effect would be intensified by the abnormally low expansion of these alloys, which should tend to increase the pressure upon the corpuscles when the alloy is heated.

The differences in the values of the coefficient of expansion of the metals at the two temperatures are in every case so large in comparison with the probable error of the determinations as to leave no room for doubt regarding the existence of the relation indicated.

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[1] Abstract of a paper presented at the meeting of the Physical Society held at St. Louis on September 16, 1904.

Marked variation of the cofficient of expansion with the lapse of time was observed in the case of some of the metals.

CONCERNING THE PRESENCE OF YTTRIUM AND YTTERBIUM IN FLUORSPAR.[1]

By W. J. Humphreys.

MORE than one hundred samples of fluorspar from various parts of the world, America (North and South), Europe, Asia, Africa and Australia were examined by a large concave grating spectroscope. In nearly all these yttrium in widely varying amounts was detected, and in a number of them, those that contained the largest amounts of yttrium, ytterbium was also found.

Three specimens, one from Virginia, one from Texas and one from Bolivia, South America, showed the presence of decided amounts of both yttrium and ytterbium. All three of these specimens were very sensitive to temperature effects, one of them phosphorescing even on being held in the hand. A short list of unidentified solar lines probably due to these elements is given.

THE EFFECT OF PRESSURE ON THE LIGHT FROM AN ELECTRIC ARC.[2]

By W. J. Humphreys.

AN electric arc was enclosed in a steel cylinder provided with a quartz window, and the light as produced in air of high pressure examined by means of a Rowland concave grating of 21½ ft. focal length.

The decrease in wave frequency, detected and measured by Mohler and the author some years ago at the Johns Hopkins University, has been confirmed, and the observations with the same results have been extended to many other lines and to a somewhat higher pressure.

This work is being done through the aid of the Rumfor Fund, and

ON THE CONDITIONS WHICH GOVERN THE APPEARANCE OF SPARK LINES IN ARC SPECTRA.[1]

By Henry Crew.

THE intermittent arc between magnesium electrodes was examined simultaneously with the Duddell oscillograph and the spectroscope.

1. It was found that the voltage curve of the ordinary arc thus photographed showed no extra e.m.f. at break: nor did the spectroscope show the spark line at λ 4481.

2. When the break at the arc was quickened by means of an air blast the extra e.m.f. at once appeared in the voltage curve and the spark line in the spectroscope.

3. When the arc was surrounded by an atmosphere of coal gas, thus bringing out the spark lines, the voltage curve showed the extra e.m.f. at break. This gives the probable explanation of the effect of an atmosphere of hydrogen as noted by various observers, viz., the break is quickened — hence high extra e.m.f. is introduced — hence spark lines.

4. The introduction of spark lines, by use of small currents, discovered by Hartmann, is given an interpretation different from his, but in accordance with his results, with the above results, and with the modern electron theory of the arc and spark discharge.

A DIFFERENTIAL GALVANOMETER OF THE D'ARSONVAL TYPE.[2]

By J. C. Shedd.

THE principle of the differential galvanometer has long been known and is fully elaborated in most of the treatises on electrical measurements. As a matter of fact, however, this form of galvanometer is but little used. This is not from any lack in variety of method in using the instrument or lack of accuracy in the results obtained, but from other causes. One reason is that the amount of patience and trouble involved in successfully handling such an instrument, sensitive enough to be of any value, is often excessive. Also the necessity of having movable coils or of resorting to leveling screws for the purpose of delicate adjustment calls for a skill in manipulation beyond that possessed by the average student. In particular the dependence upon the earth's magnetic field limits the usefulness of the instrument to a few favored localities free from magnetic disturbance.

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[1] Abstract of a paper presented at the meeting of the Physical Society held at St. Louis September 16, 1904.

[2] Abstract of a paper presented at the meeting of the Physical Society held at St. Louis on September 16, 1904.

The advent of the D'Arsonval galvanometer has for most purposes relegated the fixed-coil galvanometer either to the shelf or to the advanced laboratory. The present state of perfection of this instrument has brought a galvanometer of fair sensibility and easy of manipulation within the reach of all. It has also made possible its use in localities heretofore entirely impossible, such as the dynamo laboratory, machine shop, power-station, or on ship-board.

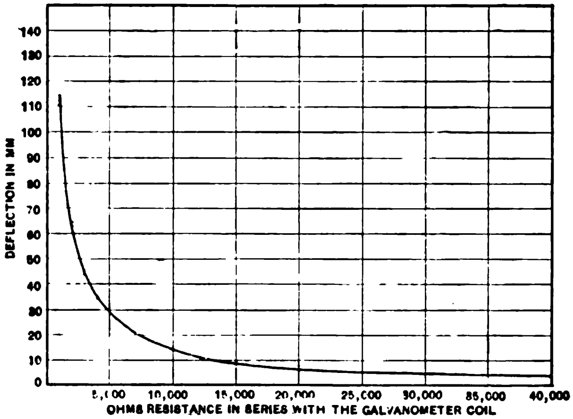

Fig. 1.

The possibility of constructing a differential galvanometer of the D'Arsonval type occurred to me some years ago but only recently has the idea taken concrete form. The work, while not complete, has progressed far enough to show that the instrument is practicable in every respect. For this purpose the coil was removed from a Rowland D'Arsonval galvanometer and a special coil substituted for it. This coil was made as follows: about 600 feet of No. 40 silk covered copper wire was measured off and then doubled back on itself. It was then run over with beeswax to render it slightly adhesive. The doubled wire was then wound by hand upon a small wooden bobbin made for the purpose, care being taken to keep the wires exactly together. The wooden bobbin was provided with two terminals insulated from each other. These were used as the terminals of one coil. To one of these terminals was mum. In bringing both terminals out at the same end in the case of one coil and at opposite ends in the case of the other coil, it is apparent that one coil will have a half turn more than the other. It was thought that with something like a thousand turns on the coil this half turn would produce no appreciable effect. On trial, however, it was found that a deflection of one mm. was produced when the coils were differentially connected in series. To eliminate this the extra half turn was brought down the edge of the wooden bobbin so as to lie in the plane of the suspension wire normal to the magnetic field. Its effect was then found to be nil.

As the wire was wound on by hand some doubt was felt lest the two coils be not exactly equivalent to each other magnetically. It was therefore a satisfaction not only to find that the deflection was zero with the coils differentially connected in series but also to obtain identical calibration curves for the separate coils. This is shown in the following table the results of which are plotted in Fig. 1.

Table showing the relation between deflection and current for the two coils of the differential D'Arsonval galvanometer.

	Coil A.		Coil B.	
Resistance in Series with Coil.	Deflection in Mm.		Deflection in Mm.	
Ohms.	Direct.	Reversed.	Direct.	Reversed.
40.000	3.3	3.8	3.8	3.5
30.000	4.5	5.0	4.3	4.8
20.000	7.0	7.4	6.0	7.3
10.000	14.0	14.4	14.5	14.4
5.000	27.8	27.5	28.0	28.0
4.000	34.0	34.0	35.0	34.0
3.000	45.0	44.3	45.0	45.0
2.000	64.0	63.5	65.0	64.0
1.000	114.2	111.0	113.5	113.0

The present trial instrument is sufficient to show that the application of the differential principle to the D'Arsonval type of galvanometer is an entire success. The exact form the instrument may take may be left for the future development. It is probable that a double suspension above will be preferable to the triple suspension below. I am confident that there need be no loss of sensibility on account of the extra suspension.

The advantage of having the D'Arsonval galvanometer capable of being also used as a differential galvanometer is at once apparent and it is hoped that the use of differential methods may receive an impetus from the introduction of this form of instrument.

The following references to the literature of the Differential Galvanometer may be found of value.

On the Theory of the Instrument:

Clerk Maxwell, Electricity and Magnetism, Vol. I., pp. 472–475.

Andrew Gray, Absolute Measurements in Electricity and Magnetism, Vol. I., pp. 374–380.

On the Practical Use of the Instrument:

W. E. Ayrton, Practical Electricity, pp. 148, 150, 171, 183, 436.

D. C. Miller, Laboratory Physics, pp. 299–302.

Carhart and Patterson, Electrical Measurements, pp. 40–45.

Parham and Shedd, Dynamo and Motor Testing, pp. 174–182.

Physical Laboratory,
Colorado College,
July 9, 1904.

Note Added September 10, 1904.

An examination of the files of the La Lumieer Electrique and LeEclarage shows that a little work along this line was done in 1889 (III., p. 545) by M. Ledebor and M. Carpentier. Their attempt was to wind two independent coils upon the same core. This procedure must have resulted in a very cumbersome instrument. The insulation of modern wire is sufficient to allow of bifilar winding and with much better results.

Volume XIX. *November, 1904.* *Number 5*

THE

PHYSICAL REVIEW.

SENSITIVE MOVING COIL GALVANOMETERS.

BY WALTER P. WHITE.

THE following article deals primarily with the principles underlying the construction of sensitive moving coil galvanometers and with various experimental devices for increasing their sensibility. Under this latter head are included tremor-shielding devices. Incidentally, by treating of the galvanometer as a motor some of its fundamental properties are conveniently exhibited; the results obtained lead naturally to a brief discussion of the relative sensibility of different types of instrument.

I. FUNDAMENTAL FORMULÆ.

The essentials of the mathematical theory of the moving coil galvanometer are comparatively well known.[1] It therefore seems sufficient here to merely state as much of the theory as will be needed in the present discussion.

In any galvanometer whatever:

Let K be the moment of inertia.

Q, the moment of torsion of the controlling force.

I, the current.

ϑ, the angular deflection.

R, the resistance of the whole circuit.

G, the dynamic constant, that is, the deflecting torque produced by unit current, this depends on the strength of the magnets and the arrangement of the coils.

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[1] F. Kohlrausch, Lehrbuch der praktischen Physik, 9th Ed, p. 450; Th. Des Coudres, Zeitschrift fur Electrochemie, 3, 1896–1897. O. M. Stewart, PHYS. REV., 16, p. 158, 1903. W. Jaeger, Zeitsch. f. Instrumentenkunde, 23, pp. 261, 353, 1903.

$T/2$ the undamped half period.

Then

$$\frac{T}{2} = \pi \sqrt{\frac{K}{Q}} \qquad (1)$$

The sensibility,

$$\frac{\theta}{I} = \frac{G}{Q}. \qquad (2)$$

The counter electromotive force developed in the coil by its own motion is

$$G\frac{d\theta}{dt}$$

the corresponding current is:

$$\frac{G}{R}\frac{d\theta}{dt}$$

the damping effect of this current is a torque,

$$\frac{G^2}{R}\frac{d\theta}{dt}, \quad = D'\frac{d\theta}{dt}$$

whence D', a damping coefficient, $= G^2/R$. (3)

It will presently appear that the damping due to this cause involves no loss of energy during deflection; it is, in a somewhat special sense, conservative damping.

For the moving coil galvanometer in particular:

Let r be half the width of the coil.

l, the total length of vertical wire in the coil.

H the field strength.

For simplicity the mirror and the "dead horizontal wire at the ends of the coil may be neglected, since these can easily be brought in as corrections to the simplified formulæ.

Then

$$K = lr^2 \times \text{(mass of unit length of wire)}, \qquad (4)$$

$$G = Hlr. \qquad (5)$$

that such motion dies down most rapidly when the damping is critical — that is, just barely aperiodic, and the condition for this critical damping is[1]

$$D = 2\sqrt{QK}, \tag{6}$$

and the moving coil galvanometer is actually found to work best when so damped. Moreover, for maximum sensibility it is evident that no energy should be wasted, hence the damping should be all conservative, and D may be taken equal to D'. This condition can be approximately realized with the moving coil galvanometer.[2] Then, assuming critical damping, we have, from equations (3), (5), (6),

$$G^2 = H^2 l^2 r^2 = 2R\sqrt{QK}, \tag{7}$$

$$H^2 = \frac{2R\sqrt{QK}}{l^2 r^2}. \tag{8}$$

The expression for the sensibility, (2), can now be transformed by eliminating G and inserting T by (1), (7)

$$\frac{\theta}{I} = \sqrt{\frac{2R\,T/2}{\pi Q}}, \tag{9}$$

and similarly,

$$\frac{\theta}{I} = \sqrt{\frac{2R(T/2)^3}{\pi^3 K}}. \tag{10}$$

These formulæ can be expressed in more convenient units. If R is in ohms, I in microamperes, θ in two-thousandths of a radian — that is, millimeters at a meter distance, allowing for the doubling effect of the mirror — then the sensibility in megohms — which is numerically the same as millimeters per microampere at one meter — becomes:

$$S = 5.02\sqrt{\frac{R\,T/2}{Q}}, \tag{11}$$

or

$$S = 1.60\sqrt{\frac{R(T/2)^3}{K}}. \tag{12}$$

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[1] O. M. Stewart, loc. cit.

[2] W. Jaeger, op. cit., p. 355.

A corresponding expression can be found for H by eliminating T from (8) and changing units. For critical damping,

$$H = \frac{25{,}200\sqrt{RQ\dfrac{T}{2}}}{lr}. \tag{13}$$

The above formulæ are sufficient for the designing of moving coil galvanometers, given the properties of the magnets and the wire. Another relation will prove convenient a little later. From equation (7) it appears that in *any* galvanometer if T is increased by diminishing Q, the value of G required for critical damping becomes less.

$$G \propto \frac{1}{\sqrt{T}}. \tag{14}$$

That is, it is easier to get critical damping as T increases.

For the moving coil galvanometer the required value of H diminishes also, according to (8), if T is increased by increasing l. In this case

$$H \propto T^{-\frac{1}{2}}. \tag{15}$$

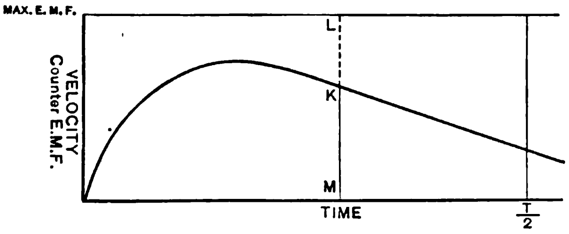
2. The Galvanometer as a Motor.

One great advantage of the moving coil galvanometer is the ease with which large values of G are obtainable. These at first sight promise great sensibility, and suggest the use of powerful electromagnets to secure it. It is now well known, however, that the possibilities in this direction are limited by the overdamping which results when the field strength exceeds the critical value. This damping is usually represented as the result of a current generated by the motion of the coil, and opposite to the current which is being measured. In reality, of course, there can not be two currents in the same circuit, and it is simpler to regard the damping during deflection as due to a counter electromotive force which diminishes the driving current. A galvanometer is a motor, and overdamping is simply the inevitable result of an attempt, in trying to turn a heavy coil too far in a given time, to get more energy out of the current source than it is capable of giving.

Some notion of the magnitude of the counter E.M.F. in a critically damped galvanometer may be obtained from Fig. 1. The upper curve represents the variation in the angular velocity, as computed from the equation of motion, during the time of a free half swing. Since the counter E.M.F. is proportional to the velocity it also is represented by this same curve if a suitable scale be taken. If, now, the upper horizontal line shows to the same scale the value of the driving E.M.F., the dotted ordinates, measured downward from this line, represent the resultant E.M.F., and are therefore proportional to the actual current. It may be noted that this falls at one time to about one fourth its maximum value. Since the upper and lower ordinates at any point, as *LK*, *MK*, are proportional to current and counter E.M.F. respectively, their product will be proportional to the power expended on the coil. Values of this product are shown in the power curve, and the upper line shows to the same scale the maximum possible power obtainable from the current, or $C^2R/4$. The area under the curve corresponds to the total work done on the coil during the time $T/2$, and this evidently bears a very high ratio to the maximum possible work.[1]

Fig. 1a.

Velocity-time curve for a critically damped galvanometer, which is also counter-electromotive-force-time curve. The dotted ordinates are proportional to current.



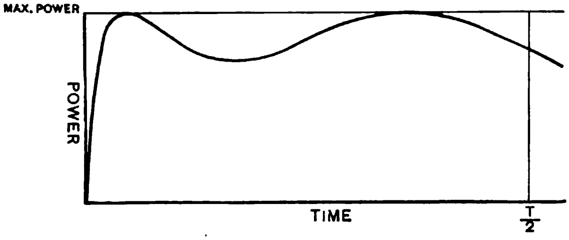


Fig. 1b.

Power-time curve. The area under the curve is the current energy utilized in the first half-period; the area of the parallelogram is the maximum available energy.

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[1] The exact ratio, readily obtainable by analytic methods, is 87.4 per cent.

The above curves represent any galvanometer which has critical, conservative damping, hence all such galvanometers utilize during corresponding periods the same proportion of the energy obtainable from the current. This conclusion, like the curve itself, follows from the equations of motion of the galvanometer, and can be shown to be involved in equation (9). Similarly it can be shown that the proportion of the energy utilized is a maximum for the critically damped galvanometer. This galvanometer is thus somewhat analogous to the Carnot engine. It is, in a way, an ideal galvanometer.

The notion of an ideal galvanometer is often convenient. For instance, such a one — that is, generally speaking, a good moving coil galvanometer — will be more sensitive than any other galvanometer of the same moment of inertia, period, etc., and will usually prove decidedly superior if any heavy mass, such as a pointer or large mirror, is to be moved. Moreover, the sensibility of a corresponding ideal galvanometer may often be advantageously used to determine the limit of possible sensibility of any other. For example, C. H. Abbott has given very full data in regard to a sixteen-coil instrument in use at the U. S. Astrophysical Observatory in Washington.[1] The weight of the magnet system is 10 mg., its moment of inertia, 6×10^{-6}, the resistance is 1.6 ohms, and the (computed) sensibility at 10 seconds half period is 20,000 megohms. From formula (12) the ideal sensibility is 26,000 megohms. For 1.5 seconds the sensibility is 450, which is now only 30 per cent. of the ideal — a decrease accounted for by (14). A similar 4-coil instrument, too delicate for use under ordinary conditions, whose system weighs 2 mg., gives a sensibility of 96,000 megohms at 10 seconds half period. Its ideal sensibility for 10 seconds is in the neighborhood of 50,000. Thus the instrument at 96,000 megohm sensibility would be overdamped, and really inefficient, unless the total resistance were rather high; and if this were nearly three times that of the galvanometer the damping would be aperiodic from magnetic action alone. We thus demonstrate from energy considerations that for moderately long periods this galvanometer has attained at ordinary cooking conditions the maximum sensitiveness possible without a diminution in the size of the moving mass.

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[1] C. H. Abbott, Astrophys. Jour., 18, p. 1, 1903. S. P. Langley, Astroph. Obs. Annals, 1, pp. 59, 244, 1900.

The energy available in a galvanometer is a function of current, total resistance, and time, and these are in one sense quantities independent of the instrument itself, representing the conditions it has to meet. Given these conditions, sensibility in an ideal galvanometer depends on the deflection obtainable with a given amount of energy — that is, upon the smallness of the controlling force and coil — a conclusion also derivable from equations (9) and (12). In galvanometers of the Kelvin type special effort is also needed to increase the value of G, which, as we have just seen, is almost never as great as is desirable. The various improvements in this type of instrument have generally been secured by attention to the quality of the magnets and to the coils as well as by reduction in the size of the moving parts.

3. Comparisons of Sensibility.

If any galvanometer has its period lengthened by a change in Q the sensibility will increase as the square of this change. Moreover, the number of turns in a coil of given size is approximately proportional to the square root of the resistance. Hence, when a galvanometer is used under different conditions sensibility will be proportional to $T^2\sqrt{R}$. This formula, or others similar to it, has become familiar[1] as a means of ranking galvanometers by comparing their supposed performance under like conditions. Derived as it is from a consideration of the peculiarities of the Kelvin type it gives misleading results when applied to the moving coil galvanometer, for three reasons.

1. The sensibility of the moving coil instrument is not affected by the internal resistance, except as this modifies the total resistance. This is due to the fact that we can compensate for lack of wire in the coil by increasing the strength of the magnet — a thing usually impossible with the Kelvin type. (See equation (16), below.)

2. While a change in Q will of course alter the sensibility of a moving coil galvanometer, just as of any other, yet it will also destroy the condition of critical damping. The work required for the deflection varies as $\vartheta^2 Q$, and therefore, by (2), as $1/Q$ for given current, and the increased work necessitated by a reduction in Q

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[1] W. E. Ayrton and T. Mather, Phil. Mag., 46, p. 350, 1898.

can therefore not be provided for by increasing the period only as $\sqrt{1/Q}$. The correct result is shown in equations (10), (12).

This objection does not apply to the Kelvin instrument, because that, being magnetically underdamped, is necessarily inefficient; a large part of the energy of its current is usually wasted, and we are at liberty to recover as much of it as we can. Increasing the period does this, as appears from (14).

3. The assumption that the sensibility of a moving coil galvanometer can be increased at all by diminishing Q is in many cases practically an impossible one. The limit of skill in the Kelvin instrument is reached in making the magnet system small; there is usually no difficulty in reducing the controlling force to correspond. The coil in the other type, however, is never as small as could easily be made, since the limit to progress in this direction is set by the relatively stiff suspending wire. The only way to increase the period of an instrument of maximum sensitiveness, therefore, is to make the coil heavier. Accordingly, the formula for comparing sensibility should be one in which K may be varied at will, but in which Q is fixed — that is, formula (11).

From this formula it appears that under the same external conditions the sensibility varies inversely as $\sqrt{Q}$ — that is, it depends ordinarily simply and solely on the size, elasticity, and length of the suspending wire. On the other hand, when T is diminished the sensibility diminishes only as the square root of T. The moving coil galvanometer is thus relatively more sensitive than the Kelvin at short periods.

Jaeger[1] calls attention to this difference between the two instruments. It may be worth noting, in regard to his and similar comparisons, that a Kelvin galvanometer is most serviceable when the whole resistance of the circuit is double its own, so that a reduction of such a galvanometer to a one-ohm resistance is equivalent to a reduction of the circuit to two ohms. Accordingly, since R in the moving coil formula applies to the whole circuit, it should be taken twice as great as the resistance of a Kelvin galvanometer in making

This question is independent of the further fact, to which Jaeger calls attention, that since it is not necessary in the moving coil instrument to add wire in order to increase the value of G, its resistance should be made not equal to the rest of the circuit, but as low as possible.[1]

4. Wide vs. Narrow Coils.

(*a*) *As to Sensibility.* — The effect of coil width on sensibility has been the subject of an amount of discussion which is rather surprising, considering that, as shown by the formulæ already given, coil width does not necessarily have any effect on the sensibility. This is due to the fact that it is the value of the dynamical constant, $G(\doteq Hlr)$, that is important, and this can, in general, be kept constant by modifying H in case $l\,r$ changes. The contrary opinion seems to be based mainly on a demonstration by Mather,[2] which has been widely quoted. A glance at this demonstration shows that in it H is assumed constant, an assumption evidently unnecessary. Another assumption, which lurks concealed in the demonstration, is that the galvanometer is underdamped, since otherwise there is no object in increasing G. Moreover, as will be shown presently, the increase of G here obtained by narrowing the coil can also be produced by diminishing the size of wire. Mather's demonstration, then, amounts to this: In an underdamped galvanometer, where the field cannot be made stronger nor the wire in the coil smaller, it is well to increase G by making the coil narrow.

b. As to Resistance. — If the resistance per unit length of insulated wire is taken inversely proportional to the mass, the approximation is much closer than in the more familiar case where the ratio of resistance to cross section is considered. Then if m is the mass per unit length, the resistance per unit length is c/m, where c is a constant,[3] and x the resistance of the coil.

$$R' = \frac{lc}{m}. \qquad (15)$$

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[1] These two sources of gain are mutually exclusive, which is perhaps the reason why Jaeger omits mentioning one of them. But in any case, unless the galvanometer furnishes more than half the total resistance, the gain is at least as great as indicated above.

[2] W. T. Mather, Phil. Mag., 29, p. 434, 1890.

[3] c was found to be .000023 for a sample of 2-mil wire (diameter, .005 cm.), the units being ohms, grams, centimeters.

Moreover, we have [(4) and (5)] $K = lmr^2$, and $G = Hlr$. Eliminating m and l, we get:

$$G^2 = \frac{H^2R'K}{c}. \tag{16}$$

If we consider alterations in the coil which do not affect the principal constants of the instrument — period, sensibility, circuit resistance — we have: $H^2R' =$ constant. That is, field strength and coil resistance may be regarded as mutually replaceable elements; for a given value of G either one of them may be diminished at the expense of an increase in the other. Here, again, the result is independent of the shape of the coil.

The increase of resistance here contemplated can be effected in two ways; by making the coil narrower, thereby (4) increasing l, or by using more turns of finer wire. Hence, electrically, a narrowing of the coil is equivalent to a decrease in the size of wire.

The last conclusion suggests that a narrowing of the coil may be important when the finest possible wire is already in use and a further increase in the power of the coil is desired. This is, of course, the special but important case treated by Mather.

5. DETRIMENTAL MAGNETIC IMPURITIES.

Ayrton has pointed out[1] that the magnetic impurities in the coil are acted on by the field with the production of a directive force which may greatly reduce the sensibility. To diminish this difficulty coils are usually made of materials as pure, magnetically, as possible. The detrimental effect can never be wholly eliminated, as has been shown in the case of the radiomicrometer. It is partly, if not wholly, due to residual magnetism, since a uniform field will not exert any disturbing action on a coil unless that coil is already magnetized in a direction different from that of the field. Accordingly, investigation of the detrimental effect seemed to require direct substances, in uniform fields, the detrimental magnetic effect is proportional simply to the square of the field strength and to the mass of the coil. Neither width nor shape had any perceptible influence, nor the direction of the wires, whether vertical or horizontal.

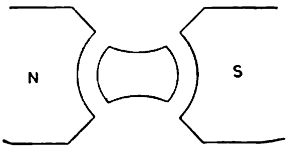
From these results it readily appears that if we consider changes which affect the coil alone, wide coils, being lighter, will give less trouble from magnetic impurity. In this case the wide coil will have finer wire. If the width of the coil is altered without changing the size of wire the magnetic effect also is unaffected, for in this case it is easy to show from (4) and (5) that the change in H offsets change in mass.

The coil is thus in unstable equilibrium, and as it rotates the couple tending to displace it further increases at about the same rate as the force of restitution due to the twisting of the wire.

6. The Radial Field.

The detrimental magnetic action in a uniform field is purely a directive force. Therefore in a radial field—that is, a field whose direction is at every point perpendicular to the direction of motion of the coil as it rotates—the detrimental effect should be largely eliminated. Such a field can be approximately realized by the arrangement shown in Fig. 2 (where the amount cut out on the sides of the central armature is probably a matter of minor importance). Its usefulness was tested in the following way: A coil, made sufficiently magnetic by a coating of soft wax and carrying a mirror, was suspended by a phosphor bronze wire, first in a radial field and then in a field of such uniformity that the strength varied less than one per cent. from the center to the side. The directive force of the field was found by observing the position of the coil with the magnet unexcited, and then noting the change when the current was put on. This was done for different initial positions of the coil. Repetitions of the same determination seldom varied more than one per cent. In general, with the radial field, a slight deflecting

Fig. 2.



Horizontal section of a radial field. The lines are everywhere perpendicular to the motion of the coil, so that this is unaffected by magnetic impurity.

force was found. This force can be compensated by twisting the suspending wire, and if constant is of practically no importance. It is the variation of the force as the coil turns which counts. That produces the effect of a directive force resisting the rotation. This variation in the case tested was less than one sixtieth the directive force of a uniform field of the same strength. The ratio would vary in different cases, depending on the accuracy with which the poles and armature were constructed, etc., but it is evident that the magnetic trouble may be very greatly diminished by the use of a radial field.

7. Description of a Galvanometer.

An application of the above formulæ was made in the construction of a galvanometer of relatively high sensitiveness, intended to be used in a circuit of from 1 to 100 ohms' resistance, especially with null methods. It seemed best to limit the length of the suspending wire to 10 cm., both for compactness and for low resistance. The finest wire obtainable was some listed by Hartmann and Braun as .0208 mm. in diameter. Silver was chosen on account of its low resistance. The moment of torsion of the upper and lower wires together was .17 c.g.s. units, and the resistance, 10 ohms. To secure a radial field of the proper proportions the least admissible coil width was thought to be 12 mm., and this was selected. The length of the coil was made 22 mm., in order to reduce sufficiently the effect of the dead wire, which now contributes about one fourth of the whole resistance, and one sixth the moment of inertia. On account of the intended use in null methods a period of 1 second was first planned, but it was found on calculation that this would involve a value of H so great as to make an electromagnet necessary. To avoid this complication the period was lengthened to 1.3 seconds, and for the same reason the number of turns of wire in the coil was made rather large (47) at the cost of a somewhat increased resistance. The total resistance is 36 ohms, and it could not well have been made less than half this, as long as the suspension was which Jaeger[1] assigns as the probable limit at the present time. This result, however, is merely the natural consequence of using the fine wire.[2] It involves no special difficulties, for the suspension is amply strong, and has stood some unreasonably severe shocks without once breaking.

Several special features of the galvanometer may prove interesting. The radial field has already been mentioned. In an earlier galvanometer the presence of magnetic impurity nearly doubled the controlling force, and rendered the zero very inconstant. In the present instrument no shift of zero can be detected after ordinary deflections, nor any variation in the time of swing due to removing or replacing the magnets.

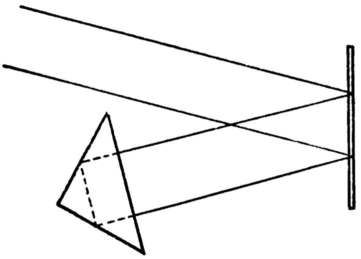
The coil is kept in shape by being bound with silk and varnished with collodion, a substance comparatively free from magnetic impurity. Coils 2 cm. square, and as light as 20 mg. can be made in this way.

The coil and suspension wires were permanently soldered together, and mounted on a glass rod, by means of which they are handled.

Ordinary solder seems to be a solvent for silver, and it is difficult to use it on these fine wires without melting them off. Fusible metal seems entirely free from this difficulty, and leaves the joints much stronger.

The pole pieces project from the case, and different magnets can be slipped upon them. An electromagnet could thus be used at any time if desired.

Fig. 3.



Showing a total reflecting prism used so that the effect of horizontal deflections of a mirror is doubled, and of vertical, eliminated.

The deflection is doubled according to a well-known method,[3] involving the use of a stathe moving mirror. For the stationary mirror is substituted a right-angled prism, with its hypothenuse toward the incident light and its elements horizontal (Fig. 3). Such an arrangement has two advantages: (1) the reflection is total; (2) such a prism behaves like an ordinary mirror toward horizontal deviations of the beam of light, but in the case of small vertical deviations the light is always returned parallel to itself and therefore the effect of all oscillations of the mirror in a vertical plane is neutralized. As magnet- and other suspended systems when disturbed by tremors of their supports frequently have their most energetic oscillations vertical, this method of eliminating such oscillations may, perhaps, be found generally useful. It is not important with a coil as steady as the one used in this instrument.

The scale is at two meters' distance. The mirror is 7 mm. wide; its moment of inertia is only one eighth that of the coil. This width of mirror, according to Einthoven,[1] would admit of a scale distance three times that actually used. As the instrument has ample sensibility for its present work, I have not sought to reach the limit in this direction.

The actual working sensibility of the present arrangement is 450 megohms at 62 ohms' resistance. This is .3 as sensitive as the sixteen-coil galvanometer at the Astrophysical Observatory (for equal period and resistance) and a little more sensitive than Dubois and Rubens' most sensitive "ironclad" galvanometer.[2]

8. Further Increase of Sensibility.

The above results are such as can be easily obtained with materials now available. It remains to inquire what further gain may attend special labor or skill.

1. A diminution in the size of the suspending wire would seem promising in the light of the large factor of safety evidently possessed by the wire of .02 mm. It seems unlikely that finer wire can be drawn directly, but if it could be drawn inside a copper coat-

silver. Whether, given the wire, the increase in sensitiveness would be nearer four- or fifteen-fold would depend on various circumstances. The gain would be considerably less if the suspension were shortened so as to keep the same resistance. Moreover, an increase in period would be almost necessary, as the small value of lr in a very light coil would require inconvenient values for H — unless indeed, the radial field were given up, and the coil made narrow, and this would introduce other difficulties.

Platinum wire is of course obtainable of sufficient fineness, but its rigidity and resistance are such that for low resistance it is no better than sizes of silver now available, while none of these extremely fine wires can carry a coil large enough to use in a high resistance circuit.

2. The suspending wire may be left as it is, and its torsion counterbalanced by another force, very much as one magnet of an astatic pair balances the other. The simplest way to do this is to make the coil one-sided, and hang it in an inclined position with the heavy side uppermost.[1] There are also other and better, because more stable, methods of accomplishing the same result.[2] Somewhat varied trials of two of these led to the conclusion that it is easy to increase the sensibility about three times, and apparently impossible to go much further. The main difficulty appears to lie in the imperfect elasticity of the wire, which renders all these methods practically useless, as every time the zero of the wire shifts the other element of the pair has to be adjusted to correspond.

3. The relatively heavy and stable moving coil naturally lends itself to a certain amount of optical magnification. The limits of this, however, are soon reached, since the resolving power of the mirror (and therefore the permissible scale distance) increases as its breadth, while its moment of inertia increases more rapidly than the cube of the breadth.

4. The main disadvantage of the moving coil galvanometer is the necessity of passing a current into the moving portion, which compels the use of relatively stiff wire. There is no need of passing a current into the mirror, however, nor, if this is suspended separately,

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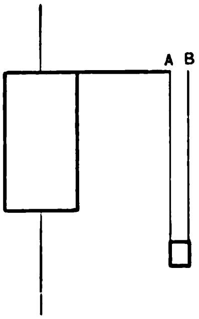
[1] H. Abraham, L'Éclairage Electrique, 7, p. 276, 1896.

[2] W. Stroud and J. H. Oates, Phil. Mag., 6, p. 711, 1903.

is there any lack of energy for turning it. A method due to Kelvin has been used by Prof. C. E. Mendenhall to produce a large deflection of a mirror from a small movement of the coil. The device is shown in Fig. 4, where the mirror is suspended by fibers from the points *A* and *B*, one of them fixed, the other at the end of an arm carried by the coil. This arrangement evidently requires great stability, for it responds to movements of translation, as well as rotation. This difficulty is avoided by the arrangement shown in Fig. 5. The mirror is suspended by the single fiber, *A*, so as to be drawn slightly forward against the two fine hooks, *H*, *K*, which are connected with two corners of the coil. I have tested this method (without the hooks) on a blank coil; it works well, and is extremely stable. It did not occur to me till after the galvanometer was finished.

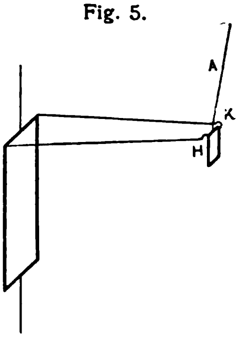
Fig. 4.



Device for multiplying the motion of a coil by a separate mirror.

Since the energy of a rotating body is proportional to the square of the angular velocity, the work required to turn a separate mirror will be multiplied by the square of the ratio in which the sensibility is increased. The moment of inertia of the mirror must be correspondingly diminished if the work expended in turning it is to be kept constant. For instance, a mirror whose motion is multiplied ten times should have one hundredth the moment of inertia of one attached directly to the coil. That would give, in the case of the galvanometer described above, a mirror width of 2 mm. The corresponding sensibility, using the reflecting prism, would be five times what it is at present, and greater than could be obtained by any of the other methods here described.

Fig. 5.



More stable arrangement for the same purpose. The mirror is held at three points and is not rotated by translatory motion of the coil.

Increasing the size of the coil has precisely the same effect upon the allowable multiplication as decreasing the mirror — that is, the multiplication varies as the square root of the moment of inertia of the coil. It is thus (12) inversely proportional to the sensibility of the coil. The sensibility attained by the mirror, then, is constant — that is, it depends upon the mirror and not at all upon the coil. This result also follows directly from the assumption used in getting it — viz., that a certain proportion of the energy should be expended on the mirror.

The size of the coil would really be immaterial if there were no question of stability involved. But in practice a greater ratio of multiplication would involve a greater magnification of the tremors of the coil. Since, however, rotations of the coil are the only motions which will trouble the mirror appreciably, and since a large coil can be balanced more truly than a small one, there is evidently a point beyond which increased multiplication is preferable to a decrease in the size of the coil. It appears not improbable that the fitting of a multiplying device to such galvanometers as are now on the market may usually be the most advantageous method of securing high sensibility.

9. The String Galvanometer.

An ingenious modification of the moving coil principle is found in Einthoven's recently described "String Galvanometer,"[1] in which the advantages of a mirror are given up, and the resulting loss more than compensated by the exceeding lightness of the moving system thus made possible. The galvanometer consists of a silvered quartz fiber, 2.3 μ in diameter, stretched like a piano string in a field of 2,300 lines, its movement magnified 660 times by a microscope.

This instrument is easily brought within the scope of the above equations. (Not those for critical damping.) If we assume that the fiber, fixed at the ends and swinging in the middle, is equivalent to a straight fiber of half the length moving uniformly it then corresponds to a coil of this length and equal weight, of 3 mm. radius, since a coil of that radius would have its linear motion magnified 660 times by a mirror with a scale at a meter's distance. The silver, estimated from its resistance, 10,000 ohms, would appear to furnish about one twentieth the volume of the whole fiber. The average density of the fiber may then be taken as not far from 3, which makes its weight, in round numbers, 1×10^{-6}. The greatest sensibility obtained by Einthoven is 100,000 megohms. This gives, in accordance with equation (2), $Q = 8 \times 10^{-5}$, taking $G_{,} = Hl r_{,} =$ 41,000. This gives for $T/2$, .1 second. The observed value was 20 seconds, two hundred times as great, which shows a truly enormous amount of air damping. This damping gives the readings great steadiness, an important advantage of this form of galvanometer. At the same time the advantage of the very light moving system is so far neutralized that the sensibility is, considering the resistance, lower than that of the best Kelvin galvanometers. This is very far, however, from saying that the instrument is less useful. At shorter periods, as might be supposed, the air damping is less important, and the sensibility surpasses all other galvanometers.

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[1] W. Einthoven, Drude Ann., 12, p. 1059, 1903.

On the basis of the performance at one two hundredth of a second Einthoven computes a normal sensibility — that is, a sensibility at one ohm and five seconds — of from 900,000 to 2,380,000 megohms — more than he was able to get at 20 seconds with 10,000 ohms. This discrepancy between theory and practice illustrates the idea suggested on page 318, above, that the normal sensibility of a galvanometer has little value when computed in accordance with the properties of a radically different instrument. Einthoven takes pains to make his computation, first, on the assumption that the normal sensibility varies as the square root of the resistance, and then that it varies as the three-fifths power. Now the question which of these two ratios should be used has to do, among other things, with the thickness of the insulation of wires. It originated in a discussion of the Kelvin galvanometer, and was in its origin a thoroughly practical question, resulting from the attempt to make the formula conform as far as possible to the physical conditions obtaining in the construction of that instrument. But whether the ratio of reduction should be the one half or the three-fifths power is surely a minor consideration in the case of an instrument where the sensibility actually varies more nearly as the first power. (The justification of this last statement is found in the fact that in this galvanometer a decrease of resistance is in general accompanied by an increase of the moving mass.)

10. STABILITY — THE JULIUS SUSPENSION.

Any device, optical or other, for magnifying the deflection of a coil manifestly depends for its success on a high degree of stability. Accordingly, the work on the galvanometer above described has been accompanied by an attempt to secure a specially stable support.

The most popular method of securing stability at present is undoubtedly the Julius suspension,[1] in which the galvanometer support, hanging from long wires or cords, is only slightly affected by lateral tremors of the building to which it is attached. This very simple and efficient arrangement has two disadvantages: (1) The apparatus is liable to be set swinging by air currents; (2) there are always oscillations about a horizontal axis. A good deal of attention has been paid to these latter. Since they are oscillations about the center of gravity their effect is largely neutralized by attaching at that point the fiber which bears the magnet system. This adjustment is one of the special points of the Julius suspension. It is evidently desirable to diminish these oscillations also. To secure this result, Julius directs that the suspended frame be made symmetrical, and the suspension wires as nearly alike as possible. The impulses coming down the different wires will thus be very nearly alike, and so will have the least tendency to produce rotations. The absence of this condition Julius assigns as the probable cause for the failure of attempts to secure stability by suspending from springs, etc.

There are, nevertheless, reasons for thinking that springs may be an advantage. The equal wires are never able to exclude altogether the impulses that produce rotation, and those which do arrive act upon a system capable of very energetic rotation with vibrations of a period comparable with those of the disturbances. If the system were made incapable of these vibrations it would be much steadier, even if the arriving impulses were somewhat increased. Hamy,[2] who has also treated the question mathematically, has obtained good results with springs, but finds that some precautions are necessary to get the best results. Damping devices are especially important. The neglect of some of these precautions may account for the failures alluded to by Julius. Suspensions supported by springs are reported as "very efficient" at Johns Hopkins University. I have found that three different systems were improved by springs, especially if these were long. Some years ago a very crude suspension in use at Cornell University, in which the magnet system was not well protected from the oscillations about the center of gravity, was improved by the substitution of cords for wires, and still further when the cords were saturated with linseed oil. Here there seems to be little doubt that the improvements were due to the elasticity of the cord, and the damping effect of the viscous material. In all such cases discrepancies between the results obtained by different observers are apt to arise from the fact that there are, so to speak, various kinds of stability, and different apparatus and conditions require different treatment.

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[1] W. H. Julius, Wied. Ann., 56, p. 151, 1895; Zeits. Instrkunde, 16, p. 267, 1896.

[2] M. Hamy, Compt. Rend., 136, p. 990, 1903.

The Julius of course answers all ordinary requirements[1] — the above discussion applies rather to the problem of securing specially high stability. When this is desired, however, there is an evident profit in making some effort to secure the steadiest possible foundation to begin with. This will generally be a basement floor; accordingly, the writer tried to make use of devices which did not need to be supported from above.

II. STABILITY OBTAINED BY FLOTATION ON MERCURY OR OIL.

This method has been studied by Einthoven[2], who used mercury. Although his results as announced bear no apparent resemblance to those of Julius, a little examination shows that the properties of the two tremor-shielding devices are essentially similar. Einthoven found that for good results the float should be flat and the apparatus low down upon it. The flat float, offering a small lateral surface to the fluid, acts like the long wires of the Julius in conferring freedom from purely horizontal disturbances. That is, upon the center of gravity; since the float must follow the vertical motions of the support at every point it will rotate like the Julius suspension (though, in general, to a less extent), and points above or below the center of gravity will have horizontal motion. Putting the apparatus to be shielded low is simply putting it near the point of least motion.

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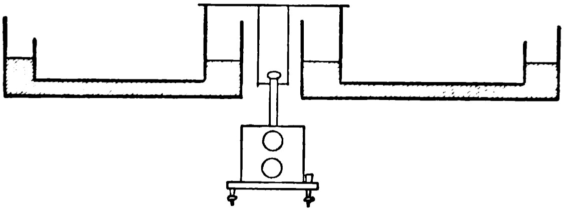
[1] Dr. A. L. Day has called the writer's attention to the fact that, in addition to the adjustments described by Julius, the regulation of the dash pots is important — so much so, that changes in the temperature of the room, by altering the viscosity of the damping fluid, may materially affect the stability.

[2] W. Einthoven, Wied. Ann., 56, p. 161, 1895.

It would manifestly be still better if the apparatus could be put exactly at the center of gravity, as in the Julius suspension. This result can be secured by making both tank and float in the form of hollow squares, as shown in vertical section in Fig. 6. As the apparatus is now suspended from the float, it may be of any shape we please.

Fig. 6.



Tremor-shielding device. Vertical section through a tank and float, made in the form of hollow squares so that apparatus can be supported at the center of gravity, the point of least disturbance.

Another alteration would be the substitution of heavy mineral oil[1] for mercury. By this several advantages are gained. The oil is cheaper, and non-poisonous; on account of its viscosity it tends to damp out the waves produced by tremors of the tank; it does not require so careful leveling, on account of the thicker layer admissible underneath the float, resulting both from the greater cheapness of the oil, and its greater damping power.

Comparing this arrangement as to its efficiency with the Julius, we have the tank offering to the oil a lateral surface not much greater than would be exposed in a set of dash pots, and this is, with the exception of the air, practically the only medium for communicating tremors to the float. It might be expected to behave, therefore, like a Julius system in which the horizontal disturbances from the dash pots were slightly increased, and those from the wires entirely removed, while the oscillations of the support about a horizontal axis were not augmented, as they are apt to be with the Julius. The tank takes up more floor space, and instruments on it are less accessible. It costs less, and can be more easily shifted from one part of the room to another. It is especially adapted to that large class of cases where a pier is present, or where the floor itself is steadier than the ceiling.

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[1] D. B. Brace, Astroph. Jour., 5, p. 214, 1897.

Abbott, at the U. S. Astrophysical Observatory,[1] used a combination of wire and fluid support, putting the float at the top. There are two advantages in putting it below. (1) We can use either larger masses or a smaller amount of fluid, since the float has only the galvanometer to sustain. (2) The instrument will remain level in spite of changes in the suspension wires. This latter advantage, indeed, might prove a powerful reason for the use of the flotation method, aside from its additional tremor-shielding power, in cases where the use of springs caused troublesome changes in the level of the Julius alone.

12. THE JOINTLESS TABLE.

If a rod or bar of wood or metal is stood upright on one end, then for any small horizontal vibrations of the supporting surface the center of percussion of the bar — two thirds of the way up, if it is uniform — will have very little horizontal motion. If three equal upright bars have cleats fastened to them at the center of percussion a board or frame laid upon the cleats will form a table which is relatively independent of tremors. Such an arrangement, like the tank and float, is evidently most useful where the floor is steadier than the walls or ceiling. It may possibly be the same arrangement which Abbot calls a "rickety table,"[2] but I am not aware that it has been particularly described.

One great advantage of the arrangement is that, like the fluid tank, it is self-damping. It needs to be guarded in some way against shielding devices. For continued use it should be of metal, as the warping of wood gives endless trouble. In the preliminary tests of the jointless table, it was found that an ordinary table magnified the tremors of the floor even more than the other diminished them. Hence, if a table or stand must be used on a pier the substitution of a jointless table for an ordinary one gives a specially great gain.

13. EXPERIMENTAL RESULTS.

The three forms of support just described were tested, singly and in various combinations. An exact comparison of tremor-shielding devices would require that they be supported at identically the same points, and then the results would apply to the particular apparatus used to test the amount of shielding. No such exact comparison was attempted, and the conclusions are somewhat in the nature of estimates. The steadiness was judged in all cases by the use of mercury, and usually by various galvanometer systems also.

The estimate made above of the relative affectiveness of the Julius and the tank seemed confirmed. The center of gravity of the tank was much steadier than points not 20 cm. above it, though the difference was less than with the Julius, which seems to magnify the oscillations which its support performs about a horizontal axis.

The combination of a tall jointless table supporting a Julius does not seem worth while under ordinary circumstances, considering the liability to disturbance from air currents and the complication generally. On the other hand, a low jointless table seems less effective than either of the two other supports. Nevertheless it would be superior to the Julius if the floor were more stable than the ceiling. It is also an advantageous support for a tank.

The stability finally attained was limited by the effect of air currents. The disturbance from them is so great that it has become necessary to use nothing but the float (which is somewhat protected by the walls of the tank around it), and to hold this in place by relatively stiff springs. That is to say, since the whole object of a tremor-shielding arrangement may be said to be rendering a piece of apparatus independent of the building in which it is, the air currents have in this case compelled a deliberate sacrifice of the steadiness otherwise attainable. Even as it is, while a violent stamping on the floor near the apparatus produces no perceptible effect, the waving of an arm at the same distance gives a millimeter deflection. This result is partly chargeable to the galvanometer, which is but slightly affected by tremors, but has, on account of its damping being mostly magnetic, small protection against the production of a pendulum-like swaying of the coil, caused by slow motions of its support. In any case, it is clear that one condition of high stability in the presence of tremors is a very thorough shielding against air currents. The necessity for such shielding is often mentioned, but the extent of it may prove surprisingly great. A Julius support carried on a jointless table was markedly disturbed if anyone walked by at three meters' distance, outside a closed door. There was no question that the effect was due to the movement of the air, and not to ground tremors.

14. SUMMARY.

1. In all galvanometers a counter E.M.F. is produced by the motion of the coil or magnet, which seems to cause a damping effect. When there is no appreciable waste of energy and when the counter E.M.F. is sufficient to make the motion just barely aperiodic, the galvanometer has the maximum possible effectiveness, as far as utilizing the energy of the current is concerned, and this effectiveness is the same for all such galvanometers.

2. The power of the magnet in moving coil galvanometers renders easy the securing of the above condition, in ordinary cases.

The sensibility for given resistance and period then depends solely on the deflection produced by a given quantity of energy. Moreover, for such moving coil galvanometers as are generally used the limit of delicacy of construction is set by the size of the suspending wires. It follows that the normal sensibility depends solely on the directing force of the coil, and is proportional to the square root of that.

3. When resistance or period is changed the sensibility is proportional to $\sqrt{RT}$, where R applies to the whole circuit. To employ instead of this formula the one derived from the Kelvin galvanome-

vanometer is the ability to replace part of the wire of the coil by other masses without loss of sensibility. This type is thus by far the best when pointers, etc., are to be moved, as in lecture table apparatus.

5. For the same reason the width of the coil has, in general, no effect on the sensibility.

6. A change in the width of the coil has, electrically, the same effect as a change in the size of wire in the coil.

7. Coil resistance and magnet strength may to a large extent be substituted one for the other. The dynamic constant is proportional to the product of the field strength by the square roots of the coil resistance and of the moment of inertia.

8. Trouble arising from magnetic impurities in the coil is reduced to one sixtieth by using a radial instead of a uniform field.

9. It is easy to make a simple and "robust" moving coil galvanometer that shall have an *effective* sensibility of 560 megohms for 100 ohms resistance and a period of 1.3 seconds. (This slightly excels the most sensitive Dubois-Rubens ironclad galvanometer, for the same conditions.) The internal resistance can not then be brought much under 20 ohms.

10. A separate mirror promises a very considerable increase in this sensibility.

11. Attempts to increase sensibility by decreasing the control below the limit naturally set by the wire have not been successful.

12. In securing high stability a device which can be supported by a basement floor is preferable to any arrangement, in itself equally good, which hangs from a wall or ceiling. A tank and float may thus be of greater advantage than a Julius suspension.

13. It is very much better, when a float is used, to suspend the apparatus from its center of gravity. This is readily accomplished by making the float and tank annular.

14. A jointless table permits taking advantage of a stable floor, and excels in simplicity and portability.

Cornell University.

TEST OF THE LIQUID AIR PLANT AT WESLEYAN UNIVERSITY.[1]

By W. P. Bradley and A. W. Rowe.

THE plant was installed in the summer of 1902. It consists of a 15 H. P. Mietz and Weiss kerosene motor, a four-stage compressor built by the Norwalk Iron Works Company, of South Norwalk, Conn., and a liquefier constructed in our laboratory.

The cylinders of the compressor are single acting, and are 7¼, 3⅞, 2, and 1 inch in diameter respectively. The air is taken from under the eaves of the building into a forty-gallon tank or reservoir, and thence directly into the low pressure cylinder. The compressor is provided with three intercoolers in the usual way.

From the compressor, the air passes successively through an aftercooler; a separator; two cylinders, containing potassium hydroxide in sticks and calcium chloride respectively, in order to remove the carbon dioxide and water vapor; and finally through a precooler charged with broken ice or snow — reaching the liquefier at a temperature of about 2°. The unliquefied air which escapes from the liquefier is carried back through the reservoir already mentioned into the compressor again.

There is an obvious advantage in using over again the air which has once passed through the purifiers. As it is practically impossible however so to adjust the valve of the liquefier as to accommodate exactly the amount of air which is being delivered from the compressor, it frequently happens that this purified air returns for a short time somewhat too rapidly from the liquefier to be taken up at once by the compressor. The reservoir is intended to store this lengths of this pipe, of somewhat more than 50 meters each, are wound in parallel so as to form a series of discs, fifty-nine in number, lying one above another and forming a cylindrical column 11.75 cm. in diameter and 41 cm. in height. A space of 1.6 mm. separates the discs and an equal space separates the turns of pipe in each disc.

Two types of liquefier are in somewhat general use. In one of these, that of Linde, the current of compressed air is divided at the expansion valves, the greater portion returning to the compressor at an intermediate pressure, while the rest, minus the liquid formed, leaves the liquefier at atmospheric pressure. In the Hampson liquefier, on the other hand, all the air which passes the valve drops at once to atmospheric pressure. The liquefier here described belongs to the latter, or single-circuit type.

The measurements of the various factors involved in the tests were made as follows:

The horse-power developed in the engine was taken in the first instance from indicator cards in the usual way. At the close of the tests, the accuracy of this indicated horse-power was checked once for all by a brake test, during which cards were taken.[1]

The kerosene used in the motor had a specific gravity of 0.775, and its combustion value was 11.072 calories.[2] Accordingly, one quart of the oil represents 12.66 H. P.

To measure the amount of air delivered by the compressor, an arrangement was made whereby the air current could be diverted at will from the liquefier into a large metallic cylinder or eudiometer. The method is illustrated in Fig. 1. The valve of the liquefier having been closed, the throttling valve *A* is opened and so adjusted that the high pressure gauge of the compressor remains stationary at the pressure required. During the establishment of equilibrium in this way, the valve *B* is maintained in the horizontal position B_1, and the blast escapes through it into the open air. When a measurement is to be taken, *B* is suddenly turned downward into the position B_2 for a measured interval of time, during which the air flows through the valve *G* (position G_1) into the eudiometer *C*, forcing the water over into the reservoir *D*.

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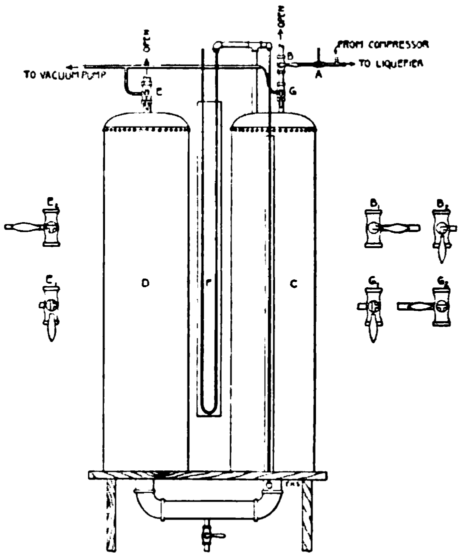
[1] Two indicators were used during the tests. They were obtained respectively from the Star Brass Manufacturing Co. and from the Crosby Steam Gauge and Valve Co., both of Boston, Mass. The former was of the usual type, including the inside spring, while the latter was of a new design with the spring mounted above the arm. Both were accurate and gave concordant results. The new indicator is somewhat more convenient for use upon an explosive engine, as it is more easily taken apart for cleaning.

[2] We are indebted to Mr. Ernest M. Swett for this determination of the thermal equivalent of the oil, which was carried out with the aid of the Atwater modification of the Berthelot bomb calorimeter. Three tests were made, giving the values 11.057, 11.092 and 11.066 respectively.

In the calculation of the horse-power equivalent, the U. S. quart is considered equal to .9464 liters.

Fig. 1.



If a suitable time interval has been chosen, the water will have been nearly all driven into the reservoir and the air in *C* will therefore stand under a certain slight pressure, which will be indicated by the gauge *F*. To bring the air to atmospheric pressure before reading, the reservoir is connected with a small suction pump by means of the valve *E* (position E_2), until the liquid in the arms of the gauge *F* is leveled, the top of *E* being covered meanwhile by the hand.

To prepare for the reception of a new charge of air from the compressor, the suction pump is switched from *D* to *C* (with the valves in the positions B_1, G_2, E_1).

A carefully calibrated tell-tale tube on the outside of *C* shows the volume of air delivered. Under each set of conditions employed, several concordant readings of volume were taken and averaged.

The approximate horse-power equivalent of the air delivered, which is designated below as compressor horse-power, is calculated on the basis of isothermal compression by the formula: .

$$HP = \frac{P_1 V_1 \log_e P_2/P_1}{33{,}000}$$

where

P_2 = absolute pressure of compressed air in pounds per square foot,

P_1 = atmospheric pressure in pounds per square foot (14.7 × 144),

V_1 = volume in cubic feet per minute at atmospheric pressure, P_1.

The amount of liquid air produced was measured by drawing it into a large vacuum test-tube of about 600 c.c. capacity. It was drawn at regular intervals of six minutes, being allowed to accumulate in the reservoir of the liquefier between drawings.

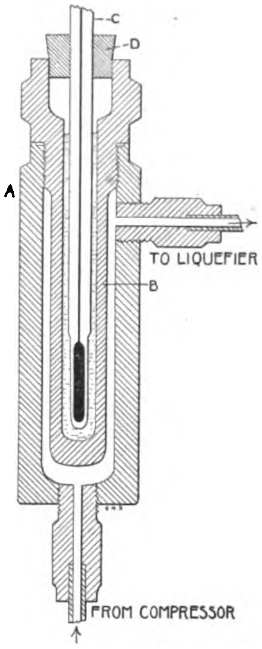
However the liquid is drawn, a certain loss by evaporation is unavoidable. This loss is greater if the liquid flows continuously as fast as formed, rather than in full stream at not too short intervals. Of course the liquid evaporates also on standing, whether in the liquefier or in the vacuum vessel. Probably evaporation is more rapid in the liquefier. But it is worth nothing that while outside evaporation involves a total loss of the energy represented in the portion of liquid evaporated, inside evaporation does not. Even if it be granted that the heat required to convert liquid into vapor of its own temperature comes in part from without, that which serves to raise that vapor to room temperature is taken from the coils of the interchanger, and so from the incoming compressed air. In so far, at least, liquid which evaporates inside the liquefier tends to replace itself.

The yield of liquid, as recorded for any given set of conditions, represents the average of a considerable number of drawings. Of course no drawings were recorded until the liquefier had reached thermal equilibrium.

In calculating the horse-power equivalent of the liquid, its total energy per gram was assumed to be 97.5 calories,[1] and its specific gravity, 0.932.[2]

During all the tests given in this paper, careful note was made of the initial and final temperatures of the air, as it was delivered to, and escaped from the liquefier. The final temperature is easily obtained by the use of an ordinary thermometer whose bulb lies in the direct path of the air as it emerges from the top of the interchanger. The initial temperature on the other hand gave us some trouble until the adoption of a simple device which is illustrated in Fig. 2, and which scarcely needs any explanation otherwise. The compressed air, just before entering the liquefier, passes through the block *A* in such a way as to completely envelop the pocket *B*, which contains the bulb of the thermometer *C*. A little water introduced into the pocket serves to convey the temperature of the air to the thermometer more quickly than would otherwise be the case. If the initial temperature were below 0°, kerosene or some other suitable liquid could be substituted. The thermometer is held in place by a rubber stopper *D*.

Fig. 2.



We pass now to the results obtained.

In Table I. are presented complete records of four runs, made at the ordinary working pressure of 2,600–2,700 pounds. One other run is included, in which no determination was made of the amount of air delivered by the compressor, and still another, in which no cards were taken from the motor. These two runs were made before the arrangements for the tests were complete. This table gives a fair representation of perhaps over one hundred runs made with the plant for the purpose of obtaining liquid air simply, and of

TABLE I.

Normal Runs.

Pressure.	Air.			H. P.			Cu. Cm. Liq. per Eng. H. P.	Per cent. Air Liq.
	Cu. Ft. per Min.	Liters Liq. per Hour.	Oil.	Eng.	Comp.	Liq.		
2,665	14.9	2.84			5.0	.40		8.6
2,670		2.35	98.7	5.2		.33	452	
2,650	15.5	2.80	86.1	7.1	5.2	.40	394	8.0
2,650	14.4	2.60	83.6	6.3	4.8	.37	413	8.1
2,660	15.0	2.88	81.0	5.7	5.0	.41	505	8.7
2,670	13.6	2.66	83.6	6.2	4.5	.38	429	8.8
Av. 2,661	14.7	2.69	86.6	6.1	4.9	.38	439	8.4

A study of the data reveals the following facts:

The Motor. — The efficiency of the motor averages 7.1 per cent. This is a low efficiency for a motor of this type, but it is sufficiently accounted for by the fact that the load is less than half of that for which the motor was designed. It is well known that explosive engines, though they exhibit great efficiency when run at full load, as compared with those driven by steam, fall off in this regard with especial rapidity when they are either overloaded or underloaded.

The Compressor. — The horse-power equivalent of the compressor output averages 77.7 per cent. of the indicated horse-power of the motor.

It should be said in this connection that the compressor was run at a speed of 133 revolutions per minute, and not at its normal speed of 180. Something more will be said on this point below.

The efficiency of the compressor varies considerably in the different runs. Thus in the fourth, only 14.4 cubic feet were delivered, with an expenditure of 6.3 H. P. in the motor, while in the fifth, 15 cubic feet required but 5.7 H. P. Not all of the apparent variation is to be charged to the compressor, for the motor friction, and slip of the belt cannot be supposed to remain constant from day to day. A large share is due however to the compressor, as was shown by a study of the pressures registered by its three lower gauges. The details of the matter would be out of place at this

The Liquefier. — The air liquefied averages 8.4 per cent. of the air delivered from the compressor.

The average yield of liquid air per horse-power per hour is 439 c.c.[1]

In the fifth test the yield is 505 c.c. This is 15 per cent. higher than the average, and is considerably the highest yield per horse-power per hour which we have ever obtained with this liquefier. In spite of that fact, there is considerable reason to think it more nearly representative of the normal functioning of the plant than the average yield itself. The record was made after the plant had been running over three hours in the production of liquid air. All of the peculiar fluctuations of pressure on the three lower gauges, which have already been alluded to as characteristic of the early stages of a run, had disappeared, indicating the establishment of thorough and uniform lubrication. The motor, which always runs somewhat stiffly when first started, had smoothed down and was consuming an unusually small quantity of oil. And the purifiers were working very perfectly, with the result that the liquid air, as it flowed from the reservoir, was entirely free from turbidity and sparkled like fresh spring water.

All of the other tests were made as soon as thermal equilibrium was fairly established in the liquefier, and therefore under comparatively less favorable circumstances. Indeed, in the third test especially the motor piston was known to be working with much friction, owing probably to the deposition of carbon in the cylinder, while in the fourth, the compressor gauges revealed the fact that the piston leakage was greater than usual during the entire run, so that we were not at all surprised on measuring the air delivered at the close

lubrication of the cylinders, particularly the third and fourth. These receive no direct, independent lubrication. The third cylinder depends upon the oil which is brought along mechanically from the second, while the fourth draws similarly from the third.

[1] Some previous records are:

Standard Hampson Liquefier: 340 c.c. (Hampson, Scientific American Supplement, July 1, 1899). 200–300 c.c. (Travers, Study of Gases, pp. 192–4). 166 c.c. (Olszewof the run to find it less than usual. Both of these comparatively unfavorable tests are presented nevertheless with the rest, because they represent conditions which are likely to be encountered at any time in connection with the short runs which are often made with the plant.

The average efficiency of the entire plant is 6.2, that is, the horse-power equivalent of the liquid air produced averages 6.2 per cent. of the indicated horse-power of the motor. In this respect also, as might have been expected, the fifth run is well in the lead, with an efficiency of 7.2 per cent.

THE EFFECT OF INITIAL TEMPERATURE.

Among the recorded yields at other laboratories, which have already been given, it will be noticed that the last three represent precooling through the aid of freezing mixtures. Müller used ice and salt; Linde recommends calcium chloride and ice; while Olszewski uses a still more powerful cryogen, solid carbon dioxide and ether. In all three cases, the yield is larger than the ordinary, as was to have been expected.

It seemed of interest to test the effect of initial temperature on the output of our liquefier. The results embodied in the following table were secured, so far as temperatures above 20° are concerned, by substituting hot water for the ice of the precooler. The desired temperature was maintained by leading into the water a suitable current of steam. The data were all obtained in a single continuous run, so as to secure as nearly uniform conditions otherwise as possible, and after each change of temperature the record began only when thermal equilibrium had been secured, and the drawings of liquid became concordant.

The pressure was 2,650 lbs. by the gauge. The horse-power amounted to 7.1, and though rather high, remained quite constant.

Diagram I. exhibits the manner in which the yield of liquid air per horse-power per hour decreases with rising temperature.

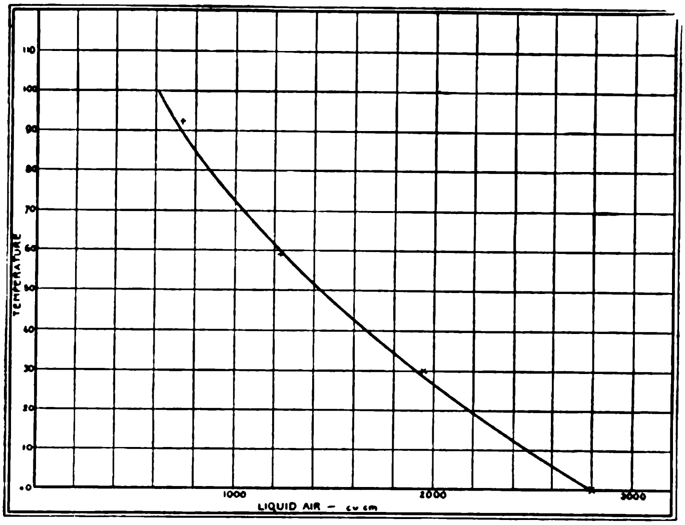
Whether at a sufficiently high initial temperature the liquefier would cease to produce liquid air cannot be decided by these data. Much less is it possible, if so, to form any conclusion as to the location of such a thermal limit. In any event, the great economy of

TABLE II.

Effect of Initial Temperature.

Initial Temp.	Air.		Eff. of Plant.	Per cent. Air Liq.	C.c. Liq. per Eng. H. P.
	Cu. Ft. per Min.	Liters Liq. per Hour.			
+2	15.5	2.80	5.6	8.0	396
30	15.3	1.94	3.9	5.7	274
59	15.0	1.23	2.5	3.7	174
92	15.0	.74	1.5	2.2	105

Fig. 3.



power which is effected by precooling is made very clear by the diagram.

We have as yet made use of no cooling agent other than broken ice or snow. It is our intention soon to extend our tests to low initial temperatures also. Even on the conservative assumption that the efficiency curve is rectilinear at temperatures below 0°, efficient precooling by means of solid carbon dioxide and ether, as employed by Olszewski should increase the average yield of our liquefier to five liters or more per hour, which would be equivalent to 700 c.c. per horse-power.

EFFECT OF INITIAL PRESSURE.

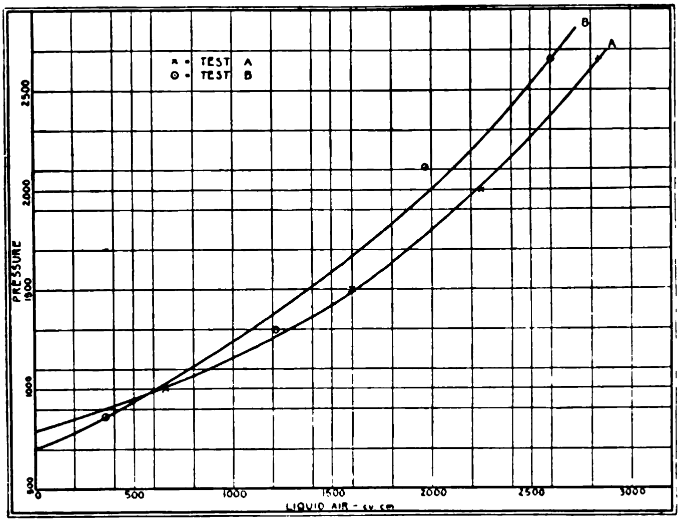
Another question upon which some data have been secured is the relation between the initial pressure of the air supplied to the liquefier and the yield of liquid air. Two representative tests are given, which were made about a year apart. In the first, no indicator cards were taken. In the second, the horse-power of the motor is given for the higher pressures only. When the compressor is being driven at pressures in the neighborhood of 1,000 lbs. or lower, the explosions in the cylinder of the motor become irregular, and it is

TABLE III.

Effect of Initial Pressure.

Test.	Pres.	Air.		H. P.			Eff. of Plant.	Per cent. Air Liq.	$t_1^\circ - t_2^\circ$.
		Cu. Ft. per Min.	Liters Liq. per Hour.	Eng.	Comp.	Liq.			
A	2,665	14.9	2.84		5.0	.42		8.6	0.0
	2,000	15.8	2.25		5.0	.33		6.4	0.25
	1,500	16.2	1.60		4.8	.23		4.5	0.50
	1,000	17.1	.66		4.6	.10		1.7	3.25
B	2,650	14.4	2.60	6.3	4.8	.38	6.0	8.1	0.0
	1,980	14.6	2.11	5.5	4.6	.31	5.6	6.5	1.0
	1,300	15.8	1.21		4.6	.18		3.5	2.5
	850	16.4	.37		4.3	.05		1.0	5.5

Fig. 4.



difficult to obtain cards which are fairly representative. Each of the tests, *A* and *B*, forms a continuous run, in which, after each change of pressure, no records were made until equilibrium was established throughout the plant.

Diagram II. exhibits the effect of pressure on the yield.

At low pressures the yield falls off very rapidly indeed. In fact, it is evident that not far from 700 lbs. no liquid would accumulate at all, though the temperature of liquefaction would be maintained. The cold produced at the valve would be exactly offset by the various unavoidable losses.

Two of the factors may be mentioned which contribute to the impaired efficiency of the liquefier at low pressures. In the first place, if the generally accepted view be correct,[1] that the fall of temperature at the moment of expansion is due to the Joule-Thompson effect, the degree of refrigeration produced at the valve should decrease with the pressure. In the second place, the larger volume occupied by a given mass of air at low pressure necessitates a much more rapid passage through the liquefier, inside the pipes. This rapidity of flow becomes increasingly an obstacle to perfect thermal interchange between the inflowing and outflowing currents. This is eloquently attested by the figures in the last column of the table, which show the difference between the temperature of the air as it enters the liquefier ($t_1°$), and that at which it escapes ($t_2°$). At the ordinary working pressure of 2,600–2,700 lbs. this difference is so slight as to be negligible, while at the lowest pressures it increases very rapidly.

At 850 pounds, for instance, the escaping air was 5.5° colder than when it entered the liquefier. Of course this is a very serious matter when taken in connection with the constant loss due to imperfect insulation. The yield at low pressures would be smaller even than it is, if it were not for the fact that the compressor pumps more air as the pressure falls, as is shown in the third column.[2] It will be shown below that under otherwise equal conditions, the yield of liquid air increases with the amount of air pumped.

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[1] Quite recently this view has been vigorously assailed by Pictet. (Zeitschrift für comprimirte und flüssige Gase, 7, 1903, 1.)

[2] Due to decreased piston leakage, to the diminishing effect of clearance in the last three cylinders, to the increasing efficiency of the water jackets and intercoolers, and, in very slight degree indeed, to increased speed in the motor, as it responds to the lighter load. This increase of speed only amounts to about 1 per cent.

THE CAPACITY OF THE LIQUEFIER.

Finally, in order to determine the capacity of the liquefier, it was supplied with varying amounts of air, while all other conditions were maintained as nearly constant as possible.

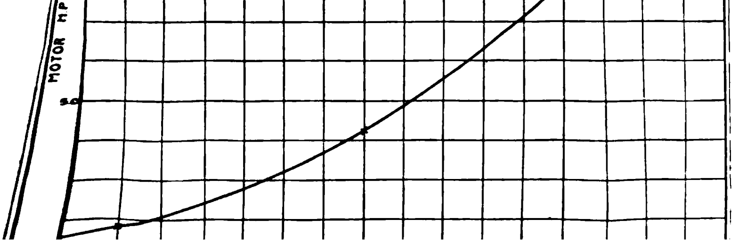
Obviously there are two ways of changing the amount of air delivered by the compressor. One way is to vary the speed at which the compressor is driven. The only practical way of securing this result, except within narrow limits, is to change the size of the driving pulley on the motor. But this involves a separate run for

TABLE IV.

Effect of Variation in the Air Supply.

Pres.	Air.		H. P.			C.c. Liq. per Eng. H. P.	Per cent. Air Liq.
	Cu. Ft. Per Min.	Liters Liq. Per Hour.	Eng.	Comp.	Liq.		
2,650	13.6	2.66	6.2	4.5	.39	429	8.8
	10.8	2.00	4.8	3.6	.29	416	8.3
	8.1	1.40	4.2	2.7	.20	333	7.8

Fig. 5.



each speed employed, and the conditions of friction, piston leakage, etc., in the motor and compressor are never the same in different runs, as is amply illustrated by the data already given. The other way is to throttle the intake. This can be done to any extent, and as often as may be desired, without any interruption in the operation of the plant, and this method therefore was adopted.

Diagram III. shows the relation between the indicated horse-power of the motor and the yield of liquid air.

One may see from the diagram how very unfavorably the throttling of the intake effects the plant as a whole. This of course was to have been expected. On the other hand, the data given make it clear that the liquefier has not yet been driven either to its full capacity or to its maximum efficiency. In fact the percentage of air liquefied, as given in the last column of the table, gives no indication that the point of maximum efficiency is close at hand. It is our purpose to attach a larger driving pulley to the motor so as to give the compressor a speed of 180 revolutions per minute — which indeed is that for which it was designed — instead of 133 as in all the tests so far. The yield of liquid air will thus be much increased, and it may be confidently expected that the efficiency of the plant also will be improved. The motor especially should develop ten horse-power much more efficiently than it now does six, which is but little more than one third of its normal load.

SUMMARY.

The chief results of this inquiry may be summarized:

1. The average quantity of liquid air per horse-power per hour in normal runs is 439 c.c. The maximum variation from this average is 15 per cent.

2. The average percentage of air liquefied is 8.4, and the maximum variation is only 4¾ per cent.

3. The horse-power equivalent of the liquid air produced averages 6.2 per cent. of the indicated horse-power of the motor. The efficiency of the plant may be considerably improved by increased compressor speed.

4. Liquid air would cease to accumulate in the reservoir of the liquefier if the initial pressure of the air were to fall to about 700 lbs.

5. The importance of precooling is very great indeed. Apparently, in runs of sufficient duration, and with present arrangements, the plant might yield 700 c.c. of liquid air per horse-power per hour, if solid carbon dioxide and ether were used as a cryogen.

WESLEYAN UNIVERSITY, MIDDLETOWN, CONN.,
June 15, 1904.

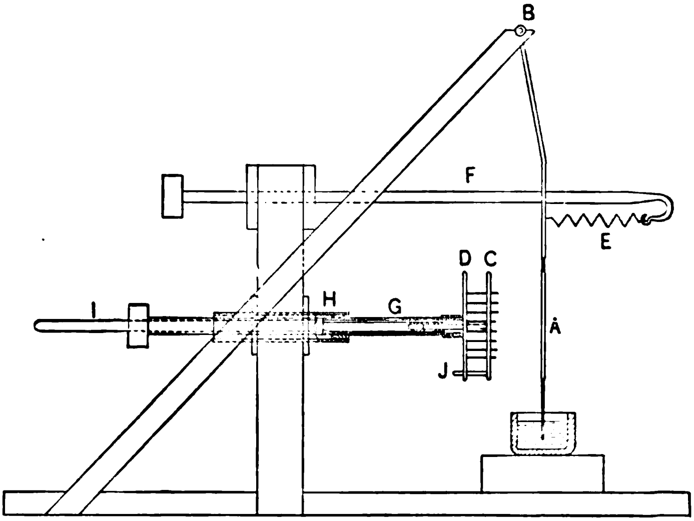
AN AUTOMATIC POTENTIAL REGULATOR.

By Arthur W. Gray.

As the writer had occasion to maintain potentials from about 5,000 volts on, which were kept up by a Toepler-Holtz machine driven by an electric motor, more constant than was possible with an ordinary point-overflow to earth, he designed the following simple arrangement, which answered the purpose very well.

A vertical projection of the regulating device is shown in the figure. A light disk *A* of polished metal is suspended by means of an insulating rod from a suitable support so that it can rotate with

Fig. 1.



diluted glycerin contained in a glass trough resting on a block of paraffin. The glycerin has dissolved in it a little calcium chlorid or other salt to render it conducting, and is prevented from creeping over the edge of the trough by slightly greasing the upper part of the glass. A wire dipping into the glycerin connects *A* with the positive terminal of the electrical machine; while another connects *C* and *D* with the negative.

The needle points opposite *A* form a number of ordinary point overflows which are all in operation at the same time; so that the potential difference between *A* and the needles is maintained at a fairly definite value, depending on the distance between them, as long as the source of electricity furnishes a sufficient supply. While the main advantage of this type of regulator over the device ordinarily used consists in the large number of points not too close together, which secures promptness in lessening any increase in the potential due to irregularities in the supply and demand, another gain is effected by the movement of the plate *A*. The attraction due to the electrostatic field between *A* and *C* is counterbalanced by the delicate spiral spring *E*, whose tension is adjusted by pushing the glass rod *F* through the cork that supports it in the framework until *A* is parallel to the plane of the needle points. Any increase in the potential now causes the disk *A* to move towards the points, thus hastening the overflow; while a decrease causes the spring *E* to draw *A* away and lessen the action of the points.

To afford an easy means of adjusting the potential to any desired value, *D* is fastened to a glass tube *G*, which slides with gentle friction through slightly greased corks fixed in the ends of a wider tube *H* fastened to the main framework. By giving the cork handle at the farther end of *G* a slight rotary motion while shoving the tube through *H*, the adjustment may be made almost as delicately as if a screw were used.

With this regulator properly adjusted and connected with a Leyden battery and a good electrical influence machine driven by an electric motor, a potential of twelve thousand volts or more could be maintained for hours within about one per cent. without giving it the slightest attention, although the speed of the driving motor remained by no means so constant; and if the regulation be aided by slightly moving *G* with the hand as the electrometer may indicate, the fluctuations may be made much less.

It is not intended that the movement of the disk *A* should follow sudden fluctuations of the potential due to such causes as erratic breaking down of insulation; its purpose is rather to readjust automatically the distance between the plate and the points in order to accommodate the slower changes that come from slight irregularities in the rate at which the machine keeps up the supply of electricity. To secure the best results, it is necessary to adjust properly the tension of the spring *E* and also the damping of *A*. The former should be very delicate and well stretched, that it may contract promptly on the slightest reduction in the strength of the field between *A* and *C*. The damping is easily regulated by varying the level of the glycerin. But in addition it is necessary to regulate roughly the distance that the needle points project beyond the plate *C*. This is done by moving *C* to or from *D* by means of the glass tube *I*, which slides through a cork in the end of *G* and is fastened to a rod soldered to the back of *C* and projecting through a hole in *D*. The short rod *J*, projecting backward from *C* through *D*, is merely to prevent breaking the needles by rotating *C* while making the adjustment. If the disk *C* is not far enough behind the points when rather low potentials are employed, *A* will be set in motion so as to cause rapid fluctuations in the potential; whereas, when high potentials are used, *C* should be pushed up close to the points so as to increase its influence on *A*. When *A* hangs so that it is parallel to the plane of the points, the latter will glow uniformly in the dark. The points should, of course, be sharp and always negative.

Berlin, Physical Institute of the University,
April, 1904

ON THE PRODUCTION OF OZONE IN THE SIEMENS GENERATOR; AND ON AN IMPROVED CONSTRUCTION OF THIS APPARATUS.

By Arthur W. Gray.

1. The writer has described elsewhere[1] some preliminary experiments which he made to compare the quantity of electricity passed through the oxygen in a Siemens ozone generator with the mass of the ozone formed. As was there mentioned, two main sources of error were present, whose effect on the results could not then be estimated, conduction of electricity along the glass surfaces of the generator and failure of the total charge to pass through the galvanometer. In the determinations described in the present paper both of these difficulties were avoided and in addition other improvements conducive to accuracy were introduced. These included a greatly improved key[2] for automatically sending a single charge or discharge through the galvanometer while the generator continued to be charged and discharged in rapid succession, an automatic potential regulator,[3] the use of more concentrated oxygen freed with greater care from water vapor, carbon dioxid, and organic impurities, and the substitution of Schönbein's method of determining the mass of ozone formed by absorbing with potassium iodid and titrating with sodium thiosulfate for Thénard's method of absorbing with sodium arsenit and titrating with iodin.

Greater accuracy was also secured by employing the expression used in calculating the quantity of electricity sent through the gas in a form independent of the shape and dimensions of the generator, so that all the quantities involved could be determined by purely electrical measurements.

In addition to investigating the formation of ozone from concentrated and dry oxygen, and the capacity of the generator at various electrical potentials, experiments were made to determine in a general way the influence of water-vapor, air, and the lengthening, the duration of the charging current (*i. e.*, reducing the current strength) by the insertion of a very high resistance in the circuit.

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[1] A. W. Gray, Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin, p. 1016, 1903; Ann. d. Phys., 13, p. 477, 1904.

[2] A. W. Gray, Phys. Rev., Vol. XIX., p. 293.

[3] A. W. Gray, Phys. Rev., Vol. XIX., p. 344.

Brief Outline of the Method of Investigation.

2. A Toepler-Holtz machine driven by an electric motor served as the source of electricity. One pole was earthed while the other was connected with a large Leyden battery and a Braun electrometer. The potential was maintained uniform by means of a specially designed point overflow to earth. While the outer electrode of the generator always remained earthed, the inner one was connected alternately with the Leyden battery and with the earth. From time to time the quantity of electricity used in charging or discharging the generator was measured. To accomplish this the writer constructed an automatic commutator and galvanometer key. By depressing a lever attached to this a *single* charge was caused to pass from the outer electrode through a ballistic galvanometer instead of direct to earth, while the charging and discharging continued in rapid succession without interruption. By depressing a second lever the current on discharging could be measured at will.

On leaving the generator the ozonized gas passed through a three-way cock lubricated with concentrated sulfuric acid and through either one of two ozone-absorbing solutions into the atmosphere. The first was merely to prevent the ozone from escaping into the room, the second for determining the amount formed during an experiment.

The number of times the electricity passed through the gas was recorded by a specially constructed rotation counter attached to the charging and discharging commutator.

In order to make a determination the electrical machine and the to divert the gas stream into the second solution and the counting device was set in operation. From time to time the quantity of electricity used in charging the generator was measured as described above. After 12,000 to 65,000 passages of the electricity through the gas, the number depending on the potential employed and on the other conditions of the experiment, the three-way cock was turned to its original position, the counting device thrown out of operation, and the mass of ozone produced determined by analyzing the second solution.

CALCULATION OF THE QUANTITY OF ELECTRICITY PASSED THROUGH THE GAS.

3. As ordinarily constructed, the effective part of a Siemens ozone generator consists of two coaxial cylindrical shells of glass separated by a layer of the gas to be submitted to the action of the electrical discharge — the whole forming a compound condenser of three layers. As long as the potential difference between the electrodes is insufficient to cause a conduction current through the gas, the electrostatic capacity remains constant; but when the stress becomes sufficient to break down the insulation offered by the gas, the capacity is increased by the effect of the conduction. In his previous paper[1] the writer showed how to calculate the quantity of electricity that flows through the second layer of a condenser composed of three infinitely long coaxial shells, when the potential difference between the outer surfaces is changed from any given value V to any other $V + \Delta V$; the first and third layers being assumed to insulate for all values of V, the second to insulate for low values, but to conduct partially or wholly if V exceeds a certain amount. If we substitute in the expressions there given

$$C_1 = \frac{K_1 l}{2 \log_e r_1/r_0}; \quad C_2 = \frac{K_2 l}{2 \log_e r_2/r_1}; \quad C_3 = \frac{K_3 l}{2 \log_e r_3/r_2},$$

at the same time remembering that

and

$$\frac{1}{C_1}+\frac{1}{C_3}=\frac{1}{C_\infty},$$

it follows that

$$\frac{1}{C_2}=\frac{1}{C_0}-\frac{1}{C_\infty},$$

or

$$C_2=\frac{C_0C_\infty}{C_\infty-C_0}.$$

Substituting this value in equation (5), p. 480, gives

$$Q_2=\frac{C_\infty(Q-C_0\cdot\Delta V)}{C_\infty-C_0}=\frac{C_\infty}{C_\infty-C_0}(C_v-C_0)\Delta V.$$

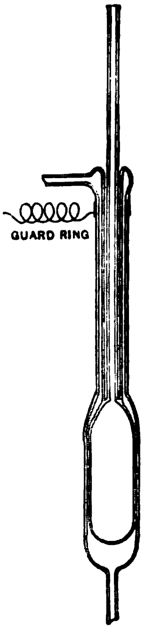
Here Q is the quantity of electricity that flows to or from the outer electrodes of the condenser, Q_2 the quantity that flows through the middle layer, C_0 the electrostatic capacity when the middle layer insulates, C_v when it conducts partially, and C_∞ when it conducts completely. In this expression all the quantities occurring can be measured electrically; and since it contains no dimensions, it is not restricted in its application to the case of coaxial cylinders, but applies to a condenser of any form whatever. It is also to be noted that the expression remains the same whether the magnitudes occurring in it are measured in C.G.S. electrostatic units, in C.G.S. electromagnetic units, or in coulombs, farads, and volts.

IMPROVED CONSTRUCTION OF THE SIEMENS TYPE OF OZONE GENERATOR.

4. The ozone generator used in the writer's preliminary work was of the usual construction with only a very short length of glass surface separating the walls bounding the gas space, so that it seemed not at all improbable that a portion of the electricity in passing from one side to the other took this path instead of going *through the gas*. As this portion would be measured by the galvanometer, although ineffective in producing ozone, the values obtained for the mass of ozone per coulomb sent through the gas would be too low. A similar error would be caused if the surface separating the electrodes of the generator failed to insulate sufficiently. The

conduction here could be decidedly reduced by coating with fused shellac as was done then while the oxidizing action of the ozone made such an application within entirely out of the question.

Fig. 1.



To overcome these difficulties the writer constructed the generator represented in longitudinal section by Fig. 1. The effective portion that comes between the electrodes is of large diameter, with thin walls as close together as possible, and short enough to permit a flow of gas sufficiently rapid to prevent the concentration of ozone from becoming so great that the deozonizing action of the electric discharge pointed out by Warburg[1] would have an appreciable effect. The remainder of the generator is of thicker glass. Interior insulation is secured by extending the gas-space above, and by reducing the diameter there; and the flow of gas is facilitated by increasing the distance between the walls in the upper portion as well as by providing a considerable space at the bottom. That the insulation thus obtained was effective, even when considerable water-vapor was present in the oxygen, seems clearly indicated by the results obtained.[2] External surface insulation is increased by carrying the conductor to the inner electrode down through a narrow glass tube sealed into the neck as indicated. As an additional precaution a metallic guard-ring connected to earth surrounded the neck of the generator just below the gas exit. The electrodes were of silver deposited on the glass in order to avoid strains which would be caused by the employment of a liquid.

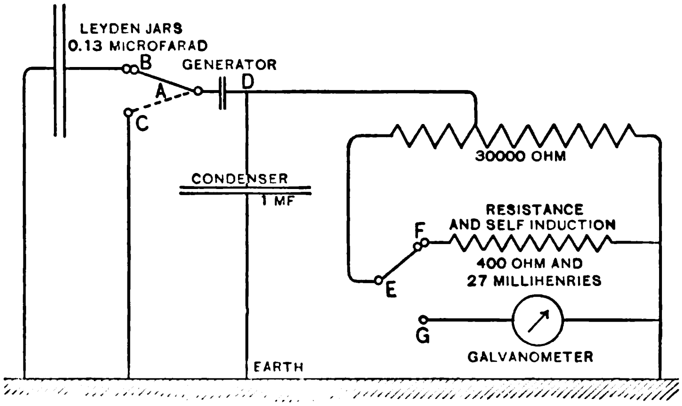
Great care was given to the construction of the generator in order to get the walls bounding the narrow part of the gas space as nearly parallel as possible. The interior was also carefully cleaned by boiling concentrated acids in it for about half an hour. The length of the uniform narrow portion was a little over 5 cm. and the mean special determinations of the mean thickness of the gas layer gave $r_2 - r_1 = 0.064$ cm. with a probable error of less than 0.001 cm.

Arrangement of the Electrical Circuit for Avoiding Leakage.

5. Owing to the great self-induction (about 27 millihenries) of the galvanometer employed, which was of the Deprez-d'Arsonval type constructed by Siemens and Halske, considerable difficulty was experienced in getting all of the current to be measured to pass through the entire length of the coil. Attempts were first made in the direction of improving the insulation of the circuit and by inserting a high resistance immediately after the generator, so as to obtain as low a potential as possible in the rest of the path through the galvanometer to earth. But when high potentials were used, the suspended coil behaved as if it were not swinging freely; instead of giving a prompt deflection, it would quiver for a few seconds, and then drift, sometimes slowly, sometimes suddenly, perhaps to the right, perhaps to the left. This behavior had also previously been observed with the similar galvanometer employed in the earlier investigation; but after removing the coil and wiping the pole pieces with the finger, the trouble, for some reason or other, apparently disappeared, so it was assumed that the coil had been catching against a bit of thread or something of the kind; but when the trouble recurred with a different instrument, its cause was searched for until found. Upon removing the brass plate across the front of the magnet and observing the coil while attempting to get a deflection, it was noticed that the coil was violently drawn aside against one of the pole-pieces and held there, while a series of small sparks appeared. Thus the difficulty was evidently due to an electrostatic charge accumulating on the insulating material of the coil on account of the great self-induction. And this effect persisted even when the resistance in series was increased to over forty megohms. Attempts to avoid the self-induction by substituting a Dubois-Rubens iorn-clad galvanometer failed because the brief but strong current changed the controlling magnetic conditions sufficiently to produce a large shifting of the resting-point after every deflection.

The difficulty was, however, finally removed completely by adopting the arrangement indicated in Fig. 2. The essential point was to place a condenser of large capacity in parallel with the galvanometer, so that the current from the generator flowed first into the condenser, which then discharged itself under very low voltage through the galvanometer at a rate governed by the self-induction in the circuit. The motor-driven key *A* connected the inner electrode of the generator alternately through *B* with the Leyden battery, which had a capacity of about 0.13 microfarads, and through *C* with the earth, making about five cycles per second. At the point *D* in the wire leading from the outer electrode of the generator a branch lead to one terminal of a microfarad condenser, whose other terminal was connected with the earth. The wire from the generator also lead to an Ayrton shunt of 30,000 ohms, one end of which was earthed, while the other lead to the key *E*, which was ordinarily connected through *F* with a resistancc and self-induction equal to that of the galvanometer (about 400 ohms and 27 millihenries); but which could, by means of an automatic device attached to *A*, be connected during the course of a single charging or discharging with *G*, so that the current would flow through the galvanometer. With this arrangement no resistance was needed in series with the galvanometer.

Fig. 2.



Since the capacity of the generator was very small compared with the other two, and since the arrangement was in series, the total capacity of the circuit through which the generator was charged, and also of that through which it was discharged, was a trifle less than its own capacity. Also, the resistance and self-induction of these circuits, being merely that of the connecting wires, was extremely small, affording a very sudden passage of the electricity. To investigate the effect of lengthening the duration of this current (see 10), a resistance of about 42 megohms was inserted between the generator and the point *D*.

The galvanometer was calibrated by substituting for the generator an Elliott standard condenser charged with a single storage cell and removing the microfarad condenser attached at *D*. While the effect of the residual charge introduced a slight error, it was of no special consequence for the present purpose, and would have no effect whatever upon the comparisons of results made later. (9 ff.)

The length of time that the key *A* remained in contact with *B* or with *C* was about 0.047 seconds. Lengthening this by decreasing the speed at which *A* was operated made no perceivable difference in the galvanometer deflection, even when 42 megohms were in the circuit of the generator.

Results.

9. As the writer had observed in his preliminary experiments,[1] a flash of light occurred in the generator when it was charged *suddenly* as well as when it was discharged. Also, with dry oxygen or dry air flowing through it, the same quantity of electricity passed through the circuit on discharging as on charging; but an exception (12) to this was observed when the oxygen contained considerable water-vapor. The results of the measurements, collected in Tables I., II. and III., and represented graphically in Figs. 3, 4 and 5, show, however, not only the unreliability of the previously used method of connecting the galvanometer for the purpose of measuring the quantity of electricity passing through the circuit, but also the serious deficiency of the ordinarily constructed Siemens generator as a means both of carrying out such investigations as are here described, and of applying to the production of ozone the greatest fraction of the electricity employed.

The writer considers all of the determinations described in the present paper as reliable to within about one per cent., while in

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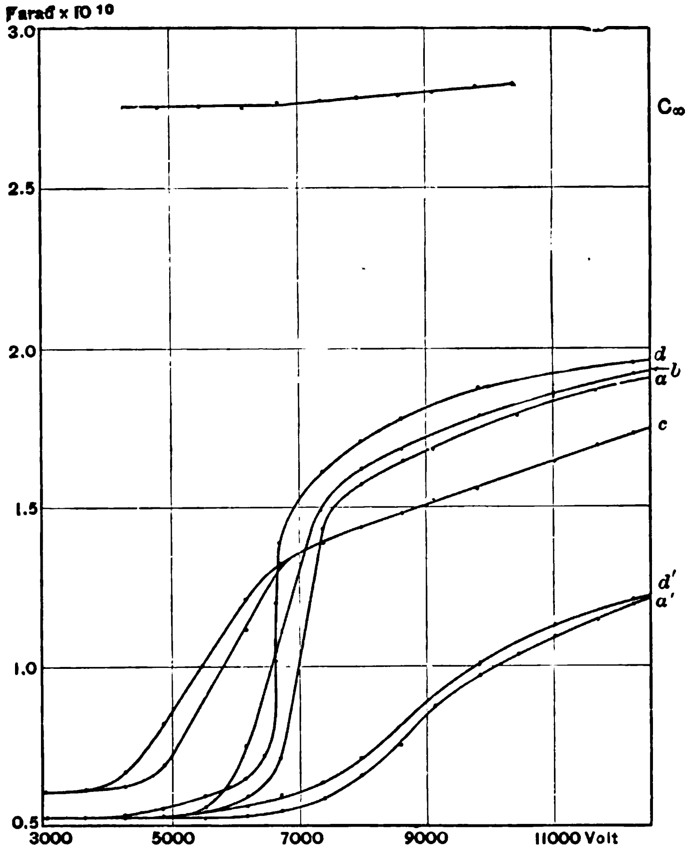
[1] A. W. Gray, Ann. d. Phys., 13, p. 486, 1904.

Fig. 3.



many cases greater accuracy was undoubtably reached. Among the thousands of galvanometer deflections that were obtained, not a single unusual one was observed, while in the previous work abnormal ones would appear every now and then.

On account of lack of time for perfecting a means of reliably measuring potentials beyond 12,240 volts, which was the limit of the Braun electrometer employed, it is to be understood that the general statements that follow concerning the description and interpretation of the results are intended to apply only to cases where the potential is less than this value, although some of them, at least, are undoubtedly applicable beyond the range investigated.

The Capacity of the Generator at Various Potentials.

10. Table I. shows the influence of the potential and other conditions on the electrical capacity of the generator. The first column, V., gives in volts the potential difference applied between the electrodes; the other columns give the corresponding values of the capacity, $C_v \equiv Q/V$, expressed in units equal to 10^{-10} farads. The results recorded under *a* and *a′* were obtained when oxygen, more than 98 per cent. by volume pure, and given its final drying by passing over fresh phosphoric anhydrid, was passed through the generator. Those under *b* illustrate the effect of a very slight amount of water-vapor with the same oxygen, being obtained after the phosphoric anhyrid had been somewhat used up and was coated with a slight liquid film. Renewing the P_2O_5 caused the return of the values under *a*. In obtaining the results under *c* the same oxygen was again used, but the soda lime and phosphoric anhyrid were removed and the gas caused to bubble through distilled water contained in an absorption apparatus of five chambers in series, and then to pass through a considerable length of cotton wool to prevent spray from being carried along into the generator. The temperature of the water was slightly lower than that in the neighborhood of the generator, so the vapor was not quite saturated. The results for air dried with phosphoric anhydrid appear under *d* and *d′*, and those for mercury filling the gas-space under C_∞. While making the measurements for *a*, *b*, *c*, *d* and C_∞, the only resistance in the charging and discharging circuits was that of the conducting wires, whereas while obtaining the last two columns, *a′* and *d′* the duration of the electric current was considerably increased by inserting between the generator and the point *D*, Fig. 2, a resistance of about 42 megohms composed of a solution of cadmium iodid in amyl alcohol. All of these results are represented graphically in Fig. 3, each curve being designated by the letter heading the column from which it was plotted. Except for the low potentials, *a* and *a′* are the mean values taken from Tables II. and III.

During the various experiments in which oxygen was used, the pressure in the generator ranged from 76.2 to 78.4 cm. of mercury, while with air it was about 73.5 cm. A thermometer suspended about four centimeters from the generator indicated a mean temperature during any one series of measurements ranging from 15.6 to 19.7° *C*. A careful inspection of the results revealed nothing that could be traced to these slight variations of either pressure or temperature.

TABLE I.

V	a	b	c	d	C_∞	a'	d'
3070	0.522		0.609				
3660	0.524		0.615			0.519	
4250	0.525		0.62 0.66	0.53	2.76		0.528
4850	0.523		0.69 0.82	0.55	2.76	0.524	0.533
5500	0.552	0.554	0.91 1.00	0.60	2.76	0.520	0.541
6160	0.588	0.750	1.12 1.21	0.65	2.75	0.530	0.555
6430				0.72			
6600				1.20			
6710	0.717	1.112	1.32	1.40	2.77	0.540	0.600
7380	1.42	1.490	1.39	1.61	2.78	0.587	0.633
7960	1.57	1.637	1.44	1.71	2.78	0.659	0.710
8590	1.64	1.670	1.48	1.78	2.79	0.751	0.801
9220	1.68		1.53		2.80	0.877	
9810	1.76	1.790	1.55	1.87	2.82	0.966	1.002
10410	1.79				2.82	1.034	
11680	1.86		1.69			1.144	
12240	1.90	1.912	1.73	1.95		1.197	1.205

$$C_0 = 0.523. \quad C_\infty = 2.76. \quad Q = \frac{C_\infty}{C_\infty - C_0}(C_v - C_0)V = 1.23(C_v - C_0)V.$$

11. It is to be noticed that none of these curves show the discontinuity observed in the preliminary experiments; but instead, a very rapid rise between certain potentials, provided the electric current is of very short duration, the capacity soon reaching a value between three and four times what it had before the gas began to conduct. When, however, the duration of the current is increased by the insertion of a large resistance, the rise in the capacity begins at a greater potential, takes place much more slowly, and, at least within the range of the potentials here investigated, is very much less, indicating greatly reduced conductivity of the gas, and also that the minimum potential difference between the boundaries of the gas-space, which plays an important part in Warburg's theory of the Siemens generator,[1] depends on conditions other than its construction.

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[1] E. Warburg, Verhandl. d. Deutschen Phys. Gesellsch., 22: p. 382, 1903.

12. A comparison of curves *a* and *b* shows that the presence of a slight amount of water-vapor in the oxygen increases its conductivity a little, but causes the rate of most rapid rise in the capacity to be reduced somewhat. Curve *c*, however, shows some rather remarkable effects of the presence of considerable water-vapor. In column *c* of Table I. it is to be noticed that for values of V between 3,660 and 6,710 volts two values of C_v are given. Those on the left were obtained when the generator was being charged, while those on the right while it was being discharged. Each value given represents a series of consistent galvanometer deflections, and as a check, the same measurements were repeated several hours afterward with the same result of finding that between these potentials a greater deflection was obtained on discharging than on charging; while outside of this region the direction of the current seemed to make no difference.

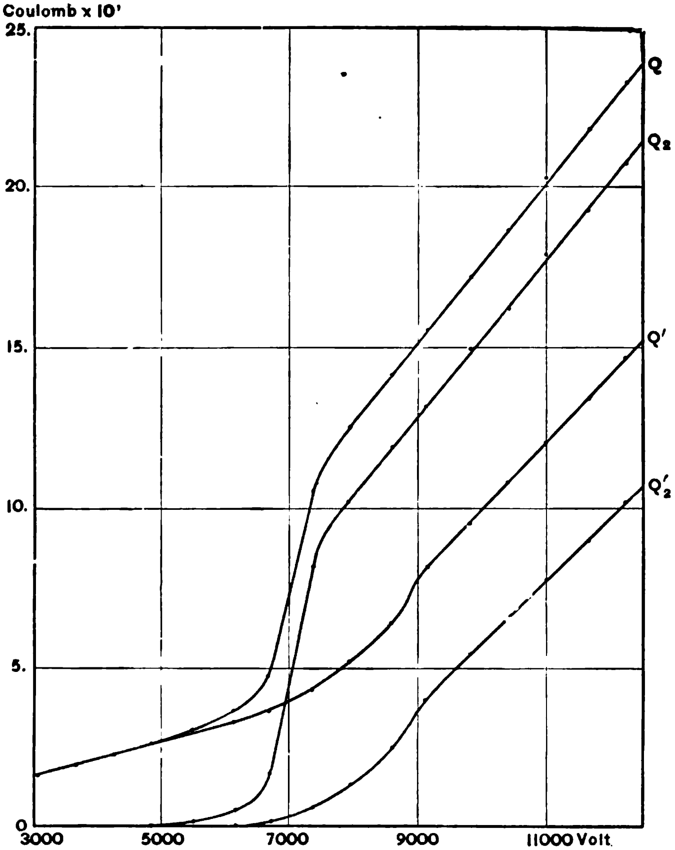
Another point exhibited by curve *c* is that while the presence of considerable water-vapor causes conduction through the gas to take place when the dry gas would insulate, and to reach a much higher value for low potentials; on the other hand, it retards conduction as soon as the potential exceeds a certain value, and from here on the increase in the capacity seems to be directly proportional to the increase in the applied potential. Just what correction may be necessary for the effect of any conduction there may have been along the internal surfaces is not apparent.

It should be remarked in passing that curve *c* is only one of a series of many similar curves representing the conditions for the same generator when water-vapor was present. In a series of twenty consecutive galvanometer deflections, the greatent deviation from the mean would be less than one half per cent.; in fact, the uniformity was greater than with dry oxygen; but an equally consistent series obtained with the same potential several hours later might differ from the first by five per cent. or even more. Just what part was played by differences in humidity, temperature, pressure, or other influencing conditions was not apparent. The results represented in curve *c* were all secured within a short space of time, and represent merely a typical case.

13. Examination of curves *d* and *d′* show that dry air begins to

conduct sooner and conducts better than oxygen; also that the difference between the air and the oxygen becomes less when the electric current is retarded by great resistance or when the potential difference applied between the electrodes of the generator is increased.

Fig. 4.



Air also shows the most rapid rise in the capacity in the neighborhood of the critical potential.

14. When the gas space was filled with mercury, the capacity remained constant up to about 7,000 volts, and then seemed to increase slowly, proportionally to the increase in the potential, at the rate of about ⅔ per cent. for every additional 1,000 volts. This is apparently due to conduction through the glass.

15. Some interesting information is imparted by Fig. 4, which presents from another point of view the results recorded in columns *a* (and *a′*) of Table I. The ordinates of curves Q and Q' represent the quantities of electricity used in *charging the generator* at various potentials, those of Q_2 and Q'_2 the quantities that passed as *conduction current through the oxygen.* Q and Q_2 are from the same data as *a* and are taken directly from Table II.; Q' and Q'_2 correspond to *a′* and are from Table III.

Until reaching the critical potential at which conductivity in the gas begins Q and Q' follow the same straight line from the origin of coördinates; similarly Q_2 and Q'_2 both remain zero. In the neighborhood of the critical potential each curve suffers a rapid rise along a curved path and then continues along a straight course which is considerably steeper than before the critical potential was reached; showing that after a certain potential is attained, the increase in the quantity of electricity is directly proportional to the increase in the potential. These lines are all slightly divergent although very·nearly parallel. Fig. 4 also makes clear that the lengthening the time required in charging and in discharging the generator reduces not only the *total quantity* of electricity that passes through the circuit, but also the fraction of this (Q_2/Q) that passes as *conduction current through the gas.*

Corresponding curves (not shown here) were plotted for the other cases investigated. All showed the same general characteristics exhibited in Fig. 4. The values of $\frac{dQ}{dV}$ and $\frac{dQ_2}{dV}$ after these curves become straight lines at the higher potentials are given in the following table, where the letters have the same significence as before.

	$\frac{dQ}{dV}$		$\frac{dQ_2}{dV}$	
a.	2.51	$\times 10^{-10}$	2.47	$\times 10^{-10}$ coulombs per volt.
b.	2.54	"	1.91	"
c.	2.25	"	1.72	"
d.	2.42	"	1.92	"
a′.	2.11	"	1.89	"

THE PRODUCTION OF OZONE.

16. The results of the measurements made to determine the relation between the mass of ozone produced under given conditions

and the quantities of electricity involved are collected in Tables II. and III. Both series were obtained when commercial oxygen, more than 98 per cent. by volume pure, and passed in succession through a solution of potassium permanganate, glass beads drenched with concentrated sulfuric acid, solid potassium hydroxid, soda lime, phosphoric anhydrid, and cotton wool, flowed through the generator rapidly enough to exclude any possibility of the ozone reaching such a concentration that the deozonizing action of the silent electrical discharge would have an appreciable effect. Nor could any

TABLE II.

V	$C_v \times 10^{10}$.	$C_v - C_0 \times 10^{10}$.	$Q \times 10^7$.	$Q_2 \times 10^7$.	$M \times 10^7$.	M/Q_2.	M/Q.
5,500	0.552	0.029	3.02	0.196			
6,160	0.588	0.065	3.62	0.492			
6,710	0.717	0.194	4.81	1.60			
7,380	1.42	0.901	10.5	8.18			
7,960	1.56	1.04	12.4	10.1	1.47	0.145	0.120
"	1.58	1.06	12.6	10.3	1.43	0.138	0.114
"	1.57	1.05	12.5	10.2	1.45	0.142	0.117
8,590	1.67	1.15	14.3	12.1	1.91	0.157	0.133
"	1.61	1.08	13.8	11.5	1.74	0.152	0.126
"	1.64	1.12	14.1	11.8	1.77	0.150	0.126
"	1.65	1.12	14.1	11.8	1.78	0.151	0.126
"	1.64	1.12	14.1	11.8	1.80	0.152	0.128
9,220	1.67	1.15	15.4	13.0	2.18	0.167	0.142
"	1.69	1.17	15.6	13.2	2.11	0.160	0.136
"	1.68	1.16	15.5	13.1	2.15	0.159	0.139
9,810	1.76	1.24	17.2	14.9	2.64	0.177	0.153
"	1.75	1.23	17.2	14.8	2.58	0.174	0.150
"	1.76	1.23	17.2	14.9	2.61	0.176	0.152
10,410	1.79	1.27	18.7	16.3	2.99	0.184	0.160
"	1.78	1.26	18.6	16.2	2.99	0.185	0.161
"	1.79	1.27	18.6	16.2	2.99	0.184	0.161
11,020	1.85	1.33	20.4	18.0	3.55	0.197	0.174
"	1.83	1.31	20.3	17.8	3.50	0.197	0.173
"	1.84	1.32	20.3	17.9	3.53	0.197	0.173
11,680	1.85	1.33	21.7	19.1	3.90	0.204	0.180
"	1.85	1.33	21.7	19.1	4.06	0.212	0.187
"	1.88	1.36	22.0	19.5	4.03	0.206	0.183
"	1.86	1.34	21.8	19.2	4.00	0.208	0.184
12,240	1.90	1.38	23.3	20.8	4.62	0.222	0.198

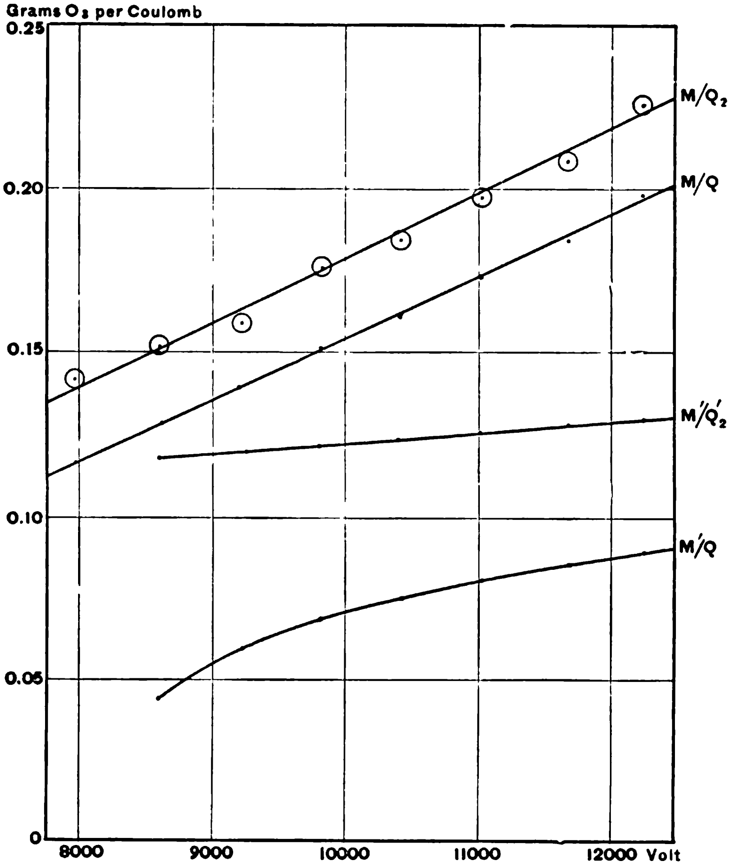
measurable quantity of ozone have escaped absorption; for, while the solution of potassium iodid contained in the first chamber of the absorption apparatus became darkly colored with the liberated iodin, that in the second chamber never exhibited more than a very faint tinge of yellow, and that in the third, fourth, and fifth remained perfectly colorless. In obtaining the results in Table II., the only resistance and self-induction in the charging and discharging circuits was that of the conducting wires, which were for the most part of polished brass about 3 mm. in diameter. Table III. gives the results secured when about 42 megohms were inserted between the generator and the point *D*, Fig. 2. *V*, C_v, *Q*, and Q_2 have the same significance as in 3, and are expressed in volts, farads, and coulombs respectively. The primed letters refer to Table III. *M* is the number of grams of ozone formed by a single passage of the electricity through the circuit. Just below each single horizontal line are given the means for the same potential of the quantities immediately above.

TABLE III.

V	$C'_v \times 10^{10}$	$C'_v - C'_0 \times 10^{10}$	$Q' \times 10^7$	$Q'_2 \times 10^7$	$M' \times 10^7$	M'/Q'_2	M'/Q'
6,160	0.530	0.007	3.26	0.053			
6,710	0.540	0.017	3.62	0.140			
7,380	0.587	0.064	4.33	0.580			
7,960	0.659	0.136	5.24	1.33			
8,590	0.756	0.233	6.49	2.46	0.292	0.119	0.0449
"	0.746	0.223	6.40	2.35	0.273	0.119	0.0435
"	0.751	0.228	6.45	2.41	0.285	0.119	0.0442
9,220	0.877	0.354	8.07	4.01	0.480	0.120	0.0595
9,810	0.966	0.443	9.47	5.33	0.650	0.122	0.0687
10,410	1.032	0.509	10.74	6.53	0.819	0.125	0.0762
"	1.022	0.499	10.64	6.40	0.778	0.122	0.0732
"	1.037	0.514	10.80	6.59	0.812	0.123	0.0752
"	1.046	0.523	10.89	6.70	0.825	0.123	0.0757
"	1.034	0.511	10.77	6.56	0.808	0.123	0.0751
11,020	1.096	0.573	12.09	7.78	0.981	0.126	0.0812
11,680	1.144	0.621	13.38	8.94	1.140	0.128	0.0854
12,240	1.197	0.674	14.67	10.15	1.296	0.128	0.0885
"	1.198	0.675	14.68	10.16	1.316	0.130	0.0897
"	1.198	0.675	14.68	10.16	1.306	0.129	0.0891

17. Inspection of the columns headed M/Q_2 (represented graphically in Fig. 5) shows beyond question that, contrary to what the

Fig. 5.



preliminary investigation seemed to indicate, the mass of ozone produced per coulomb sent through the gas increases with the potential difference applied between the electrodes of the generator, and that the increase in the former is apparently directly proportional to the increase in the latter. In addition, it is evident that the insertion of the high resistance not only decreases the quantity of electricity that passes as conduction current through the gas, but also decreases a great deal the mass of ozone produced per *coulomb of what is conducted.* And again, the presence of the resistance lessens decidedly the rate at which M/Q_2 increases with the potential. The circles around the points representing the mean values of M/Q_2 are intended to indicate approximately the degree of reliability that could be reasonably expected. Similar straight lines (not shown here) represent M/Q_2 as a function of Q_2.

18. In Fig. 5 the values of M/Q and M'/Q' are also represented. Here it is to be noticed that, in addition to what is apparent from the discussion of Fig. 4, the mass of ozone produced per coulomb of the *total quantity* of electricity used in charging the generator increases almost, if not quite, in direct proportion to the increase of the potential, provided the resistance in the circuit is negligible; but that this is not at all the case if the resistance is considerable, although proportionality is approached as the potential is increased.

19. Some experiments were made to determine the effect of considerable water-vapor on the production of ozone; but as was mentioned in 12, it was impossible to maintain constant conditions within the generator long enough so that determinations taken several hours apart could be compared with each other. However, in every one of fifteen determinations the yield of ozone per coulomb sent *through the gas* was found to be from ten to fifteen per cent. less than when well dried oxygen was employed with the same potential difference between the electrodes of the generator. And here, too, there *seemed* to be indications that, as long as the gas remained under the same conditions, the increase in the yield of ozone was directly proportional to the increase in the potential.

20. The results described in the last few paragraphs are not out of harmony with those obtained by Warburg[1] in the case of the silent electrical discharge from points. He found that the production of ozone seemed to be closely connected with the amount of illumination, and that when the positive brush discharge made its appearance, the ozone formed per coulomb increased rapidly with the current strength, although the ozone produced by the negative discharge seemed to be independent of the current. In the writer's experiments increase in potential was accompanied by increase in current. In the Siemens generator both positive and negative light are present, and it also seems reasonable to expect that the amount of illumination would be influenced by such things as the resistance

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[1] E. Warburg, Sitzungsber. d. k. Akad. de Wissensch. zu Berlin, p. 1011, 1903; Ann. d. Physik., 13, p. 464, 1904.

in the circuit, the nature of the gas, and the presence of water-vapor. The Siemens generator, however, gives a much greater yield of ozone for a given quantity of electricity sent through the gas. It is possible that the great difference in current density in the two cases may have considerable influence.

COMPARISON WITH THE RESULTS OF THE PRELIMINARY EXPERIMENTS.

21. It remains to point out a few things which seem to offer plausible explanations for certain differences between the results here described and those of the preliminary experiments. The discrepancies appear to arise mainly from the two sources of error referred to in 1; and these errors in the earlier work were evidently much greater than was then suspected. But, in addition, inserting a high resistance while using the galvanometer[1] probably changed the conditions within the generator; and it is possible that some differences are to be referred to the reduction of the distance between the walls bounding the gas-space from 0.8 mm. to 0.64 mm.

Poor insulation along the glass surfaces would account for the irregularities and discontinuity previously observed in the neighborhood of the critical potential and would cover up slight differences in conductivity that depended on the condition of the gas used. Surface leakage would also tend to mask the increase in M/Q_2 with the potential. And good evidence of the inadequate internal insulation of the generator employed in the earlier experiments is afforded by some attempts to use moist air and oxygen, which gave values for the capacity only five or ten per cent. less than those obtained when the gas space was filled with mercury.

It is also to be noticed that the greatest value of M/Q_2 that appears in Table II. is considerably less than the smallest value to be found in the corresponding table of the writer's earlier paper. While part of this difference is possibly due to the differences in the generator referred to above, it seems much more reasonable (5) to refer the large values previously obtained to failure of a portion of the electricity to be measured to pass properly through the galvanometer.

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[1] A. W. Gray, PHYSICAL REVIEW, Vol. XIX., p. 293.

TABLE IV.

V	*a*	*b*	*c*	*d*	*e*	*f*
3,070	1,240		1,190			
3,660	1,480		1,420		1,490	
4,250	1,720		1,640 1,610	1,720		1,720
4,850	1,960		1,820 1,710	1,940	1,960	1,950
5,500	2,200	2,190	1,840 1,750	2,160	2,230	2,210
6,160	2,420	2,240	1,830 1,740	2,360	2,490	2,460
6,430				2,380		
6,600				1,870		
6,710	2,480	2,010	1,750	1,660	2,700	2,630
7,380	1,790	1,700	1,840	1,540	2,900	2,840
7,960	1,720	1,650	1,910	1,520	3,030	2,960
8,590	1,740	1,700	1,950	1,530	3,120	3,050
9,220	1,780		2,060		3,140	
9,810	1,790	1,730	2,150	1,580	3,190	3,120
10,410	1,830				3,250	
11,020	1,840	1,800	2,240		3,320	3,250
11,680	1,910		2,270		3,420	
12,240	1,910	1,890	2,290	1,800	3,470	3,440

APPLICATION OF WARBURG'S THEORY OF THE SIEMENS OZONE GENERATOR.

22. Warburg[1] has given a theory of the Siemens ozone generator in which an important part is played by what he terms the "minimum potential," that is, the potential difference between the boundaries of the gas space when conduction through the gas ceases. Designating this by $\overline{V}_2$ and using the notation employed in the present paper Warburg expresses this in the form

$$\overline{V}_2 = \frac{V}{2}\left[1 - \left(1 - \frac{C_0}{C_2}\right)\frac{C_v}{C_0}\right];$$

this can also be expressed (see p. 350) in the form

$$\overline{V}_2 = \frac{V}{2}\left(1 - \frac{C_v}{C_\infty}\right).$$

This form is much better adapted to the determination of $\overline{V}_2$ since all the quantities can be easily measured electrically, whereas in calculating C_2 from the dimensions of the generator, very slight errors in measurement have an enormous influence.

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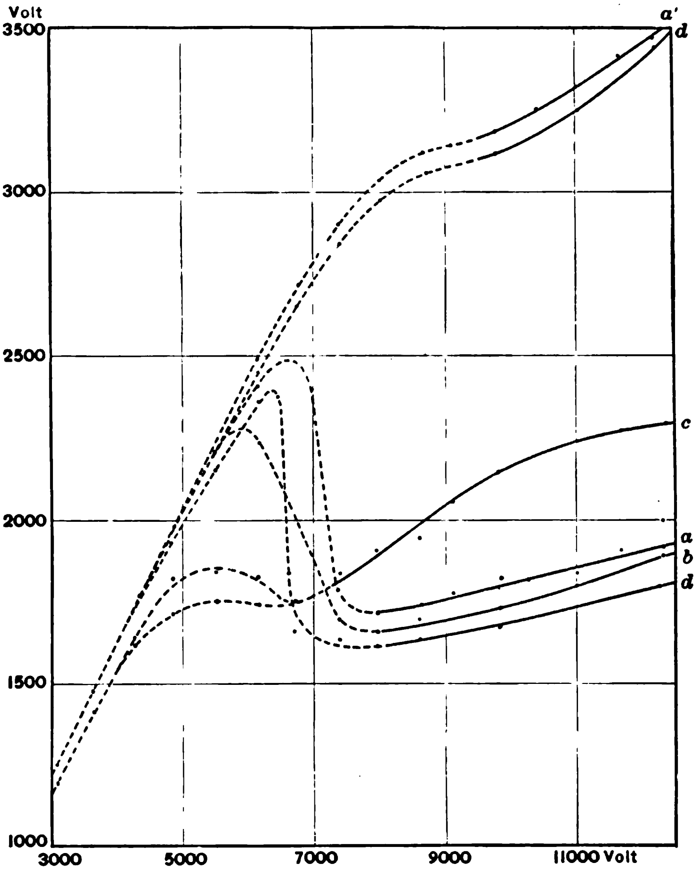
[1] E. Warburg, Verhandl. d. Deutschen Phys. Gesellsch. zu Berlin, **22**, p. 382, 1903

From the data in Table I. V_2 has been calculated and recorded under corresponding headings in Table IV. Fig. 6 represents the same facts graphically, the abscissæ being V and the ordinates V_2.

Fig. 6.



The maxima in the curves seem to mark the potential differences between the electrodes at which conduction through the gas begins to take place pretty generally.

Warburg also gives an expression for the potential difference, $(V_2)_0$, between the boundaries of the gas when conduction begins. It is difficult to decide what value of V should be used in this expression to calculate $(V_2)_0$ for a layer of gas 0.064 cm. thick enclosed between glass walls. According to data given by Paschen[1] the sparking potential difference between metallic spheres 1 cm. in radius and 0.064 cm. apart in ordinary air is about 3,360 volts. This value is obtained for $(V_2)_0$ when a point at about the middle of the steep descent of curve *d* in Fig. 6 is chosen as the basis of the calculation. This corresponds to the point of inflection of curve *d*, Fig. 3.

CONDITIONS FAVORABLE TO THE ECONOMICAL PRODUCTION OF OZONE.

23. From what has been brought out in this paper, it would seem of importance to those engaged in the preparation of ozone for technical purposes to secure the following conditions:

1. Good surface insulation of the generator, both internal and external, by means of some such device as described in 4.
2. Dry oxygen.
3. Charging and discharging as sudden as possible. For this reason the resistance and self-induction of the circuit should be reduced to a minimum, and it is also possible that operation by connection with a Leyden battery somewhat as described might yield better results than the ordinary method of employing an induction coil or an alternating current dynamo, as these machines could not give such a rapid change in the potential difference between the electrodes. Merely placing the generator in parallel with the spark-gap of an electrical machine, as was done by Shenstone and Priest,[2] is inefficient, because the charging takes place so slowly that no illumination,[3] and consequently no ozone,[4] is produced.

In conclusion, the writer wishes to acknowledge his obligations to Professor Warburg for his kindly interest and helpful criticisms, and also to the University of California for generous financial aid extended through the Whiting Fellowship in Physics.

BERLIN, PHYSICAL INSTITUTE OF THE UNIVERSITY,
April, 1904.

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[1] Paschen, Wied. Ann. 37, p. 79, 1889.

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THE

PHYSICAL REVIEW.

THE ELECTRICAL CONDUCTIVITY OF VERY DILUTE HYDROCHLORIC AND NITRIC ACID SOLUTIONS.

By H. M. Goodwin and R. Haskell.

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I. Purpose of the Investigation.

It is well known that there is great difficulty in obtaining an exact value for the equivalent conductivity of completely dissociated acids, owing to the large effect of the impurities of the water on the conductivity of even strong acids in very dilute solutions. These give rise to a maximum value of the conductivity at a dilution of from 500 to 2,000 liters, and to a marked decrease in its value at greater dilutions. Yet a knowledge of the theoretical limiting value is of great importance, since upon it is based the assumed value of the equivalent conductivity of the hydrogen ion and the calculated degrees of dissociation of acids in general.

Kohlrausch[1] in 1898 derived values for the conductivity of strong acids at extreme dilution by adding to the observed value at 0.001 normal a quantity equal to the product of the observed increase between 0.001 and 0.00001 normal in the case of neutral salts by the ratio of the increase between 0.1 and 0.001 normal in the case of the acids to the corresponding increase in the case of neutral salts, the conductivity of the water having been first subtracted in all cases. By subtracting from these values the conductivity of the negative ions, which had been derived from the conductivity and transference values of neutral diionic salts, he arrived at the value 318 for the equivalent conductivity of the hydrogen ion at 18° C. — a value, however, which he regards only as a rough estimate. Three years later[1] he deduced also the temperature change of this conductivity, expressing it by the equation :

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[1] Wied. Ann., **66**, 794 (1898).

$$A_t = 318[1 + 0.0154(t - 18) - 0.000033(t - 18)^2].$$

Quite recently Noyes and Sammet[2] have deduced the equivalent conductivity of the hydrogen ion by an independent method; namely, by combination of the transference number of moderately dilute hydrochloric acid with the equivalent conductivity of the chlorine ion derived from transference determinations and conductivity measurements of potassium chloride. The value thus obtained is 330 at 18° or 3.8 per cent. greater than that of Kohlrausch. The authors consider that this divergence cannot be due to experimental errors, but that it must arise either from an error in the extrapolated values of the conductivity of acids at extreme dilution, or from some real defect in the theoretical considerations from which the identity of the results of the two methods is predicted.

As it seemed possible to eliminate to a great extent the effect of the impurities in the water by appropriate measurements of the conductivity of dilute acids and a treatment of the data by certain methods to be described below, we undertook an investigation in this direction, both with nitric and with hydrochloric acid, and desire now to present the results which have been obtained. It may be stated in advance that these results are in substantial agreement with the extrapolated value of Kohlrausch.

II. PREPARATION OF THE SOLUTIONS.

In the preparation of the solutions, water was used which had a specific conductance of $0.8 - 1.5 \times 10^{-6}$ reciprocal ohms and which had been purified by redistillation of ordinary distilled water to which alkaline permanganate was added, the first and last portions of the distillate being rejected. The acid solutions were made by adding to definite portions of this water weighed portions of a standard acid of strength about 0.01 normal. The standard hydrochloric acid solution was obtained by absorbing in water hydrogen chloride gas made from twice-precipitated sodium chloride and pure sulphuric acid. Its strength was determined gravimetrically by precipitating with silver nitrate and weighing the silver chloride in Gooch crucibles. The several determinations made showed an average deviation from the mean value of less than 0.05 per cent. A standard nitric acid solution approximately 0.01 normal was made by diluting chemically pure acid through which air had been drawn to remove any lower oxides of nitrogen dissolved in it. This was standardized against barium hydroxide solution, whose strength had been determined by two independent methods. Five 0.01 normal solutions of potassium chloride were used to obtain the cell-constant; these were made from a sample of the salt which had been twice precipitated with pure hydrochloric acid and then crystallized from water, dried, and ignited to incipient fusion. A 0.01 normal sodium chloride solution made from salt prepared in the same way was used as a check.

In all cases, for the sake of greater convenience and accuracy, the content of the solution was determined by weighing, and not by volume measurement; but the values thus obtained of the number of milli-equivalents of solute in 1,000 grams of water were invariably (both in the calculation of the cell-constant and of the specific and equivalent conductivity of the acid solutions) considered to be numerically identical with the concentration in milli-equivalents per liter of solution. That no appreciable error (not even one as great as 0.05 per cent.) is introduced by this assumption will be readily seen, since none of the solutions were stronger than 0.01 normal.

III. THE CONDUCTIVITY APPARATUS AND METHOD OF MEASUREMENT.

The conductance of the solutions was measured by the Wheatstone slide-wire bridge arrangement in the form recommended by Kohlrausch, using an alternating current produced by an induction coil, and a telephone as an indicator of zero potential. The bridge consisted of a marble drum on which was wound five meters of manganine wire. Five or six settings were made for each solution, so as to eliminate as far as possible any personal error in determining the exact point corresponding to the minimum sound in the telephone. The average deviation of the separate values from the mean was less than 0.1 per cent. except in the case of the extreme dilutions, where it amounted in a few cases to 0.3 per cent. The probable error of the mean was always less than 0.1 per cent.except in the case of water. Both the slide-wire and the rheostat used were carefully calibrated, and the corrections were found in no case to exceed 0.1 per cent. The resistance vessel for holding the solution whose conductance was to be measured was a platinum cylinder 15 cm. high and 5 cm. in diameter. Into this was inserted, so as to rest on the bottom of the cylinder, an inverted glass bell 4.4 cm. in diameter attached to a glass tube sealed near the bottom, as shown in Fig. 1.

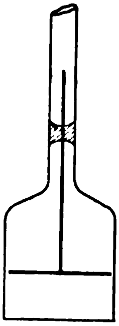
Fig. 1.

Into the bell an unplatinized but mechanically roughened platinum disk was sealed by means of a stiff platinum wire. The electrode was not platinized, so as to reduce as far as possible adsorption effects. It had an area of 15 sq. cm., and the distance between its lower surface and the bottom of the platinum cylinder was 1.9 cm. There was enough space along the side of this disk to allow the liquid to pass, and near the top of the bell a large hole was blown so that raising and lowering the glass plunger gave an efficient obtained, as there was less chance for the absorption of carbon dioxide. The plunger passed through a vulcanite cap which fitted the top of the cylinder, and thus kept the solution from much contact with the air. The cylinder was immersed almost to the top in a thermostat kept always within 0.02° of 18°. The temperature was measured by a thermometer that had been standarized and corrected to the scale of an air thermometer. The room was also kept nearly at the same temperature to prevent condensation at the top of the cell. Contacts were made by mercury.

The solutions were made by adding successively a number of portions of standard acid (approximately 0.01 normal) from a weighed Beckman pipette to a weighed quantity (about 180 grams) of water in the vessel. The weight of the added portion could easily be determined to less than 0.5 milligram, or 0.05 per cent. in the case of the smallest portion. In this way a series of solutions, usually varying in concentration from about 0.0001 normal up to 0.001 or 0.002 normal, were prepared, and their conductance measured.

Several experiments were made to determine whether the conductivity of dilute acid solutions changed with time upon standing in the covered conductivity vessel in the thermostat. The change upon standing over night amounted to only 0.1 per cent. of the conductivity of a 0.0002 normal solution.

In order to confirm the accuracy of the method, similar series of measurements were made with potassium chloride solutions at the same dilutions as in the case of the acids. The results, which are communicated in the Section V., agree within 0.1 per cent. with those obtained by Kohlrausch and Maltby,[1] in their very accurate determinations published in 1900.

IV. METHODS OF CORRECTING FOR IMPURITIES IN WATER AT GREAT DILUTION.

We have employed two methods for eliminating the influence of *already produced when a relatively small quantity of acid has been added to the water, after which further additions of acid remain unaffected by the impurities.*

In the first method of computation this principle is applied in a direct manner by simply subtracting the conductivity of a very dilute acid solution from that of a more concentrated one, and regarding the ratio of this difference to the difference in the respective concentrations as the equivalent conductivity of the acid at the higher concentration; that is, if κ_1 and κ_2 are the conductivities at the concentrations C_1 and C_2 respectively, C_1 being a very small concentration, it is assumed that

$$\frac{\kappa_2 - \kappa_1}{C_2 - C_1} = \Lambda_{C_2},$$

the equivalent conductivity at the concentration C_2.[1] Thus, the underlying idea is the preparation, by the addition of a little acid, of a sample of solvent the impurities in which shall have no appreciable influence on the conductivity of subsequently added portions of acid. Theoretically, it seems highly probable that this must be practicable; for any basic constituents of the water like ammonium or other alkaline hydroxides or carbonates would be completely neutralized, and any weakly acid constituent like carbonic acid which Kohlrausch[2] has shown to be undoubtedly the chief source of impurity, would have its dissociation reduced nearly to zero, as soon as the concentration of the strong acid added reached a value as great as, or a few times greater than, the concentration of the impurity. It is true that even though the impurities may have been thus rendered harmless, the increase in conductivity produced by the added portion of acid will not be strictly identical with the conductivity which it would have if it were added to pure water; for the acid originally present will be somewhat reduced in dissociation, and will itself reduce the dissociation of the added acid: but this effect is known to be very small at high dilutions, and the last-mentioned part of it is eliminated by regarding the calculated equivalent conductivity as corresponding to the final concentration rather than to the difference in concentration.[1]

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[1] In the calculation of Λ_{C_2} it is necessary to apply a slight correction to the observed conductance (κ_1'), since when the second portion of acid is added the volume of the solution is somewhat increased, whereby the observed conductance and concentration would be decreased in the same proportion. Thus if m_w, m_1 and m_2 are the weights of

In regard to the choice of the initial concentration, it may be pointed out that this must be taken so large that the maximum effect of the impurities in the water and of the acid on the conductivity of each other shall be attained, and that this condition is shown to be fulfilled when the equivalent conductivities calculated from two successive values of the initial concentration are in substantial agreement. The fact that such an agreement is actually found to exist even with relatively small values of the initial concentration is evidence of the correctness of the principle on which the computation is based.

The second method of computation is based on the assumption that the true values of the equivalent conductivity increase regularly to a maximum at the highest dilution, and that the decrease in the observed values from the maximum obtained at a moderate dilution is due to a *constant* error in the specific conductance arising from the effect of the impurities in the water. Therefore it is justifiable to add to the observed values of the specific conductance such a quantity as will cause the maximum in the so-corrected equivalent conductivity values to occur at the lowest concentration at which the error just referred to has attained its constant value. The method of procedure is best illustrated by a reference to the accompanying diagram (Fig. 2). In this the curve AB represents the "observed" values of the equivalent conductivity Λ, of the acid computed from the results of an actual series of experiments (first series, in nitric acid, p. 381) by subtracting from the observed specific conductivity of the solution $\varkappa_s$ the observed conductivity of the water $\varkappa_w$, and dividing the difference by the concentration C; that is,

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[1] For example, it is readily calculated from Kohlrausch's mean values (Wied. Ann., 66, 794, 1898) of the change of the conductivity at 18° of the ions of neutral salts with the dilution that the mean error in their equivalent conductivity would be 0.42 per cent. at 0.0002 normal and 0.25 per cent. at 0.0005 normal if computed by the method here described, taking 0.0001 as the initial concentration. In the case of the strong acids, whose conductivity changes much less rapidly with dilution, the error from this source would be about one half as great.

$$\Lambda_{obs} = \frac{\varkappa_s - \varkappa_w}{C}.$$

Now the error in the value Λ at A must be at least as great as the ordinate AC, C being equal to the maximum ordinate of AB. If we therefore increase its value by this amount and increase the observed value of Λ of every other point as B' by this amount multiplied by the inverse ratio of the concentrations corresponding to B' and A respectively, a new series of values corresponding to the curve CD is obtained. The value C is still seen to be in error by an amount as great as CF (F being equal again to the maximum ordinate of the new curve CD); the other values on this curve are in error by an amount less in proportion to the greater concentration to which they correspond: we therefore increase all the values in the same way as before, producing a third curve, and this process is repeated till the maximum point on the resulting curve lies on the ordinate through ACF and the curve has the form of GH. This method may also be described algebraically as follows. The true specific conductivity $\varkappa_s$, of the acid (that which it would exhibit in

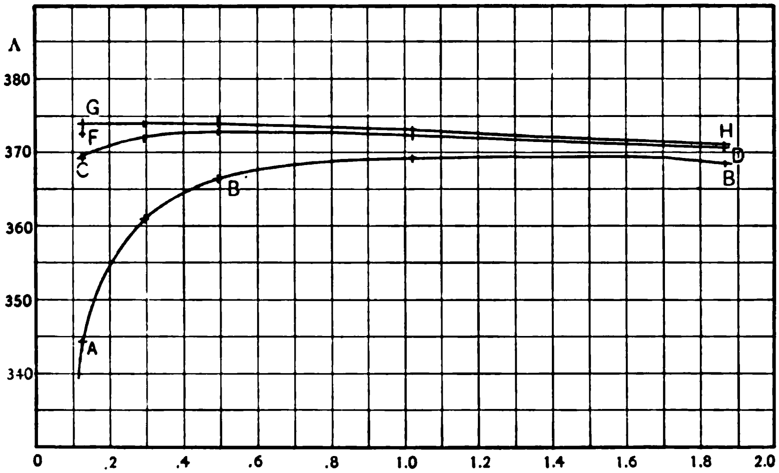

Fig. 2.

pure water at any concentration C), is equal to the observed conductivity of the solution κ_s, diminished by that of the water κ_w, and increased by a quantity δ, constant at all concentrations and equal to the reduction in conductivity produced by mutual influence of the acid and impurities; that is, $\kappa_c = \kappa_s - \kappa_w + \delta$, or expressed in terms of equivalent conductivities,

$$\Lambda_c = \Lambda_{Obs} + \frac{\delta}{C}$$

where Λ_c is the true equivalent conductivity of the acid and Λ_{Obs} is the "observed" equivalent conductivity as above defined. The value of δ is then determined from this equation by a series of approximations as described above until finally

$$\frac{\delta}{C} = 0$$

and Λ_0 coincides with Λ_{Obs}.

It should be understood, however, that this method gives only a minimum value for the equivalent conductivity at the highest dilution, for it is only in case the Mass-Action Law equation holds that the conductivity curve approaches asymptotically to a maximum with increasing dilution. If the empirical formula of Kohlrausch $\Lambda_0 - \Lambda_c = KC^{\frac{1}{3}}$ is applicable, the upward slope of the curve must continually increase with decreasing concentration. In fact, Kohlrausch found that the behavior of neutral salts at very low concentrations (0.001 – 0.0001 normal) was intermediate between that required by these two formulas. The values extrapolated with the help of the cube-root formula will therefore give in all probability a maximum limit for the conductivity at the greatest dilution, and the true value will lie between it and that derived by the method just described.

In applying these methods to our results, from 0.06 to 0.1 milli-equivalents per liter was taken as the standard initial concentration, although some measurements and computations have also been more dilute solution; and secondly, that the computed equivalent conductivities differ when the initial concentration chosen is varied from less than 0.03 to 0.06 millinormal, but that they no longer do so to an important extent when it is varied from 0.05 to 0.1 millinormal, showing that in the 0.03 millinormal solution the maximum influence of the water has not yet been produced.

It will be seen from the data given in the next section that the water used in different series of experiments had a conductivity varying from 0.8 to 1.6×10^{-6}, and consequently, the observed equivalent conductivity at the higher dilutions also exhibit considerable variations. In fact, the latter exist even where different samples of water with the same conductivity were used, showing that the character of the impurities varies and exerts a specific influence. Nevertheless, the corrected values of the equivalent conductivity are substantially identical — a fact which furnishes strong evidence of the correctness of the principle underlying the computations.

The above methods only serve to extend the series of reliable values of the equivalent conductivity of acids from about 0.002 normal down to 0.0002 or 0.0001 normal. To what extent the values at the latter concentration differ from those at infinite dilution can only be estimated. Probably the best basis for this estimate is the assumption of Kohlrausch which is most briefly expressed by the relation:

$$\frac{(\Lambda_\infty - \Lambda_{4000}) \text{ for acids}}{\Lambda_{4000} - \Lambda_{100}) \text{ for acids}} = \frac{(\Lambda_\infty - \Lambda_{4000}) \text{ for salts}}{\Lambda_{4000} - \Lambda_{100}) \text{ for salts}} = \frac{0.9^1}{3.0}$$

This method of extrapolation has been applied to our results. Since according to our measurements with hydrochloric acid described in Sections V. and VI. $\Lambda_{4000} - \Lambda_{100} = 377.6 - 369.3 = 8.3$, the so-determined value of $(\Lambda_\infty - \Lambda_{4000})$ for acids is only 2.5 units (or 0.7 per cent.).

V. The Conductivity Data.

The experimental data and the values calculated therefrom will now be presented

0.009074[1] normal; that of nitric acid was 0.010145[2] normal. The cell constant or capacity of the resistance vessel (that is, the ratio of the specific conductance of any solution to its actual conductance as measured in the vessel) was 0.11659.[3]

The following tables contain the results of the separate series of experiments. As headings are given, the weight in grams (m_w) of water taken, its actual conductance (L_w) multiplied by 10^6 when measured in the vessel, and its specific conductance (x_w) multiplied by 10^6. The first column gives the number of the portions of standard acid or potassium chloride solution which were added; the second gives the total weight of the added portions; the third, the concentration (C) of the resulting solution in milliequivalents per liter (see Section II.); the fourth, the conductance in reciprocal ohms measured in the resistance vessel and multiplied by 10^6; the fifth, the equivalent conductivity of the acid calculated by subtracting from the specific conductance of the solution (x_s) that of the water and dividing the difference by the concentration, *i. e.*, $(x_s - x_w)/C$. In the sixth and seventh columns are given the corrected values of the equivalent conductivity computed by the first method described in Section IV., taking for the initial concentration that corresponding to the first and second added portions, respectively. In the eighth column are given the corrected values of the equivalent conductivity calculated by the second method described in Section IV. (In the case of the potassium chloride series these are replaced by the equivalent conductivities obtained by Kohlrausch and Maltby.)

[1] This result was based upon four determinations, of which the data were as follows:

Grams HCl solution taken	346.27	328.60	347.70	356.38
Grams AgCl obtained	0.4506	0.4277	0.4521	0.4623
Equivalent per 1,000 grams sol.	0.009076	0.009078	0.009068	0.009047

The last on account of its great deviation was omitted in taking the mean.

[2] This solution was obtained by dilution (by weight), of a 0.05 normal solution which has been standardized against a 0.1 normal solution of recrystallized barium hydroxide,

HYDROCHLORIC ACID.

$m_w = 163.73$; $10^6 L_w = 11.13$; $10^6 \kappa_w = 1.30$.

Portion No.	Total Weight of Added Portions.	Concentration in Milliequivalents per Liter.	Observed Conductance × 10⁶.	Equivalent Conductivity.			
				Water Subtracted.	By First Method. I.	II.	By Second Method.
1	0.2248	0.0125	34.82	221.8			420.6
2	2.0305	0.1114	349.34	354.6	371.3		376.9
3	5.0528	0.2717	868.42	367.9	374.8	376.8	376.9
4	8.2065	0.4341	1,390.9	371.3	375.7	377.8	376.9
5	12.9987	0.6689	2,144.8	372.6	375.4	376.1	376.2
6	19.190	0.9574	3,061.0	371.4	375.4	375.7	374.0

$m_w = 170.96$; $10^6 L_w = 11.95$; $10^6 \kappa_w = 1.37$.

1	1.3360	0.07036	221.3	346.8			379.6
2	5.4563	0.2806	904.3	370.6	378.5		376.4
3	9.6226	0.4835	1,557.9	372.7	377.0		376.3
4	12.032	0.5966	1,923.8	373.6	377.0		376.4
5	17.928	0.8612	2,775.5	374.1	376.4		375.9
6	37.166	1.6204	5,214.8	374.3	375.4		375.3

$m_w = 193.83$; $10^6 L_w = 9.41$; $10^6 \kappa_w = 1.10$.

1	0.5642	0.02634	76.56	297.1			382.8
2	1.2787	0.05947	183.82	341.8	377.2		379.5
3	2.4332	0.1125	354.3	357.4	375.6	374.4	377.5
4	5.0564	0.2307	737.5	367.9	376.8	376.8	377.6
5	7.3384	0.3311	1,062.2	370.7	377.0	376.9	377.3
6	11.162	0.4942	1,588.8	372.6	376.7	376.7	377.0
7	15.184	0.6593	2,119.6	373.1	376.1	376.2	376.3
8	35.180	1.3942	4,489.5	374.6	376.0	375.9	376.2

$m_w = 184.65$; $10^6 L_w = 7.44$; $10^6 \kappa_w = 0.868$.

1	2.0218	0.09828	313.9	363.5			378.7
2	3.8508	0.1854	597.8	371.3	379.8		379.4
3	5.7913	0.2760	889.8	372.8	377.7	375.7	378.1
4	8.1604	0.3841	1,238.7	373.7	377.1	376.0	377.5
5	11.665	0.5392	1,741.3	374.9	377.3	376.8	377.6
6	18.636	0.8319	2,678.1	374.3	375.6	375.1	376.1

$m_w = 184.65$; $10^6 L_w = 7.29$; $10^6 \kappa_w = 0.854$.

1	0.9254	0.04526	141.28	345.1			371.6
2	2.5137	0.1219	392.35	368.3	381.9		378.3
3	5.4093	0.2583	835.20	373.7	379.8	378.4	378.4
4	7.8974	0.3722	1,201.8	374.2	378.1	376.9	377.4
5	11.571	0.5351	1,728.0	374.9	377.5	376.8	377.2
6	15.715	0.7117	2,298.5	375.3	377.2	376.7	376.9
7	19.915	0.8834	2,848.5	374.9	376.5	375.9	376.2

NITRIC ACID.

$m_w = 181.38$; $10^6 L_w = 13.35$; $10^6 \kappa_w = 1.59$.

Portion No.	Total Weight of Added Portions.	Concentration in Milliequivalents per Liter.	Observed Conductance $\times 10_6$.	Equivalent Conductivity.			
				Water Subtracted.	By First Method. I.	II.	By Second Method.
1	0.8480	0.0472	132.1	293.0			374.6
2	2.3534	0.1299	397.0	344.1	373.1		373.9
3	5.4994	0.2985	937.7	360.9	373.4	373.6	373.9
4	9.3059	0.4951	1,568.1	366.1	373.5	373.6	373.8
5	20.477	1.0291	3,273.2	369.3	372.6	372.6	373.0
6	40.940	1.8679	5,922.0	368.8	371.4	371.3	370.8

$m_w = 175.49$; $10^6 L_w = 13.05$; $10^6 \kappa_w = 1.52$.

1	1.0421	0.0599	183.7	332.0			374.4
2	3.2765	0.1860	588.9	361.0	374.7		374.7
3	6.5193	0.3634	1,159.7	367.9	374.7	374.8	374.7
4	9.0405	0.4972	1,586.0	368.8	373.8	373.5	373.9
5	14.931	0.7955	2,538.4	370.1	373.0	372.7	373.2
6	20.373	1.0553	3,368.2	370.6	372.8	372.6	373.0

$m_w = 169.23$; $10^6 L_w = 12.03$; $10^6 \kappa_w = 1.40$.

1	1.8262	0.1083	337.8	350.6			375.2
2	4.1137	0.2465	782.5	364.4	375.1		375.2
3	7.0453	0.4110	1,306.2	367.9	373.9		374.4
4	12.1903	0.6868	2,190.6	369.8	373.2		373.7
5	19.894	1.0720	3,421.2	370.8	372.9		373.4
6	40.299	1.9550	6,229.7	370.2	371.8		371.6

$m_w = 183.75$; $10^6 L_w = 12.64$; $10^6 \kappa_w = 1.48$.

1	1.6063	0.0879	277.1	350.7			372.1
2	3.7972	0.2054	656.0	365.2	373.9		374.1
3	5.8714	0.3141	1,005.6	369.5	375.3	374.8	374.2
4	8.8942	0.4684	1,500.0	370.2	374.6	374.1	374.1
5	14.166	0.7261	2,325.0	369.6	373.9	373.6	372.2
6	20.282	1.0085	3,224.0	371.2	373.1	372.8	372.8
7	40.360	1.8270	5,827.7	371.0	371.9	371.5	372.0

POTASSIUM CHLORIDE.

$m_w = 185.22$; $10^6 L_w = 0.862$; $10^6 \kappa_w = 1.006$.

Conc. Stand. Solution = 0.009990.							Kohlrausch and Maltby.
1	1.2147	0.0651	80.66	129.02			129.16

In connection with the above data, attention is called to the close agreement in the case of potassium chloride between our values of the equivalent conductivity, given in column five marked "water subtracted," with those of Kohlrausch and Maltby; also to the fact that the application of the first correction method to this neutral salt causes a change of less than 0.1 per cent. in the values of the equivalent conductivity. It will be seen, on the other hand, that, in the case of the acids, both methods of computation raise the equivalent conductivity on an average by about 2 per cent. in 0.25 millinormal, and by about 0.5 per cent. in millinormal solution. Although the values in column five vary quite widely in the different series owing to differences in the quality of the water used, yet when corrected by Method I. they are in substantial agreement in all cases where the concentration corresponding to the first added portion exceeds 0.05 millinormal. The agreement between the results obtained by the two methods has already been considered.

The mean of the "water-subtracted" values at the concentration millinormal (374.0 for hydrochloric acid and 371.4 for nitric acid) are nearly one per cent. lower than the corresponding values earlier obtained by Kohlrausch[1] (377 and 375 respectively). This fact led us to make a new series of measurements on the conductivity of hydrochloric acid at higher concentrations. These were made at 18° in a U-shaped apparatus with a new sample of acid prepared and standardized as before.[2] The dilutions of the stock solution (0.13747 normal) were made by weight and reduced to volume-concentrations at 18°, by means of the specific gravities of water and of the stock solution. On account of the wide range of concentrations measured it was found necessary to use two resistance vessels. The cell constants or capacities of these, determined by five 0.01 normal potassium chloride solutions, were 2.5417 and 8.5370. The average deviation of the separate values from the mean was less than 0.03 per cent. in each case.

The results are contained in the following table. Values corre-

[1] Kohlrausch and Holborn, Leitvermogen der Electrolyte, p. 160.

[2] The analytical data were as follows:

Grams HCl solution taken	112.34	106.58	91.31
Grams AgCl obtained	2.2121	2.0988	1.7994
Equivalents per 1,000 grams sol.	0.13734	0.13734	0.13744

sponding to the round concentrations are summarized in Section VII., page 385.

The values are seen to be only slightly (0.1–0.4 per cent.) lower

HYDROCHLORIC ACID.

$10^6\ L_w = 0.34$. $10^6 \kappa_w = 0.85$. *Cell Constant* = 2.542.

Concentration in Equivalents per Liter.	Actual Conductance × 10⁶.	Equivalent Conductivity.[1]	Equivalent Conductivity. (Interpolated from Kohlrausch.)
0.001917	270.12	375.0	376+
0.001937	285.87	375.1	376+
0.004744	697.25	373.1	373+
0.005525	809.79	372.5	372+
0.010124	1,470.8	369.2	370
0.01147	1,661.8	368.2	369+
0.02714	3,878.6	363.4	365
0.03137	4,468.9	362.1	364–
$10^6\ L_w = 0.10$. $10^5 \kappa_w = 0.85$. *Cell Constant* = 8.537.			
0.13747	5600.9	347.8	347+

than those of Kohlrausch. Noyes and Sammet[2] found in $\frac{1}{58}$ and $\frac{1}{252}$ normal solution values 0.5–0.6 per cent. less than Kohlrausch's. The above values were plotted against the cube root of the concentration, and between 0.005 and 0.137 normal were found to lie upon a straight line, the greatest deviation from it being less than 0.1 per cent.

VI. The Corrected Values of the Equivalent Conductivity.

In the following table are summarized the equivalent conductivity values of the dilute solutions computed by both methods from all the series of experiments. The values are interpolated for round concentrations intermediate between those involved in the experiments. In the case of the first method the values computed from the initial concentration corresponding to the first added portion were used, except in the cases where this was less than 0.05 millinormal.

[1] In these values no account is taken of the effect of the water, the correction for which is negligible, except in the first two cases in which the concentration is approximately only 0.002 normal. For this dilution the correction is + 0.2 unit, making the corrected values 375.2 and 375.3 respectively. This correction is taken as one half that computed from the data on page 378 for a 0.001 normal solution.

[2] Journ. Amer. Chem. Soc., 24, 964, 1902.

It will be seen that the results of the two methods of calculation agree almost completely in the case of both acids. The equivalent conductivity for 0.25 milli-normal hydrochloric acid is 377.6, and for nitric acid of the same strength is 374.4. The difference, 3.2, agrees well with the difference, 3.6, between the conductivities of the Cl and NO_3 ions as derived by Kohlrausch from measurements with neutral salts. It is also of interest to note that the conductivity value extrapolated for 0.25 milli-normal hydrochloric acid from those between 0.005 and 0.137 normal under the assumption of the validity of the cube-root formula ($\Lambda_0 - \Lambda_c = KC^{\frac{1}{3}}$) is 381 or only about 0.9 per cent. greater than our corrected valve.

HYDROCHLORIC ACID.

Conc. Milliequivalents.	First Method.						Second Method.					
	Separate Series.					Mean.	Separate Series.					Mean.
0.10							376.9	379.1	377.5	378.8	378.3	378.1
0.25	376.8	378.5	376.8	377.7	378.4	377.6	376.9	376.9	377.5	378.2	378.4	377.6
0.50	377.4	377.0	376.7	377.3	376.8	377.0	376.7	376.4	377.0	377.5	377.3	377.0
0.75	376.1	376.7	376.2	375.6	376.5	376.2	375.5	376.1	376.3	376.5	376.7	376.0
1.00	375.7	376.2	376.0		375.8	375.9	373.7	375.8	376.2	375.6	376.1	375.5
1.50		375.6	375.9		375.5	375.7		375.4	376.1		375.6	375.6

NITRIC ACID.

Conc. Milliequivalents.	First Method.						Second Method.					
	Separate Series.					Mean.	Separate Series.					Mean.
0.10							374.0	374.6	375.2	374.1		374.3
0.25	373.6	374.7	375.1	374.5		374.5	373.9	374.7	375.2	374.1		374.3
0.50	373.6	373.8	373.7	374.5		373.9	373.8	373.9	374.2	373.8		373.9
0.75	373.1	373.0	373.1	373.8		373.3	373.4	373.3	373.6	373.4		373.4
1.00	372.6	372.8	372.9	373.1		372.9	373.1	372.9	373.4	372.8		373.0
2.00	371.1		371.8	371.7		371.5	370.4		371.5	371.9		371.3

It remains only to apply the estimated correction of + 2.5 units, discussed at the end of Section IV., in order to obtain the most probable values of the equivalent conductivity at infinite dilution. The values so obtained are 380.1 for hydrochloric acid and 377.0 for nitric acid. The values recently derived by Kohlrausch[1] are 383.4 and 379.8 respectively, which differ from ours by less than one per cent. Unless the change with the concentration at very high dilutions follows a law radically different from that prevailing at moderate dilutions, it seems certain that these values are substantially correct. The much higher value 395.5 for hydrochloric acid derived by Noyes and Sammet from transference experiments apparently arises from some considerable error, the nature of which still remains to be explained.

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[1] Sitzungsber. d. konigl. preuss. Akad., 1031, 1901.

VII. SUMMARY.

In this article have been described measurements of the electrical conductivity at 18° of very dilute (0.0001 to 0.002 normal) hydrochloric and nitric acid solutions made by adding to a known weight of water successive portions of 0.01 normal acid. To eliminate the effect of the impurities of the water, two methods of computation of the equivalent conductivity have been used. In both of these, the assumption is involved that the total effect of the acid and impurities on the conductance of each other is produced when a relatively small quantity of acid has been added to the water. The experiments themselves furnish evidence of the correctness of this assumption. In the first method, the increase of the specific conductance over this initial value divided by increase of concentration is regarded as the equivalent conductivity at the higher concentration. In the second method, such a quantity, constant at every concentration, is added to the observed values of the specific conductance as will cause the maximum in the equivalent conductivity values calculated therefrom to occur at the lowest concentration at which the total influence of the acid and impurities on each other has been produced. In spite of considerable variations in the observed conductivities due to the use of different samples of water, the corrected values of the equivalent conductivity derived from different experiments and those computed by the two methods agree well with each other. New determinations were also made at higher concentrations with hydrochloric acid. The final values of the equivalent conductivity (Λ) at various round concentrations (C) in equivalents

Concentration in Mols.	Λ_{HCl}	Λ_{HNO_3}	Concentration in Mols.	Λ_{HCl}
0	380.1	377.0	0.005	372.6
0.00025	377.6	374.5	0.010	369.3
0.00050	377.0	373.9	0.020	365.5
0.00075	376.2	373.3	0.030	362.5
0.00100	375.9	372.9	0.050	358.4
0.00150	375.6		0.100	351.4
0.00200	375.3			

same ratio to that between 0.00025 and 0.01 normal as in the case of neutral salts; this ratio having been already derived by Kohlrausch. These values are less than one per cent. lower than those of Kohlrausch and are believed to be substantially correct. That derived by Noyes and Sammet from transference experiments for hydrochloric acid (395.5), on the other hand, is, for some reason as yet unknown, four per cent. higher.

In conclusion the authors desire to express their appreciation to the trustees of the William E. Hale Research Fund for the grant from that fund which enabled this research to be carried out. Also to Professor Noyes for many valuable suggestions received from him in the course of the work.

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THE DISTRIBUTION OF TEMPERATURE IN AN AIR LIQUEFIER OF THE SINGLE-CIRCUIT TYPE.[1]

By W. P. Bradley and C. F. Hale.

THE inquiry which forms the subject of the present paper was undertaken for the purpose of determining the nature of the thermal distribution in the air liquefier recently installed at Wesleyan. This liquefier is of the single-circuit type — that is, all of the air which enters it passes through a single throttling valve and escapes at atmospheric pressure. Its construction has been described in a previous article.

To make the inquiry exhaustive, it would have been necessary to take account of the temperature of both the inflowing and the outflowing air. This type of liquefier is constructed however of tubing so small in diameter that it is practically impossible to measure accurately the temperature of the compressed air, after it has entered the interchanger, without the use of special enlargements in the pipe at the different points whose temperature is desired. These of course could be provided only at the outset, during the winding of the coil. On the other hand, the temperature of the outflowing air, which has expanded to atmospheric pressure, may be determined very simply indeed by inserting thermometers of suitable construction between the discs of the coil.

Though the measurements actually made refer therefore in the first instance to the outflowing air only, they do afford also, as will appear, valuable indications as to the temperature of the inflowing air within the pipes.

Resistance thermometers were used, which were constructed of very pure insulated platinum wire, about 45 cm. in length and .0798 mm. in diameter, wound in the usual way upon narrow strips of gen. They were inserted into the interchanger at different distances from the throttling valve, as follows:

No. 1, between the valve and the lowest disc.

No. 2, between discs one and two, counting upward from the valve.

No. 3, between discs three and four.

No. 4, between discs seven and eight.

No. 5, between discs thirteen and fourteen.

No. 6, between discs twenty-four and twenty-five.

No. 7, between discs thirty-seven and thirty-eight.

No. 8, between discs fifty-seven and fifty-eight, the total number of discs in the liquefier being fifty-nine.

In order not to deflect the outflowing air current unduly, the thermometers were symmetrically distributed about the coil in an ascending spiral. To hold them in place, paraffin was melted upon the mica strips at a convenient spot in such a way that they fitted snugly between the discs when pushed into place. Heavy copper leads were soldered to the ends of the platinum wires, and were brought out at the top of the liquefier and connected with a Wheatstone bridge.

During the tests, the liquefier was supplied with air at 18° C., and at 1,500, 2,000, 2,500 and 3,000 lbs. gauge pressure.

The tests were made in the following manner. The motor and compressor being in normal operation, and the desired pressure having been reached, the valve of the liquefier was opened and so adjusted from time to time as to maintain the pressure practically constant. Great care was taken to make the needed changes of adjustment as minute as possible, in order to avoid unnecessary fluctuations in the volume of the outflowing current. As soon as thermal equilibrium was established in the coil, the resistance of each thermometer was measured. The valve was then adjusted successively to the other pressures, and the thermometer resistances similarly recorded at each. In this way the temperatures of eight

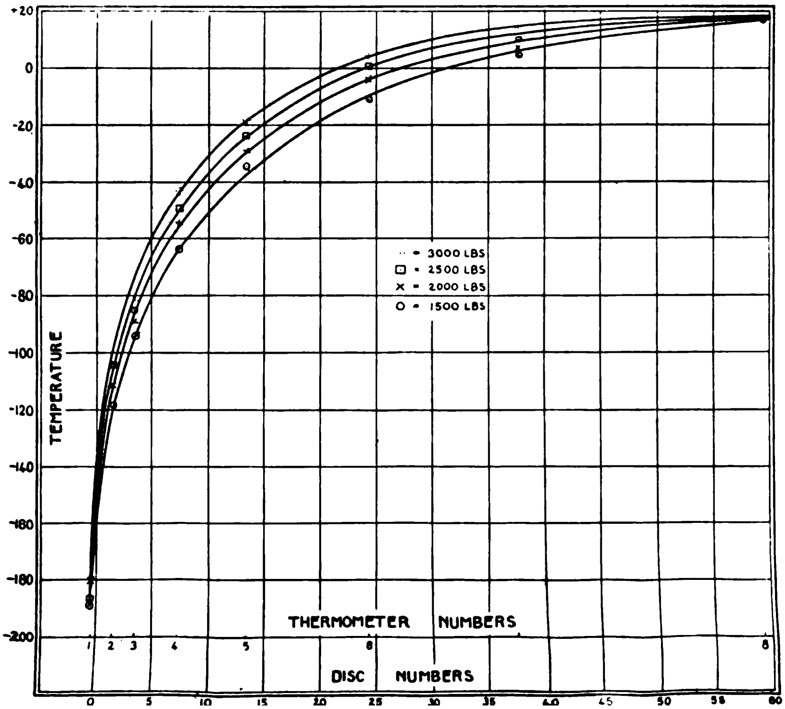
TABLE I.

Temperature Readings During the Production of Liquid Air.

Pres.	1	2	3	4	5	6	7	8
1,500	−191	−118	−94	−64	−34	−11	+ 5	+16
2,000	−180	−111	−90	−55	−28	− 4	+ 7	+17
2,500	−189	−104	−85	−48	−23	− 1	+10	+17.7
3,000	−180	−102	−82	−48	−18	+ 5	+14	+18

the coil and approximates more and more the temperature of the inflowing current. One notices at once how very rapidly this thermal interchange occurs at the bottom of the coil, and how comparatively slowly near the top.

Diagram I. in which temperatures are plotted against the heights in the coil which they represent, brings out this fact much more strikingly.



degrees in passing through a single disc only. The next rise of 80 degrees is secured by traversing the following twelve discs, while the remaining forty-six discs at the top of the coil appear just about sufficient to provide for the last 40 degrees.

To express the same thing in another way, the temperature between the third and fourth discs is almost exactly midway between the temperature of liquid air and that of the inflowing current. Three discs at the bottom effect the same interchange of heat, measured in degrees, as do the upper fifty-six. It was a matter of some surprise to us at the first, that the lowest thermometer, which is entirely below the coil, should not register uniformly the temperature of liquid air. In the light of the facts just stated, the records of this thermometer become intelligible. The thermometer was located indeed below the lowest disc of the coil, but it lay above the expansion valve, and also above six or eight inches of the pipe which connects the lowest disc with the valve. Furthermore, this thermometer lay in the region of eddy currents, as the others did not. These eddy currents consist of a mixture of air directly from the valve, and of air which has been in contact either with the metal parts just mentioned or with the walls of the metal casing which envelops the coil. The temperature of these eddy currents clearly depends on their direction. This in turn depends in part upon the adjustment of the valve, which is determined by the pressure. If the valve happens to deliver its blast directly towards the thermometer, a lower temperature is recorded. If the blast strikes first upon the metallic surfaces, its temperature will be higher on reaching the thermometer.

No such irregularity marks the records of the other thermometers. These are affected only by the steady direct current of air which sifts upward through the disc which happens to lie immediately below them.

From the diagram one may learn also that a decrease in the pressure of the air supplied to the liquefier tends to throw more of the work of thermal interchange upon the higher discs of the coil. Thus, at 3,000 lbs. the temperature of 0° is found near disc No. 22. At 1,500 lbs., the same temperature appears in the neighborhood of disc No. 33. Indeed, complete interchange was not effected at all at the three lower pressures. Except at 3,000 lbs. the temperature of the outflowing air was uniformly lower than that of the inflow, and the disparity was greatest at the lowest pressure.

This shifting of temperature appears to be due to two causes at least. In the first place, a given mass of air flows more rapidly through the pipes if the pressure be low, owing to its greater volume. The time of exposure is thus lessened and the thermal interchange becomes less complete in a pipe of given length. In the second place, the percentage of liquefaction is less at lower pressures, and in consequence, more of the air supplied to the liquefier returns unliquefied to the compressor. Thus the speed of the outflowing current also is increased. Of course the influence of this factor is comparatively slight.

While it was impracticable, as has been said, to make any direct determinations of the temperatures within the pipes, certain very interesting conclusions concerning them may be drawn indirectly from the data already given. No proof whatever is required for the statement that at any given point in the interchanger the temperature within the pipes is always higher than that outside them. Now it is shown in the table of temperatures already given, that at 3,000 lbs. the temperature of the outflowing current at a point immediately above the first disc is − 102°. As the air at this point has been warmed to its present temperature at the expense of the compressed air which is circulating within the piping of that disc, it is obvious that the temperature of the latter is still higher. But even the outside air is many degrees above the generally accepted value of the critical temperature of air. It would seem therefore to be impossible that liquefaction, in the ordinary sense at least, can occur in this disc, or in fact at any point above the valve.

This conclusion may be still further strengthened. If the very slow interchange of temperature which occurs at the top of the coil be due to the fact that the temperatures of the opposing currents at that level are not far apart, the remarkably rapid interchange at the bottom can only be accounted for by a very marked disparity in this regard. In other words, the air inside the tubing of disc No. 1 is not merely warmer, but is a great deal warmer than that outside at the same level.

It is interesting to note in this connection that if the fall of temperature at the valve is due to the Joule-Thompson effect, as held by Schrötter and Linde,[1] the temperature of liquefaction would be reached by the expanding air, if the temperature just above the valve were as high as — 80°.

Experimental data relative to the question of liquefaction above the expansion valve appear to be very meager. In fact, we have been able to find but one paper which deals at all with the subject from the experimental side — that of Schrötter, already cited. Schrötter presents a diagram showing the progressive change of temperature both immediately above and below the valve of the Linde liquefier, from the beginning of a run, until the establishment of equilibrium and the production of liquid air. At no stage in the operation, as the author takes pains to point out, does the temperature in the pipe, just above the valve, fall below — 120°.

From the theoretical side, this problem has been discussed by Pictet.[2] Travers[3] has expressed the opinion "that no liquefaction actually takes place within the jet, and that the gas, at the moment of its escape is not cooled many degrees below its critical point." The meaning of this is not clear. The facts given above appear to prove that *all* the liquefaction which occurs in a liquefier of the single-circuit type — the type which Travers is discussing — occurs "within the jet," that is, at the valve, at the moment of expansion; and that at the moment of escape, all of the air which passes the valve, whether liquefied or not, is reduced at once to the temperature of liquid air, so soon at least as the production of the liquid has commenced. How liquid air can be produced at all if the air which escapes from the expansion valve into atmospheric conditions "is not cooled many degrees below its critical point" is not easy to see. There is, to be sure, in every liquefier a slight pressure, in excess of the atmospheric, in the expansion chamber around the valve. This slight excess is due to the friction which the returning air meets on its way upward through the coil mm. of mercury. It would certainly prevent the expanding air from falling quite as low as the normal boiling point of liquid air under atmospheric pressure, but could not by any possibility keep it within a few degrees of the critical temperature.

Quite recently E. Erdmann assumes that in the Hampson liquefier there is no formation of liquid except at the valve at the moment of expansion.[1]

As stated above, the progressive change which occurs at the valve of a Linde liquefier at the beginning of a run has already been determined. It seemed desirable to repeat the determination with the present liquefier, so far at least as the returning current is concerned. To this end, temperature readings were taken about once a minute from each of the thermometers Nos. 1–3, from the moment of opening the expansion valve to the establishment of thermal equilibrium. As only one galvanometer and bridge were available however, the records of each thermometer had to be made in a separate run. This accounts for the fact that the initial temperatures are not exactly identical. Before each of the three runs was made, care was taken to see that the coil was of practically uniform temperature throughout. In order to lengthen somewhat the interval of time which elapses before liquid appears, the compressor was run at only 2,000 lbs. pressure and at the low speed of 100 revolutions per minute. Under these conditions no liquid was formed until about nineteen minutes after the opening of the valve. Under the conditions of ordinary running, the period is only eight minutes.

Diagram II. shows with what regularity, in general, the fall in temperature occurred. It is noteworthy however that thermometer No. 1, situated just above the valve, fell very rapidly indeed during the first minute or so. This is easily understood if one considers what would theoretically occur if there were no interchanger at all and the air were to escape through a valve possessing no thermal capacity, against a thermometer which likewise was without thermal capacity. In this hypothetical case, as soon as the valve was opened

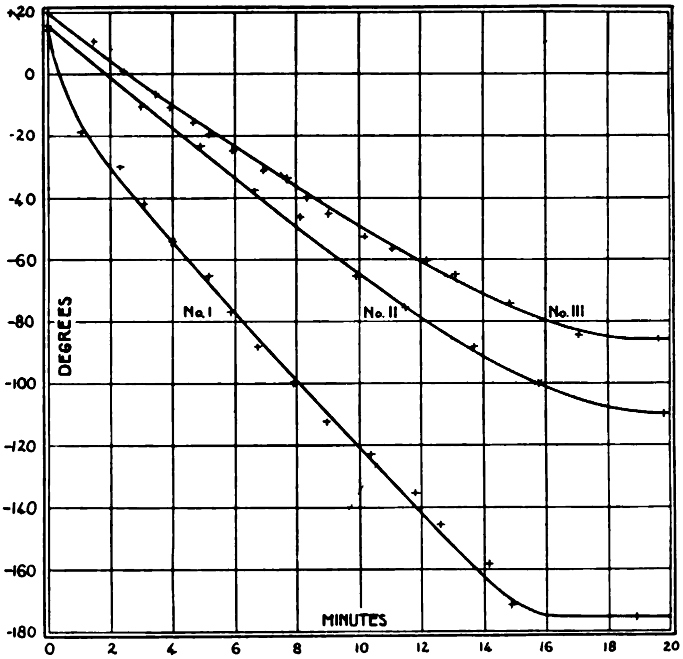


Fig. 2.

consist accordingly of two branches, one of them vertical and the other horizontal, meeting each other at a right angle. In actual practice, the thermal capacity of the valve especially and also of the thermometer, produces a lag, which causes the first branch of the curve to round smoothly into the second, instead of meeting it at an angle. This, of course, is on the assumption that the initial temperature is held constant. The introduction of the interchanger provides for a continuous fall in the initial temperature of the air just before expansion, and so, indirectly, for a corresponding fall in the temperature of the expanded air as it leaves the valve. In other words, the horizontal branch becomes a sloping one and the curve as a whole assumes exactly the form revealed in the diagram. The less the thermal capacity of the interchanger, the more steeply does the second branch fall and the sooner is the liquid formed.

Although, as has been said, no liquid flowed from the drainage pipe of the liquefier in this test for nineteen minutes, the curve of No. 1 shows that the temperature near the valve ceased its rapid fall after about sixteen minutes. There can be no doubt that liquid was forming at least two minutes before it appeared outside. In fact the actual flow of liquid from the spout was preceded for a minute or so by the appearance of a dense fog. This period of fog undoubtedly represents the time required to cool down to the temperature of liquid air those parts of the casing which lie below the direct influence of the blast, and which serve as a reservoir for the liquid whenever the latter is not drawn continuously, as fast as it is formed.

The curves of Nos. 2 and 3 are much simpler than that of No. 1, and illustrate the effect of the mass of the coils in preventing sudden changes of temperature.

The chief results of this inquiry are:

1. During the production of liquid air in a liquefier of the single-circuit type, the rise in the temperature of the returning current, as it passes upward from the valve through the interchanger, is extraordinarily rapid at the lower end of the interchanger and very slow at the top.

2. During the production of liquid, the compressed air, as it reaches the valve, is not only above its critical temperature, but apparently far above it. No liquefaction occurs inside the pipe.

Wesleyan University, Middletown, Conn.,
June 17, 1904.

STUDIES OF LUMINESCENCE.

BY E. L. NICHOLS AND ERNEST MERRITT.

IV. THE INFLUENCE OF LIGHT UPON THE ABSORPTION AND ELECTRICAL CONDUCTIVITY OF FLUORESCENT SOLUTIONS.[1]

IT is a characteristic of fluorescent substances that the light emitted during fluorescence is different in color from the light that the substance absorbs and to which its fluorescence is due. This difference may be exhibited graphically by plotting the transmission and fluorescence spectra of the substance in question upon the same diagram, as in Fig. 22. It will be noticed that the brightest fluorescence does not correspond with the region of greatest absorption, but is displaced toward the longer wave-lengths. In all the cases thus far studied by the writers[2] the two spectra overlap, and

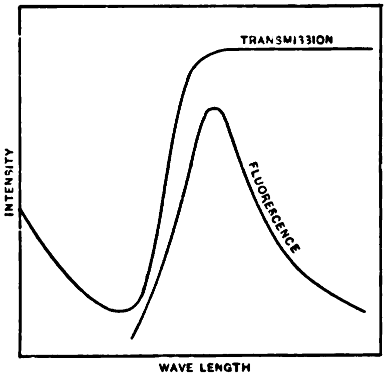


Fig. 22.

stance is thus able to emit light which, under ordinary conditions, it does not absorb; and no simple relation between absorption and emission can be recognized. In consequence of this fact fluorescent substances are usually excepted from the application of Kirchoft's Law.

This seeming lack of a simple relation between absorption and emission has made it difficult to form a picture of the molecular processes that accompany fluorescence. For, whatever may be the condition of the fluorescent substance, it does not seem possible that its molecules can be in the state of vibration corresponding to the fluorescence spectrum without also possessing the power of absorbing by resonance the same rays that they emit. In seeking an explanation of this difficulty it must be remembered that absorbing power is usually determined when the substance under investigation is *not* excited to fluorescence. The absorbing power *during* fluorescence may be quite different, for it seems not improbable that the same rays that excite fluorescence also produce a temporary change in absorbing power of a fluorescent substance. If such a change really occurs the difficulty mentioned above may prove to be only an apparent one, and the absorption, as determined during fluorescence, may be found to bear to the emission spectrum a relation which is at least similar to that called for by Kirchoff's Law.

A change in absorbing power during fluorescence has been observed in the case of uranium glass by Burke,[1] who found a considerable increase in the absorption when the glass was brightly fluorescent.

The writers have recently undertaken a study of the effect of fluorescence upon absorption in the case of other fluorescent substances, and have detected a well marked increase in absorbing power during fluorescence in each of the three substances thus far tested. The methods employed and the results obtained are described in the first part of the present paper. In attempting to interpret the results of this investigation we were led to suspect that fluorescence is accompanied by a temporary dissociation of the active molecule or ion, similar perhaps to the dissociation produced in gases by the action of Röntgen rays, and that the change in absorption during fluorescence is due to the presence in the solution of the products of this dissociation.[1] Since such dissociation would in all likelihood bring about an increase in electrical conductivity, it seemed desirable to study the possible influence of light upon the conducting power of fluorescent solutions. The negative results obtained by those who had previously attempted to detect a change in conductivity in solutions due to illumination showed that such an effect, if present at all, must be quite small. We have, however, obtained conclusive evidence of an increase in electrical conductivity during fluorescence in the case of each of the five substances thus far tested. The second and third parts of the paper are devoted to a description of the experiments on this subject and a statement of the considerations which led us to undertake them.

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[1] John Burke, Philosophical Transactions, Vol. 191A, p. 87, 1898.

I. The Influence of Fluorescence Upon Absorbing Power.

In the experiments of Burke, to which reference has already been made, the light used was that due to the fluorescence of a piece of uranium glass, and the absorption of the rays from this source by a second piece of uranium glass was determined photometrically. The absorption was found to be increased nearly 50 per cent. when the second piece of uranium glass was excited to bright fluorescence. While the method employed by Burke is a sensitive one, and well adapted to demonstrating the existence of the effect sought, it does not lend itself readily to a detailed study of the subject. It appeared to us desirable to determine not only the increase in total absorption, but also the influence of fluorescence on the form of the absorption spectrum. With this object in view we have determined the change in absorbing power for a number of different wave-lengths throughout the region in which the effect was observable, using for this purpose a Lummer-Brodhun spectrophotometer. Three fluorescent solutions have thus far been tested, namely, fluorescein in water, eosin in alcohol, and rezazurin in alcohol. Each of these solutions showed an increase in absorbing power during fluorescence, the effect being most marked in the case of fluorescein. For this reason the study of the subject has been carried further in the case of the latter substance than with either of the other two.

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[1] The dissociation here considered can hardly be of the usual electrolytic type, for it seems to be well established that with many substances fluorescence is an ionic property, and can be excited only after electrolytic dissociation has already occurred.

The arrangement of apparatus is shown in Fig. 23. The fluorescent solution, contained in a rectangular glass cell, *C*, was placed in front of the slit of the collimator. The solution was excited to fluorescence by rays from the source *E*, which usually consisted of a bank of four acetylene flames. When it was desired to measure the absorption of the solution without fluorescence a screen[*] was interposed between *E* and *C*. A block of magnesium carbonate, *M*, reflected a portion of the light from *E* through the cell containing the fluorescent solution into the collimator slit, *S*. By altering the distance of *M* from *S* the transmitted light could be adjusted to any desired intensity. In some of our earlier experiments the block of magnesium carbonate was removed and direct sunlight was used. The comparison source was an acetylene flame, *A*, whose rays were reflected from a second block of magnesium carbonate, *M'*. The spectrophotometer was used in the ordinary way, *i. e.*, without a lens in the eye piece.

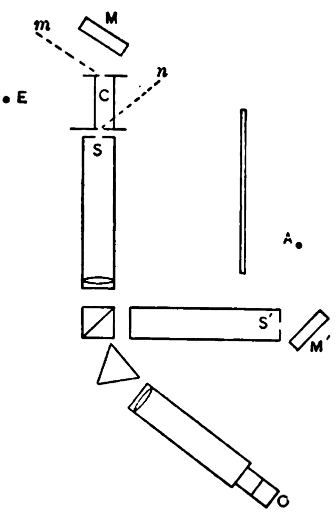

Fig. 23.

To determine the increase in absorption due to fluorescence the following three readings were required for each wave-length studied:

1. *Transmission* (*T*).—An opaque screen was placed between *E* and *C* to prevent fluorescence. The slit *S'* was then opened or closed until the two parts of the field, as seen from *O*, appeared equally illuminated. The width of the slit, as read from the drum of its micrometer screw, was then proportional to the intensity of the transmitted light. We shall designate this reading by *T*.

2. *Fluorescence* (*F*).—The screen was removed from between *E* and *C* and placed between *C* and *M*, so that the only light reaching the slit was that due to the fluorescence excited by the rays from *E*. The setting of *S'* for equal illumination was then a measure of

the intensity of fluorescence for the particular spectral region studied.

3. *Transmission and Fluorescence Combined* (C).—Both screens were removed. In this case the light entering the slit consisted partly of rays from M which had been transmitted by the solution and partly of rays originating in the solution itself by fluorescence.

The width of the slit S remained the same during all of the above readings.

If the absorbing power of the solution is not altered by fluorescence it is clear that the sum of the first two readings should be equal to the third except for accidental errors of observaton. But if fluorescence brings about an increase in absorbing power this equality can no longer hold; for, while the amount of light reaching the slit due to fluorescence will be the same in the third measurement as in the first, the light from M, which must pass through the cell, will be less intense than that observed without fluorescence. In this case therefore $F + T$ is less than C. In fact C may be looked upon as the sum of two parts F and T', where T' is the amount of light from M that is transmitted by the solution when the latter is excited to fluorescence. Since $F + T' = C$ we have

$$F + T - C = T - T'.$$

The quantity $F + T - C$ is therefore a measure of the increase in absorption due to fluorescence.

A single determination of the increase in absorption thus requires three photometric settings, each of which is subject to the usual accidental errors of observation; and the quantity sought, which is in all cases small, is obtained as the difference of two quantities that are relatively large. The results will therefore be largely affected by errors, whether accidental or systematic, in the individual readings. For this reason it was necessary to consider the possible sources of error with especial care. The recognized sources of error and the means taken to avoid them are discussed below.

Errors of Observation.—In order to reduce the effect of accidental errors each set of readings was repeated several times. In some cases as many as ten settings were made for each of the three tiveness. In making a series of determinations extending throughout the spectrum we found it inadvisable to repeat the settings more than four times. The settings were usually made by one observer, while the indications of the micrometer screw were read and recorded by the other.

Fluctuations in the Source of Light.—In our early experiments intense fluorescence was obtained by the use of the arc as an exciting source. Unfortunately the irregular fluctuations were found to neutralize entirely the advantage of great intensity. Sunlight was equally unsatisfactory. In most of our work a bank of acetylene burners was used to excite fluorescence, and with this source no further trouble due to variations in intensity was noticed.

Stray Light. — The effect of this source of error will depend upon the origin of the stray light and the manner in which it enters the instrument. Light reflected into the slit S by dust particles in the cell will cause no error in the final result; for the effect of light due to this cause will be to increase C and $T + F$ by equal amounts. Stray light from E which reaches the slit by any other route will be equally harmless, provided it is suppressed by the screen between E and C (Fig. 23) during the measurement of transmission. But light reaching the interior of the instrument in such a way as to remain nearly constant during the measurement of T, F, and C may lead to serious error. In this case each of these three readings will be too large, and the quantity $T + F - C$ will be in error by the amount of the stray light. To prevent any stray light from entering the instrument through the slit S opaque screens were arranged as shown, in part, in Fig. 23. The purpose of the large screen was to cut off all light from the comparison source. The cell C was completely covered with black paper with the exception of the face toward E and the two narrow openings, m and n, for the transmitted light. To prevent light entering the telescope after diffuse reflection from the prisms the whole instrument was covered with several layers of dark cloth, while screens were so placed as to prevent the direct rays from E or A (Fig. 23) from reaching any part of the spectrophotometer. The experiments were performed were taken it was found that when screens were placed between E and C and between M and C no trace of light could be detected in the spectrophotometer field. We feel confident therefore that the errors due to stray light are very small.

Zero Point. — The zero point of the micrometer screw used in measuring the width of the slit S' was determined by placing an acetylene flame immediately in front of the slit, so as to illuminate it brightly, and then closing the slit until no light could be detected. It is clear that an error in this zero point would make each of the readings T, F, and C incorrect by the same amount, so that the result $T + F - C$ would be subject to the same error as that made in determining the zero point. The setting for zero point was repeated many times, and the agreement of the different readings was such as to convince us that the error could not exceed half a division.

Other less important sources of error, such as irregularities in the screw, "lost motion," etc., require no special discussion.

Dependence of Fluorescence Absorption upon the Intensity of the Incident Light.

In attempting to obtain the most favorable experimental conditions for studying fluorescence absorption (*i. e.*, the increased absorption due to fluorescence), we were at first led astray by assuming that absorption in a fluorescent solution takes place in accordance with the same laws that hold in ordinary optically perfect media. We expected that the effect of fluorescence would be to increase the *coefficient of absorption* without altering the usual exponential law. If this were true the percentage absorption would be constant for a given solution and a fixed intensity of fluorescence, while the actual amount of light absorbed would increase with the intensity of the incident light. If aI is the absorption under ordinary conditions,

$I(a' - a)$, and, *if* a' *is independent of* I, it is clear that the best experimental conditions are attained when both the fluorescence and the incidence light, I, are made as great as possible.

Our early experiments, in which the conditions were chosen in accordance with the considerations just stated, gave results that were often discordant. For example, a preliminary set of readings, which happened to be made with I rather small, had indicated a well-marked increase in absorption; *i. e.*, the difference between C and $T + F$ was a large fraction of T. With the expectation of making the difference greater, and so increasing the accuracy of the determinations, the measurements were repeated with the intensity of the incident light largely increased. To our surprise the difference between $T + F$ and C was no larger than before. In fact, the errors of observation sometimes made it appear questionable whether there was any change at all due to fluorescence. We were at first inclined to ascribe the unsatisfactory results to irregular fluctuations in the intense sources of light that were used in our early experiments. But even after we had been forced to abandon the sun and the electric arc as sources, and had adopted acetylene flames throughout, no gain resulted from increasing the intensity of the incident light.

A series of measurements undertaken to determine the dependence of the effect studied upon the intensity of the incident light has since convinced us that the absorption of light in a fluorescent solution does not occur in accordance with the usual laws of absorption. The results of this series of measurements are contained in Table I. It will be observed that the quantity $T + F - C$ is nearly constant for all values of T; the variations are no greater than the possible errors of observation. Since T is proportional to I, we must conclude that the percentage absorption is not independent of I, but increases as I is made smaller. The total amount of energy absorbed, however, is constant throughout the range studied. It appears that when the solution is excited to fluorescence it acquires the power of absorbing a certain definite amount of energy, in addition to that absorbed under ordinary conditions, from a beam of light passing through it. The amount absorbed doubtless depends upon the intensity of fluorescence, the wave-length of the light, and other factors; but our observations all point to the conclusion that it does *not* depend upon the intensity of the transmitted beam.

TABLE I.

Dependence of Fluorescence Absorption Upon the Intensity of the Incident Light. The measurements were made at wave-length 0.535 μ.

Transmission. T	**Fluorescence.** F	**Transmission and Fluorescence Combined.** C	**Fluorescence Absorption.** $T+F-C$	**Percentage Fluorescence Absorption.** $\frac{T+F-C}{T}$
18.4	34.6	46.3	6.7	36.4
34.9	34.6	64.2	5.3	15.2
61.0	34.9	91.2	4.7	7.7
12.9	34.5	42.0	5.4	42.0
24.7	34.6	52.8	6.5	26.3
12.7	33.8	41.5	5.0	39.4

It was unfortunately not practicable to investigate the effect for values of T still smaller than the smallest in Table I., for with such feeble illumination of the spectrophotometer field accurate settings became impossible. It is clear, however, that the quantity $T + F - C$ cannot remain constant for very small values of T; for this would mean that the energy absorbed on account of fluorescence is greater than the total energy of the incident light. It appears to us probable that something analogous to saturation occurs in the case of fluorescence absorption. As the intensity of the incident light is gradually increased from zero, the absorption due to fluorescence probably increases also until a certain maximum is reached, after which a further increase in the intensity of the incident light produces no change in the fluorescence absorption.

Relation between Fluorescence Absorption and Intensity of Fluorescence.

Although there is every reason to expect that the absorption due to fluorescence will be greater when the fluorescence is more intense, the lack of a theory of fluorescence makes it impossible to predict the quantitative relation between the two phenomena. With the object of determining this relation experimentally we have measured the fluorescence absorption of a solution of fluorescein at 0.535 μ for several different intensities of fluorescence. The fluorescence was varied by altering the distance between the cell *C* (Fig. 23) and the exciting source *E*. By altering the position of the block of magnesium *M* (Fig. 23) the intensity of the transmitted light was adjusted in each case to approximately the same value.

The results are shown graphically in Fig. 24. In spite of the fact that each point plotted represents the average of several determinations, the curve obtained is far from being a smooth one. The results are nevertheless sufficient to show that fluorescence absorption is not directly proportional to the intensity of fluorescence. Here too we have to deal with something analogous to saturation. Although further experiments will be necessary to establish the exact relation between fluorescence and fluorescence absorption, numerous observations incidental to other phases of the present investigation serve as confirmation of the general form of the curve shown in Fig. 24.

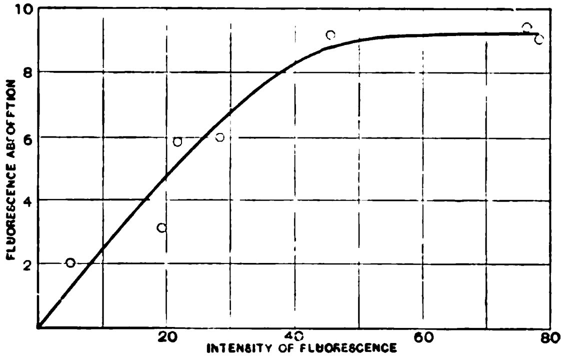

Fig. 24.

Fluorescence Absorption as a Function of Wave-Length.

In the experiments thus far described the measurements were always made in some one region of the spectrum. The wave-length of the light used, having been chosen by trial so that the fluorescence absorption was well marked, was kept constant during the whole series of determinations. To determine the florescence absorption for different wave-lengths the telescope of the spectrophotometer was shifted so as to bring different regions of the spectrum into the field, and several observations of *T*, *F*, and *C* were made for each position. During these determinations the intensity of the light exciting fluorescence and the intensity of the light used for the measurement of transmission were kept as nearly as possible constant.

TABLE II.

Dilute Solution of Fluorescein. Observations to Determine Typical Fluorescence, f, and Fluorescence Absorption, A_F. Thickness of Absorbing Layer 4.9 cm.

Wave-Length.	T	F	C	A_F	A_F Corrected.	f Computed.
.512μ	18.1	25.9	34.3	9.7	9.5	31.6
.518	19.8	27.4	40.9	6.3	8.5	32.0
.525	22.3	26.2	41.0	7.5	6.7	29
.539	27.2	20.7	43.3	4.6	4.7	20.7
.555	27.2	15.0	39.0	3.2	3.2	15
.506	12.9	20.2	24.4	8.7	7.7	28.6
.500	8.6	12.2	17.3	3.5	3.5	20.5
.494	6.5	7.4	13.4	0.5	0.5	13.8

The results of a series of observations of this kind in the case of fluorescein are given in Table II. The solution chosen for these experiments was quite dilute, preliminary experiments having shown

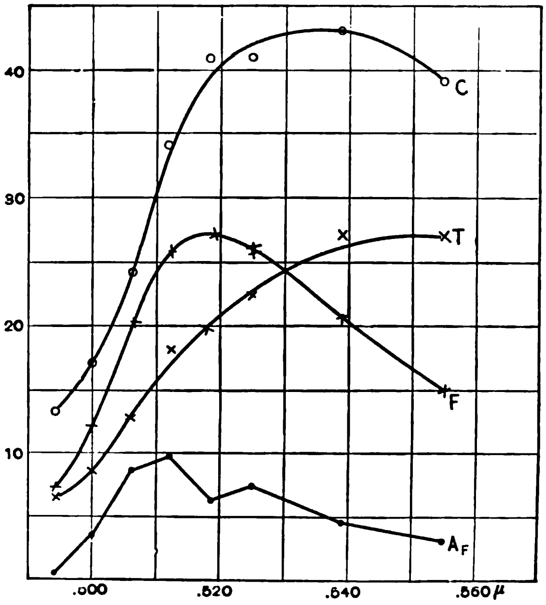

Fig. 25.

that it was difficult to obtain reliable results with more concentrated solutions, especially in the neighborhood of the normal absorption band. The data given in Table III. for *T*, *F*, *C*, and A_F are shown graphically in Fig. 25, the vertical scale being the same for all four curves.

In the case of observations made under constant conditions we should expect the successive readings for *F*, *T*, and *C* to lie upon smooth curves; and it is natural to ascribe any irregularities that appear to accidental errors. Acting upon the assumption that the conditions were constant during the observations plotted in Fig. 25, we have drawn smooth curves for *T* and *C*, as shown in the figure. By using in the computation of A_F the values of *T* and *C* that are given by these curves we obtain a curve for fluorescence absorption which is largely free from the irregularities due to accidental errors. The curve obtained in this way is plotted (to a different scale) in Fig. 26. Such a procedure would not be justified

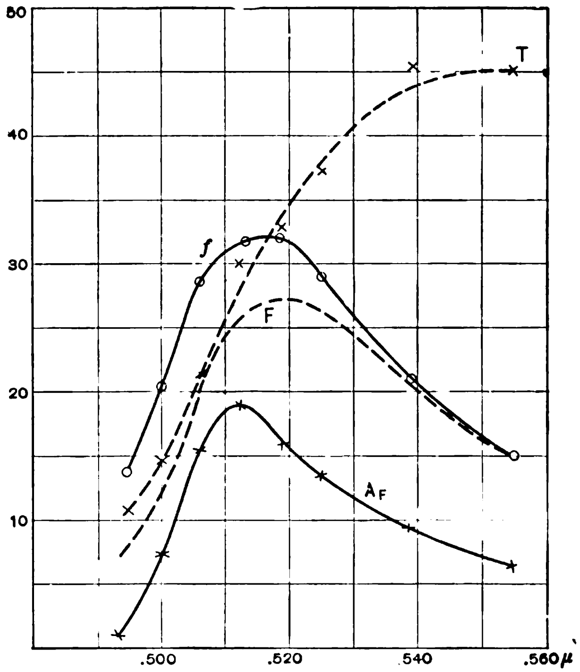

Fig. 26.

if the irregularities in the curves were due to fluctuations in the source of light or to any similar cause. But the smooth curve obtained for *F*, without adjustment, affords strong evidence that the experimental conditions were really steady, and that the irregularities in the other two curves are due to errors of observation. It has been our uniform experience that accidental errors are more common in the settings for *T* and *C* than in those for fluorescence.

In spite of the irregularities in the curve for fluorescence absorption in Fig. 25 a general resemblance will be noticed between this curve and that for fluorescence (*F*). In the case of the smooth curve for A_F the resemblance is still more marked. This similarity in form has also been noticed in the curves plotted from numerous other sets of observations, both with fluorescein and with eosin, although it was often made less striking by our inability to keep all the conditions constant during the determination of a complete curve.

It seems probable that this resemblance between the curves for *F* and A_F corresponds to some real and intimate connection between fluorescence and fluorescence absorption; apparently we have to deal not with two separate effects produced in the solution by the action of the exciting light, but rather with two different manifestations of the same molecular process. If this is true we should expect the relation between the two phenomena to be more clearly brought out by a comparison between the curve for fluorescence absorption and the typical fluorescence spectrum for the solution. The latter curve depends only upon the nature of the solution and the intensity of the exciting light, while the form of the fluorescence spectrum directly observed is modified by absorption and therefore depends upon the thickness of the solution and the method of observation.[2] The relation between the typical fluorescence, *f*, and the observed fluorescence, *F*, may be derived as follows:

Let the face of the cell containing the fluorescent solution be uniformly illuminated by the exciting light, so that all portions of amount of light of a given wave-length that is emitted in this direction per centimeter. The curve that shows f as a function of the wave-length is what has been called the typical fluorescence spectrum.

The light proceeding in the direction toward S will be of increasing intensity as we pass from m to n. If its intensity at a distance x from m is I, we have

$$dI = fdx - aIdx$$

where a is the coefficient of absorption. Upon integrating this equation we have

$$I = \frac{f}{a} + K\varepsilon^{-ax}.$$

The constant of integration K is determined by the fact that when $x = 0$, $I = 0$, so that

$$K = -\frac{f}{a}.$$

Therefore

$$I = \frac{f}{a}(1 - \varepsilon^{-ax}).$$

The light entering the slit, which determines the measured fluorescence F; is the value of I for $x = l$, where l is the thickness of the solution.

$$F = \frac{f}{a}(1 - \varepsilon^{-al}).$$

$$f = \frac{aF}{1 - \varepsilon^{-al}}.$$

Since a is readily computed from the measurements of transmission we are thus able to compute the typical fluorescence from the observed fluorescence F. In the case of the experiments corresponding to Fig. 25 the percentage transmission is most conveniently computed by dividing the value of T for the wave-length in question by the value of T for $\lambda = 0.555\ \mu$. The latter point on

$$\theta = \varepsilon^{-al},$$

$$\log \theta = - al,$$

$$f = - \frac{F \log \theta}{l(1 - \theta)}.$$

The typical fluorescence spectrum computed in this way is shown in Fig. 26. A comparison of this curve (f) with the smoothed curve for A_F adds support to the view that the phenomena of fluorescence and fluorescence absorption are intimately connected. But the relation between the two does not appear to be a simple one. Unless the observations are effected by more serious errors than we have any reason to suspect, the results show that fluorescence absorption does not bear a constant ratio to the intensity of the fluorescence with which it is associated. And yet this simplest possible relation between the two phenomena is the one which it seems most natural to expect. We are of the opinion that the question of the exact relation between the two curves is one which requires further experimental investigation before it can be looked upon as definitely settled.

A series of observations like those just described was made with a still more dilute solution of fluorescein with results that agree, in the main, with those stated above. In these experiments sunlight was used to excite fluorescence. The variations in the intensity of this source made it impossible to apply the method of smoothing described above; and without some means of reducing the effect of accidental errors the results were too irregular to lead to any reliable conclusions. These observations, therefore, serve merely as a check upon the general form of the curves shown in Fig. 25. The same statement may be made with reference to numerous other observations in which we were unable to maintain constant conditions throughout the complete series of readings.

In Fig. 27 we have plotted the results of a series of measurements made with a more concentrated solution of fluorescein. It will be noticed that the edge of the absorption band is much steeper than in Fig. 25, and the band extends further toward the red. Although the curve for fluorescence absorption (A_F) is similar in form to that shown in Fig. 25 its maximum is displaced toward the longer wave-lengths. It is clear that the normal absorption of the solution exerts a modifying influence upon the increase in absorption which accompanies fluorescence.

In addition to the experiments with fluorescein a few measurements were made also with alcoholic solutions of eosin and resazurin. In the case of the latter substance the measurements were made for only a few wave-lengths, and served merely to show that the effect was present. With eosin a nearly complete curve was obtained, although not under entirely satisfactory conditions. The curve was the same general character as those obtained with fluorescein, and occupied the same position with relation to the absorption band.

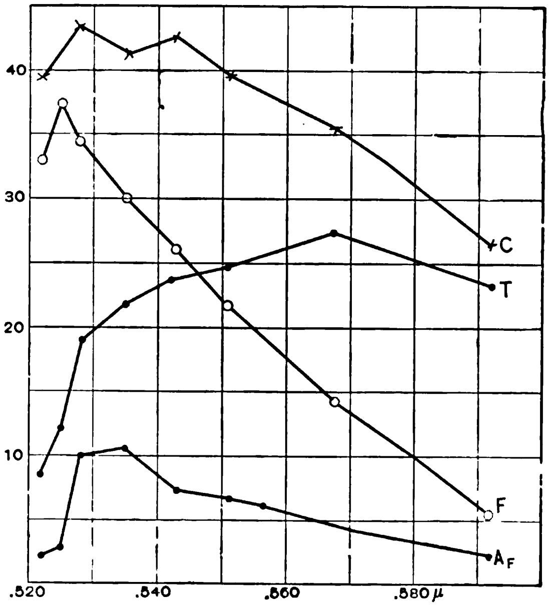

Fig. 27.

II. Theoretical.

Throughout the progress of the experiments that have been described in the present paper and in those which precede it we have naturally tried to form some picture of the underlying mechanism of the phenomena studied. It is perhaps needless to say that serious difficulties have been encountered; but although we have not succeeded in forming a wholly satisfactory picture of the molecular processes involved in fluorescence, the considerations stated below have nevertheless led us to the adoption of a working hypothesis regarding the general character of these processes which has been of considerable assistance.

In attempting to form a theory of fluorescence it is natural to turn to the other, and in some respects simpler cases of luminescence for assistance; for it is difficult to believe that the many resemblances noticeable between luminescence phenomena of different types are merely superficial. In the case of several classes of luminescence phenomena the first steps in the development of a theory have fortunately already been taken. Thermo-luminescence, for example, has been explained as the result of some chemical change in the luminescent substance, during the progress of which the molecules are thrown into such violent vibrations as to bring about the emission of light. Luminescence ceases in such cases when the change has been completed; and some outside stimulus is required, such as that furnished by cathode rays, to restore the substance to the sensitive state. Not only in this explanation of thermo-luminescence a plausible one but in several instances conclusive evidence has been found that the assumed change really occurs.

A somewhat similar suggestion has been made by E. Wiedemann in the case of phosphorescence. According to this view some change is produced in the phosphorescent substance by the action of the exciting light, and the gradual restoration of the original condition after the excitation has ceased is accompanied by luminescence. Although it has not yet been possible to determine the exact character of the assumed change, or even to demonstrate by direct tests that such a change occurs, the view suggested by Wiedemann has undoubtedly been an important aid in the more recent study of phosphorescence.

If, as seems not unlikely, the difference between phosphorescence and fluorescence is chiefly one of the duration of luminescence, the hypothesis just stated may be applied to the latter class of phenomena without essential modification, and we may assume that fluorescence is also due to some temporary change in the fluorescent substance. We must assume further that the return to the original condition is almost instantaneous; so that luminescence persists for only an inappreciable time after the removal of the exciting cause. According to this view fluorescence involves two processes (1) the alteration in the fluorescent substance brought about by the exciting light; (2) the spontaneous return of the modified substance to its original state. These two processes occur simultaneously; and one or both must be accompanied by the emission of light.

This view lends itself readily to an explanation of the increase in absorption that accompanies fluorescence, for it is clear that during the progress of such a change as is here assumed the substance may possess properties that are entirely different from those of the original substance. It is natural also to expect some intimate connection between fluorescence and fluorescence absorption, for the increased absorption during fluorescence is doubtless due, at least in part, to the same molecules or atoms whose vibrations bring about the emission of light. Without more detailed assumptions regarding the nature of the change produced by the exciting light it is not possible to predict the laws of fluorescence absorption, *e. g.*, the dependence of the effect upon wave-length, intensity of fluorescence, etc. But, on the other hand, the experimental study of these laws will afford a firm foundation for the further development of the working hypothesis.

The considerations which make it natural to expect a change in absorption during fluorescence apply equally well to all cases of luminescence in which there is a change in the active substance during excitation and a return to the original state during luminescence. We should therefore expect a change in absorbing power in the case of thermo-luminescence, chemi-luminescence, and phosphorescence. In fact, it seems not improbable that some modification of Wiedemann's original hypothesis may be found to be applicable to *all* classes of luminescence, and in this case we should ex-

A hypothesis which assumes that luminescence results from some chemical or physical change in the luminescent substance evidently implies not only a change in absorbing power during luminescence, but also temporary alteration in the other properties of the substance. The study of these changes, if such are found to occur, offers a promising means of attacking the general problem. This field of investigation also appears to be practically untouched.

It has generally been assumed, by those who regard Wiedemann's suggested explanation of phosphorescence and thermoluminescence with favor, that the change accompanying luminescence is of a chemical nature. Various suggestions have been made regarding the character of the reactions produced by the exciting light (or other cause), and attempts are not lacking to trace a connection between luminescence and chemical constitution. It appears to us that it is desirable to take into consideration changes of a different kind, which do not involve, necessarily, any chemical action. The simplest change of this kind is dissociation. In cases of photoluminescence this is just the effect that we should expect the exciting rays to produce. We can scarcely doubt that the absorption of the exciting light is the result of resonance on the part of the molecules or atoms of the active substance; and although the vibrational energy thus imparted to the molecules is rapidly transformed by collisions into translational energy (heat) yet under favorable conditions the molecules might be thrown into such violent oscillation as to be torn apart. Since chemical activity is usually increased by dissociation, chemical changes of the usual kind might occur as a secondary effect. Many instances of increased chemical activity due by the action of light on fluorescent substances might be cited.

If we look upon light as an electromagnetic phenomenon we should expect the vibrations produced by the exciting rays to be such as tend to separate the positive and negative parts of the molecule, and the resulting dissociation would either be of the usual electrolytic type or similar to that produced in a gas by the action of Röntgen rays. In the case of fluorescence the latter type of dissociation seems most probable; but in either case, if the view outlined above is correct, fluorescence should be accompanied by an increase in the electrical conductivity of the fluorescent substance. It was in order to test this conclusion that the experiments described in the third section of this article were begun. A more detailed discussion of the hypothesis outlined above, and of the experimental facts which tend to confirm it, is reserved for a later communication.

III. The Influence of Fluorescence upon Electrical Conductivity.

The effect of light and Röntgen rays upon the conductivity of solutions has been tested by Cunningham,[1] in the case of five solutions, one of which, namely, uranyl nitrate, was fluorescent. In the experiments with light an arc containing iron was used as a source, and the solutions were contained in a cell having a quartz window, the object being to obtain intense ultra-violet rays. Although some indication of increased conductivity was observed, the observations were rendered so uncertain by the heating effect of the rays from the arc that the author did not regard the results as reliable.

Experiments of a similar character were undertaken by Regner[2] with especial precautions against the disturbances due to the heating effect of the source. No change in conductivity as great as 0.1 per cent. could be detected, although the substances tested included those for which Cunningham had found indications of a positive result. Several fluorescent substances were tested by Regner, also with negative results.

In the paper by Burke[3] on the change in absorption due to fluorescence, mention is made of preliminary experiments upon the conductivity of fluorescent substances; but the author states that difficulties were met with which led him to abandon these experiments and to take up the study of absorbing power instead.

Our own experiments upon this subject were undertaken with the feeling that the increase in conductivity due to fluorescence was probably very small, comparable rather with the conducting power produced in gases by the action of Röntgen rays than with ordinary electrolytic conductivity in solutions. In attempting to detect so small an effect it seemed advisable to work with solutions in which the normal conducting power was as small as possible. In the case of all the substances thus far tested we have therefore used absolute alcohol as a solvent.[1]

The accurate measurement of the resistance of alcoholic solutions offers considerable difficulty; for while the conducting power is too great to permit the employment of an electrometer method, such as would be used with gases, it is too small to be determined with accuracy by the methods that are generally used with aqueous solutions. No benefit was to be gained in the present experiments by increasing the cross section of the liquid tested, and so reducing its resistance; for if the thickness exceeded a rather small value the absorption of the liquid made it impossible to excite the whole mass to fluorescence. After trying a number of different methods without satisfactory results we abandoned the attempt to accurately measure the total resistance, and directed our attention to the measurement of the *change* in resistance due to illumination. The most satisfactory method that we have found for this purpose is that of the ordinary Wheatstone bridge, using a direct current and a galvanometer of high resistance.

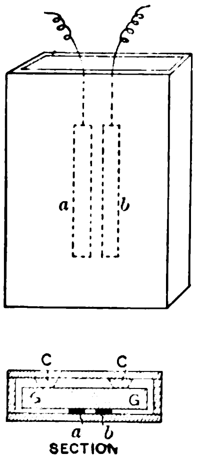

Fig. 28.

The form of the cell used to contain the fluorescent solutions is of considerable importance in its bearing upon the sensitiveness of the method. Among the galvanometers available for use with the bridge, that which seemed best suited for the work, was one having a resistance of about 10,000 ohms. It was therefore desirable to have the resistance to be tested as near to this value as other conditions would permit. On the other hand, the layer of liquid tested should be so thin that it can be excited to strong fluorescence throughout its thickness. Although we did not find it possible to satisfy both of these conditions, they were approximately met by the form of cell shown in diagram in Fig. 28.

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[1] In the experiments of Cunningham and Regner the solvent was water.

The figure shows both a perspective view and a horizontal section of the cell. A thick piece of plate-glass, *G*, was cut of such a size as nearly to fill the cell and was held in position, tightly pressed against the two electrodes *a*, *b* (of platinum foil), by the corks *C*, *C*. Although the drawing is in other respects approximately to scale and nearly actual size, the thickness of the electrodes *a*, *b* is greatly exaggerated. The thickness of the layer of liquid between *G* and the walls of the tube was determined by the thickness of the electrodes and was about 0.1 mm. In the section shown in Fig. 28 the portions occupied by the solution are left unshaded. The length of the electrodes was about 20 mm. and their distance apart 2 mm.

The fluorescent substances tested were eosin, fluorescein, rhodamin, napthalin roth, and cyanin. As already stated, the solvent was in all cases absolute alcohol. The solutions were made quite concentrated, so that fluorescence was confined to a thin layer at the surface. The concentration was so adjusted as to make this fluorescent layer approximateiy 0.1 mm. thick.

The cell containing the solution to be tested was made one arm (*C*) of a bridge. The resistance of the arm *M* in series with *C* was in all cases 9,000 ohms, while the resistance of the third arm, *R*, was 5,000 ohms. The fourth arm, *N*, was varied until an approximate balance was obtained[1] and the apparent resistance was computed by the formula $C = MR/N$. Current was furnished by two gravity cells in series.

Polarization in the fluorescent solution made it impossible to obtain the true resistance by this method, and the apparent resistance of the cell (computed as if polarization were absent), is doubtless in error by 50 per cent. or more. But the sensitiveness of the arrangement to changes in the resistance of the test cell was high. In order to avoid disturbances due to variations in the polarization E.M.F. it was found necessary to keep both the battery circuit and the galvanometer circuit closed. Even the small change made in eter was sufficiently steady for observations to be begun. During this time the cell was protected from the action of light by an opaque screen. When the conditions had become steady, or more frequently when the motion of the galvanometer needle was reduced to a slow uniform drift, the screen was removed and light from an arc was allowed to fall on the cell. After the effect of illumination had been noted, and when the needle had again settled down to a steady drift, the screen was replaced, and the throw of the needle in the opposite direction was observed.

In the measurement of so small an effect as that here considered it is clear that the heating effect of the rays from an arc is likely to produce serious errors, for the change in conductivity due to rise in temperature is unfortunately in the same direction as the change that we are attempting to detect. For this reason the use of the direct rays of the arc, even at the distance of a meter, was entirely out of the question. Even when a water cell was interposed in the path of the rays the needle was displaced through several hundred divisions. That this movement of the needle was due to rise in temperature, and not to the effect sought, was indicated by the fact that the change persisted after the rays were cut off; it was necessary to wait at least fifteen minutes before the original balance was approximately restored.

In order to avoid the disturbances due to rise in temperature we adopted the plan of dispersing the rays from the arc by a prism and using only those portions of the spectrum that were most effective in producing fluorescence. With this arrangement the effects observed were much smaller than before, but they were entirely free from any indication of temperature changes. Upon removing the screen, so as to illuminate that part of the fluorescent solution lying between the electrodes, a throw of the galvanometer needle was observed, and if originally free from drift the needle vibrated about a new position and finally came to rest. Continued illumination of

So far as we were able to judge, the effect produced by light reached its full value at once and ceased as soon as the light was cut off. The disturbance of the balance of the bridge was always such as to indicate an increase in the conductivity of the fluorescent solution. Upon repeating the experiments with rays from different portions of the spectrum it was found that a change in conductivity was produced only by those rays which were able to excite fluorescence in the solution tested. And so far as we could estimate the intensity of fluorescence by the eye, the rays which gave the most intense fluorescence also produced the greatest change in conductivity. In the case of eosin we were able to follow both effects to the extreme edge of the violet, while illumination by the red of the spectrum produced no effect.

In order to form an estimate of the magnitude of the change produced we observed in each case the throw caused by increasing or decreasing the resistance *N* by one ohm. By comparing this with the throw due to illumination, it was possible to express the observed change as a fraction of the normal (apparent) resistance. The results are given in Table III. The light used to illuminate the solution, in the case of the measurements included in the table, was that which produced the brightest fluorescence.

We have also tested in the same way one solution that was not fluorescent, namely, an alcoholic solution of fuchsin. The result was entirely negative. No change in resistance due to illumination could be found in any part of the spectrum, although the sensitiveness of the bridge was such that a change of .008 per cent. could have been detected. The solution was an old one and had prob-

TABLE III.

Increase in Electrical Conductivity due to Fluorescence.

Substance.	*N*	Apparent Resistance of Solution.	Throw Produced by Illumination.	Throw Produced by Increasing *N* by 1 Ohm.	Increase in Conductivity Due to Illumination.
Eosin.	155	300,000	57 mm.	33 mm.	1.1%
Napthalin roth.	45	1,000,000	1	39	0.05
Fluorescein.	300	150,000	10	30	0.11
Rhodamin.	360	125,000	20	40	0.14
Cyanin.	170	270,000	40	45	0.52

ably been made with commercial alcohol, since the resistance, 90,000 ohms, was much lower than that of the other solutions. For this reason we are not inclined to look upon this single experiment with a non-fluorescent solution as possessing much significance.

In the preceding description of the experiments upon fluorescent solutions we have ascribed the observed movements of the galvanometer needle to a change in the *resistance* of the solution. The galvanometer deflections undoubtedly indicate a disturbance in the balance of the bridge; but this disturbance might equally well result from a diminution in the E.M.F. of polarization. Without special modifications the method does not permit a separation of these two possible effects. In the case of eosin special experiments were made bearing upon this point; but with the other solutions we have no direct evidence to show that the observed change is not due to the influence of light upon the polarization of the cell.

The experiments referred to in the case of eosin were of two kinds. In the first of these a bridge method was employed as before. The solution was contained in a glass tube originally about 5 mm. in diameter. A portion of this tube had been drawn down to a diameter of about 1 mm. and the current passed through the liquid contained in this contracted part. The platinum electrodes were placed in the larger part of the tube at each end. We were thus able (1) to illuminate the solution about either electrode while the remainder of the tube remained dark; or, (2) to illuminate the contracted part of the tube while screening the electrodes. In the latter case the effect was observed, as in the experiments already described. In the former case only a very small change could be detected. Although this arrangement was less sensitive than that in which the cell shown in Fig. 28 was used, the results show that only a small part of the effect previously observed can be attributed to a change in polarization.

The other experiments with eosin that bear upon the question of polarization were of an entirely different character, and represent one of our many early attempts to devise a satisfactory method of attacking the general problem. Fig. 29 shows both the form of tube used to contain the solution and a diagram of the connections. The platinum wire electrodes *a*, *b* were connected to the terminals of the secondary of an induction coil, *I*, and to the two pairs of quadrants of an electrometer, *E*. The electrode *C* at the center of the tube was connected to the needle. The primary of the induction coil was excited by an alternating current of 120 cycles. When the solution was not illuminated the electrometer needle stood nearly at its zero. Under these circumstances illumination of the upper half of the tube produced a deflection in one direction, while the illumination of the lower half gave an approximately equal deflection in the opposite direction. The direction of the deflection in each case was such as to indicate a decrease of resistance. The spectrum of the arc was used in these experiments as in those made by the bridge method; and the evidence that the effect was due to fluorescence, and not to rise of temperature, was as conclusive as in the experiments already described. The method was less sensitive, however, and was therefore used only in the case of eosin.

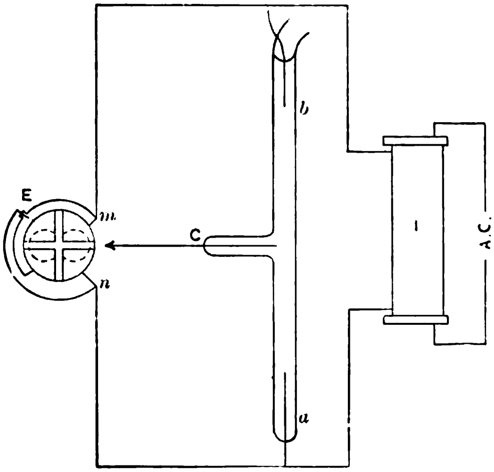

Fig. 29.

The results of these two widely different methods are conclusive in showing that in the case of eosin we have to deal with a

RADIATION PYROMETRY.[1]

By C. W. Waidner and G. K. Burgess.

In a study of some special problems in the measurement of high temperatures we have been lead to examine, in some detail, the field of radiation pyrometry, which is the line along which the most marked advances in temperature measurement have been made in recent years, and have made a study of the sources of error, necessary precautions, and limits of attainable accuracy in the use of some of the best types of radiation pyrometers that are at present available to the experimentalist, and have applied these pyrometers to the investigation of several problems that are of interest in this field of temperature measurement.

General Discussion of Radiation Pyrometry.

Under the term radiation pyrometry may be grouped all those methods in which the temperature of bodies is estimated from the radiant energy emitted, either in the form of visible light radiation or of the longer infra red waves that are studied by their thermal effects. The obvious advantages and special field of application of this system of pyrometry are, in the measurement of the temperatures of inaccessible or moving objects, the estimation of temperatures far beyond the range of any pyrometer requiring contact with the heated substance, and the rapid examination of surfaces for uniformity of heating, a problem of great importance in metallurgical processes.

A great advantage of optical pyrometry is the rapid variation in the photometric intensity of light with variation in temperature. Thus, for example, if the intensity of the red light $\lambda = 0.656\,\mu$

and Pringsheim[1] have found that at 900°, 1,400°, and 1,900°, abs., respectively, the light increases 30, 18 and 14 times as rapidly as the temperature. It is this fact alone, of the rapid change in intensity of light with change in temperature, that makes it possible for the trained eye to estimate the approximate temperature in the many industrial processes dependent on temperature control.

It would therefore appear that a system of pyrometry based on the intensity of the light emitted by incandescent bodies would be an ideal one, inasmuch as a comparatively rough measurement of the photometric intensity would fix the temperature quite accurately. This, however, is only partly true; it is limited somewhat by the fact that different bodies, although at the same temperature, emit vastly different amounts of light. Thus, the intensity of the radiation from incandescent iron or carbon at 1,000° C., for example, is many times greater than that emitted by such substances as magnesia or polished platinum at the same temperature. Consequently, if any conclusions were drawn as to the temperatures of these bodies from the light that they emit, it might lead to large errors. Thus, at 1,500° C. this difference in the intensity of the light emitted by carbon and by polished platinum would lead to a difference in the estimated temperatures of these bodies of about 90° C., and less at lower temperatures.

The possible erroneous conclusions that might be drawn from such a system of optical pyrometry is still further emphasized in considering the results to which it would lead if applied to the measurement of the temperature of flames. Here the intensity of the light would vary with the thickness of the flame in the line of sight. If applied to the estimation of the temperature of a colorless Bunsen flame the method would entirely fail.

At first sight the difficulties here enumerated, incident to the different emissivities of bodies, being a function not of the temperature alone, but dependent on other properties, such as conditions of surface and materials entering into composition, might seem insurmountable, and sufficient to condemn the use of optical pyrometry.

is based, its limitations, advantages, and the necessary precautions that must be observed in overcoming these difficulties.

Kirchoff[1] in one of the most important contributions to the theory of radiation, was led to the important conception of what he termed a "black body," which he defined as one which would absorb all radiations falling on it, and would neither reflect nor transmit any. He further pointed out the important fact that the *radiation from a black body was a function of the temperature alone*, and was identical with the radiation inside an enclosure all parts of which have the same temperature. The first experimental realization of a black body as a practical laboratory apparatus was made by Wien and Lummer,[2] by heating the walls of a hollow opaque enclosure as uniformly as possible and observing the radiation coming from the inside through a very small opening in the walls of the enclosure. It is evident that such a body will absorb practically all the radiation incident through the small opening, and this whatever the material of the walls, for unless the walls were totally reflecting, all the radiation will be absorbed except that portion which might again escape through the opening; the presence of the opening, therefore, causes a slight departure from ideal black body conditions.

Black body radiation in the interval 600° to 1,600° C., may be most conveniently obtained in the laboratory by the electrically heated black body devised by Lummer and Kurlbaum,[3] which consists of a porcelain or fire-clay tube, suitably protected by surrounding tubes and air jackets, which is wound with a platinum foil heating coil, and whose temperature can be maintained constant for long intervals by suitable regulation of the electric heating current. The radiation coming out of the end of this tube from an object at the uniformly heated region near the center, in contact with a thermo-couple, is a very close approximation to ideal black body radiation at a known temperature.

No body is known whose surface radiation is exactly that of a black body. The radiation from carbon and iron approximates fairly near to black body radiation, while the radiation from polished

Radiation pyrometers are calibrated in terms of the radiation from a black body, as this radiation alone is a function of the temperature only. They will then give true temperatures only when used on black bodies. When used to measure the temperature of other bodies ther adiation pyrometer calibrated in this way will not give the true temperature but the equivalent *black body temperature*, *i. e.*, the temperature a black body must have to send out radiation of the same intensity and of the same kind. Two bodies having the same black body temperature may differ very considerably in their true temperatures, thus in the illustration already cited for carbon and platinum at 1,500° C., the difference is of the order of 90°. The temperature of a piece of platinum at 1,500° C. (as measured by a thermo-couple), will, if measured by an optical pyrometer using red light, appear to be about 1,365° C., and if green and blue light are used in the photometry, about 1,405° and 1,425°, respectively (see p. 443), showing that platinum approximates more nearly to black body radiation for green and blue, than for red light.

The great objection to the use of radiation pyrometers, that their indications are not dependent on the temperature alone but also on the widely varying emissivities of different bodies, may then be answered as follows. These pyrometers can be used to measure the equivalent black body temperatures, and it is then possible to pass over to true temperatures by means of such experiments as those given on p. 446, giving the departure from black body radiation. Further, in many cases arising in industrial operations requiring the use of a pyrometer, the body whose temperature is to be measured is not out in the open but is found enclosed in a furnace whose walls are at a temperature not far different from that of the body, so that the radiation is a fair approximation to black body radiation, and the indications of the radiation pyrometer will not differ much from true temperatures. In many other cases, *e. g.*, when the problem at hand is the measurement of the temperature of an enclosed furnace or of a fused metal bath, the difficulty may be overcome by immersing in the bath or furnace a closed end porcelain or iron tube, and observing the radiation coming out of the open end of this tube, in which case the indications of the radiation pyrometer will give true temperatures.

Often a knowledge of the black body temperature fulfills every requirement of the problem at hand, whether it be the control of temperature in an industrial operation or a scientific experiment, and this a radiation pyrometer gives with a very satisfactory precision.

The temperature of any body as measured by a radiation pyrometer will in general (luminescence effects excluded) be lower than its true temperature. There is a source of error, however, that may act in the direction of making the pyrometer read too high, namely, due to light reflected by the body whose temperature is being measured, coming from surrounding flames and hotter objects. Experiments bearing on this point, and methods of eliminating this source of error are discussed under "Diffuse and Reflected Light."

Methods of Radiation Pyrometry.

The methods of radiation pyrometry that are available for the estimation of high temperatures, and which have been carefully studied by a number of investigators, may be broadly grouped under the following heads:

1. *Color Estimation.* — The estimation of the temperature of bodies by the color of the emitted light is still the most widely used, unfortunately so, of all pyrometric methods, especially in metallurgical operations. A comparison by Howe[1] of his results with those of White and Taylor shows that even with skilled observers, estimates by the unaided eye may differ by 50° C. or more at 800° C., and by much greater amounts at higher temperatures.

2. *Photometric and Spectrobolometric Methods.* — Becquerel was among the first to suggest the measurement of the temperature of incandescent bodies by photometric measurement of the intensity of the light emitted. As this method is now generally applied, it consists of a photometric comparison of the red light emitted by the body under observation with the red light emitted by a standard comparison source (gasoline or electric). Red light is chosen because it enables the range of measurement to be extended to the lowest temperatures measurable optically. The first practi

more recently by Wanner, Féry, Holborn and Kurlbaum, and Morse will be briefly discussed later with a view to their trustworthiness as instruments of precision.

The use of monochromatic light obviates the difficulties incident to the photometry of lights of different color, and further it makes it possible to utilize in the calibration of these instruments Wien's law for the distribution of energy in the spectrum of a black body,

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

where J is the intensity of the light of wave-length λ, T is the absolute temperature, e the base of the natural system of logarithms. and c_1 and c_2 constants.

This may be written :

$$\log J = K_1 + K_2 \cdot \frac{1}{T}$$

$$\text{where } K_1 = \log c_1 - 5 \log \lambda \text{ and } K_2 = c_2 \frac{\log e}{\lambda}.$$

If working with visible radiation it therefore requires only two photometric measurements at two known absolute temperatures to completely standardize a pyrometer. For infra-red monochromatic radiation a dispersing apparatus (non-absorbing) must be used in connection with a bolometer, thermopile, or other instrument for detecting radiation. For long wave-lengths and high temperatures, Wien's equation ceases to hold and use must be made of Planck's equation

$$J = c_1 \lambda^{-5} \left(e^{\frac{c_2}{\lambda T}} - 1 \right)^{-1}$$

which the researches of Lummer and Pringsheim,[1] Paschen,[2] and Rubens and Kurlbaum [3] have shown satisfies the experiments for black body radiation throughout the widest range of measurable temperatures (— 200° C. to + 1,500° C.), and wave-lengths (to $\lambda = 51.2\ \mu$). For the wave-lengths of the visible spectrum, how-have pointed out, for values of λT not exceeding 3,000, Wien's and Planck's equations differ by less than 1 per cent. The former may, therefore, under these conditions be used for extrapolation to the highest attainable temperatures (4,000° or more).

3. *Ratio of the Intensities of Two Wave-lengths.* — Crova[1] in 1878 devised a method based on the relative variation in the intensity of two different monochromatic radiations, red and green, but as Violle has shown, the ratio of intensities $\lambda = 0.656\,\mu$ (red) and $\lambda = 0.482\,\mu$ (blue) varies only in the ratio 1 to 4.5 over a temperature interval of 700° C., and again the lower limit of temperature determinations is given by the green (about 1,200°) so that Crova cannot be said to have made use of a practicable principle, although he actually used his spectrophotometer in some of his researches.

4. *Upper Limit of Spectrum.* — Another method, also suggested by Crova, susceptible of use at lower temperatures, was based on the use of the upper limit of the spectrum of an incandescent body. Hempel[2] has made the most extensive investigation of the possibilities of this method, using a special form of spectroscope and luminescent screen for observing when the upper limit of the spectrum is beyond the visible radiations, but compared with other photometric and radiation pyrometers only crude results can be obtained.

5. *Maximum Energy in Spectrum.* — (*a*) Displacement of λ_m. In this method use is made of the displacement, with temperature, of λ_m, the wave-length having maximum energy, as given by Wien's displacement law

$$\lambda_m T = \text{const.} \equiv A$$

or (*b*) by the maximum-energy law,

$$E_{\max} T^{-5} = \text{const.} \equiv B.$$

The constant A varies from 2,930 for a black body to 2,630 for polished platinum. A bolometer, thermopile or other instrument for detecting radiation is used in connection with a dispersing apparatus (non-absorbing) to locate λ_m or to measure $E_{\max}$. If the

the absolute temperature can then be computed. As the black body is the best radiator and polished platinum is one of the worst, it follows that if λ_m is known, the limiting temperatures between which the true temperature of the body must be found, if its radiation is purely thermodynamic and intermediate between that of a black body and polished platinum, are given by

$$T_{\max} = \frac{2{,}930}{\lambda_m} \text{ and } T_{\min} = \frac{2{,}630}{\lambda_m} .$$

Lummer and Pringsheim[1] have used this method to estimate the limiting temperatures of a number of radiants, such as the electric arc, incandescent and Nernst filaments, and others.

The displacement law is among the best established of the radiation laws, both from the theoretical and experimental side, so that this method is of considerable theoretical interest; yet it is not capable of a high degree of precision on account of the difficulty of locating λ_m, due to the flattened form of the energy curve.

6. *Energy of Total Radiation.* — Assuming the Stefan-Boltzmann law, $E = K(T^4 - T_0^4)$ as established, where E is the energy of total radiation, visible and invisible, T the absolute temperature of the radiating black body source, T_0 that of the receiving instrument, and K a constant, this method can be used to measure temperatures of bodies even long before they become incandescent.

For this purpose such instruments as the bolometer of Langley, Crooke's radiometer as modified by Rubens and Nichols, the actinometers of Pouillet and Violle, the radiomicrometer of Boys, and the thermopile may be used.

Féry has recently brought out a practical form of instrument, called a thermoelectric telescope (see page 439), based on the energy of total radiation, which is measured by the rise in temperature produced at one junction of a very sensitive thermo-couple joined to a direct reading pyrometer galvanometer.

7. *Berthelot Interference Method.* — D. Berthelot[2] has introduced a new system of gas thermometry, based on the relation between

the heated gas contained in a refractory tube, which can be kept at a high temperature by a heating coil (or by a current through the body of the tube itself) and the other through a tube of the same gas whose density can be conveniently varied; the two beams are then brought together and produce interference bands. The pressure in the cold tube is varied to prevent a shift of these bands. This is not a radiation method, but may later form an independent check on the extension of the radiation laws to very high temperatures.

Study of Instruments.

The radiation pyrometers which have come into use may be classed according to the law in terms of which their indications are given. Those based on Wien's law are the Le Chatelier,[1] and Féry's modification, known as the absorption pyrometer,[2] and the Wanner polarizing photometer.[3] Féry's thermo-electric telescope,[4] with fluorite lens is calibrated in terms of the Stefan-Boltzmann law, while the Holborn-Kurlbaum[5] and Morse[6] instruments require an empirical calibration.

All but Féry's thermo-electric telescope are photometers, the first three employing two red fields side by side, one given by a standard light and the other by the object studied, and the last two making use of an incandescent lamp filament disappearing against the object observed.

Le Chatelier's Pyrometer. — This instrument is shown in Fig. 1, in which *L* is a gasolene lamp the intensity of whose light, as viewed through a red glass after reflection from the mirror *M*, serves as a comparison standard for the light from the object studied, which passes through the iris-diaphragm *D*, whose aperture is adjusted until the photometric fields are of the same brightness.

Féry has replaced the iris diaphragm by a variable absorbing wedge, thereby decreasing somewhat the lower temperature limit and the sensibility of the instrument. He has improved it by rendering it movable about any axis, by the use of an aperture of constant angle which renders the indications independent of the focussing, and by employing a 45° mirror silvered so as to give a field as shown at *C* when looking at a crucible. The original Le Chatelier pyrometer may also be improved by substituting an electric lamp and diffusing screen for the gasolene comparison lamp.

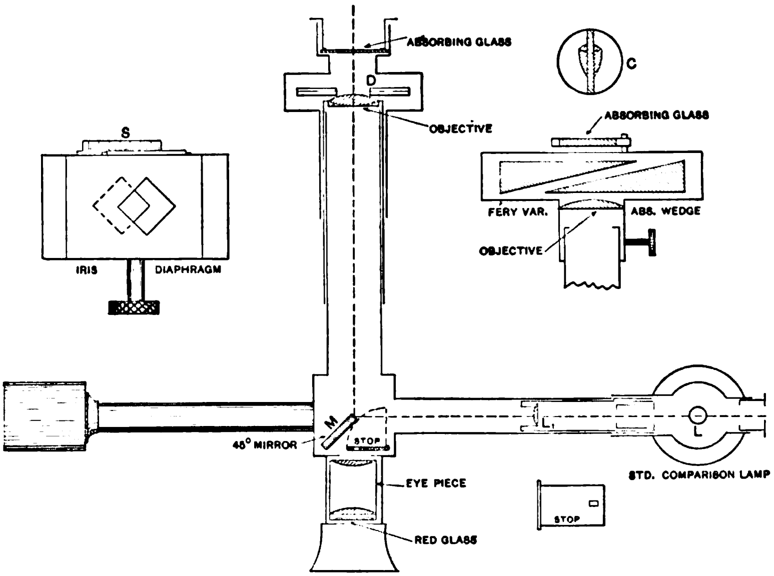

Fig. 1. Le Chatelier Pyrometer.

Le Chatelier calibrated his instrument in terms of the empirical formula

(1) $$J = 10^{a} T^{-\frac{b}{T}}$$

where *a* and *b* are constants depending on the unit of intensity, and *J*, the intensity, is given in terms of the square of the scale readings. A more simple calibration is made in terms of Wien's law, in the form

(2) $$\log J = K_1 + K_2 \cdot 1/T.$$

A good monochromatic glass (such as Jena No. 2745) is required if any calibration is to be relied on at very high temperatures. The following table shows the results of a calibration by both methods.

Observed Temperature (Absolute).	Temperature Calculated by Le Chatelier Formula.	(Le Chatelier) $t_{obs.}-t_{calc.}$	Temperature Calculated by Wien Formula.	(Wien) $t_{obs.}-t_{calc.}$
975°	975°	0	982°	− 7
995	994	+ 1	1,001	− 6
1,147	1,147	0	1,150	− 3
1,154	1,148	+ 6	1,150	+ 4
1,191	1,190	+ 1	1,189	+ 2
1,211	1,214	− 3	1,213	− 2
1,278	1,276	+ 2	1,274	+ 4
1,296	1,290	+ 6	1,287	+ 9
1,398	1,400	− 2	1,396	+ 2
1,459	1,464	− 5	1,460	− 1
1,587	1,602	−15	1,597	−10

The central portion of the flame of a Hefner standard was used as the unit of intensity, the data satisfy the following equations[1]

$$\log J = 6.697 - 9{,}983\,\frac{1}{T} \qquad \text{(Wien's Eq.)}$$

$$J = 10^{7.77}\,T^{-\frac{3660}{T}} \qquad \text{(Le Chatelier's Eq.)}$$

The calibration was carried out by sighting the pyrometer upon the bottom of a small fire-clay crucible placed at the center of a nickel wound electric furnace 60 cm. long and 2 cm. opening; temperatures were measured by two thermocouples with junctions in contact with the crucible.

The effective wave-length of the red light transmitted by the glass used before the eye piece was determined as 0.64 μ. As shown on p. 427 the value of K_2 in the Wien equation is given by

$$K_2 = \frac{c_2 \log_{10} e}{\lambda} = 9{,}983$$

$$\therefore c_2 = 14{,}700.$$

This would indicate that the radiation from the interior of the electric furnace used in this work is a close approximation to black of the furnace produced no very appreciable increase in the "blackness" of the radiation (not greater than 5° or 10°).

The gasolene comparison lamp is most satisfactory when not used where there are drafts; also the wick does not char. As only a small area of the flame is used, changes in height of the flame have but a small influence on the indications; thus we have found a variation in height of 2 mm. is equivalent to a temperature change of less than 6° at 1,000° C. The flame may be easily set to a few tenths of a mm., by inserting in the lamp holder a sight similar to that on the Hefner. The effect of time of burning on the constancy of the lamp is also small, as shown by the following observations.

	Cat's Eye Reading.	Δ in Per Cent.	Δ in Degrees C.
After 25 minutes' burning.	26.03	1.7	0.9° C.
" 45 " "	26.26	7.5	4.2
" 2 hours' "	27.03	5.0	2.7
" 2¾ " "	26.68		

Varying samples of gasolene gave results indistinguishable from each other, and even diluted to over 20 per cent. with a pure, heavy kerosene of flash point = 135° C., identical readings were still obtained.

The instrument may readily be focussed to 2 mm., corresponding to a temperature change of 1° at 1,000°, so that if the focussing is correct to 5 mm., a sufficient precision will be obtained. Varying by a fourth of its value the distance of the pyrometer without refocussing, as in rapid work, will introduce an error of only 5° at 1,000° C.

The agreement among different observers in taking readings is

	Without Absorption Glass.				With Absorption Glass.	
Observer.	1	2	3	4	1	3
Cat's eye scale reading.	7.4	7.8	7.6	7.3	25.7	25.8
	7.4	7.9	7.8	7.0	24.0	24.8
	7.2	7.7	7.6	8.0	23.6	26.0
	7.8	7.8	7.7	7.1	24.1	25.8
	7.7	7.7	7.8	8.3	25.4	24.8
	7.8	7.7	7.4	8.0	24.8	24.9

illustrated by the following observations on the determination of the coefficient of absorption of an absorbing glass placed before the objective.

Observers 2 and 4 had no previous experience with the instrument. The values of K computed from the results of observers 1 and 3 differ by 3 per cent., corresponding to a difference in temperature of less than 2° at 1,000°.

The above discussion shows the ordinary form of Le Chatelier pyrometer to possess a precision of about 1 per cent. from the lowest temperature (600° C.), which may be estimated optically.

Wanner Pyrometer. — In this instrument, Fig. 2, the monochromatic red field is produced by a direct vision prism P and slit S. The slit S_1 is illuminated by a comparison electric lamp and S_2 by the source under investigation. Both beams are divided into components polarized at right angles by the Rochon prism R, and the biprism B is so cut as to bring but two of the eight images side by side in front of the opening D before the analyzer A. The analyzer is set at the normal point by adjusting the brightness of the electric lamp, varying its current or position, to equality with a Hefner placed in a definite position.

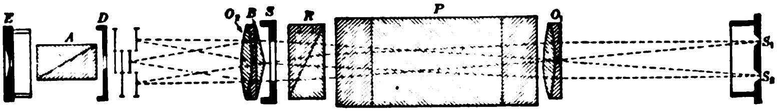

Fig. 2. Wanner Pyrometer.

In terms of the circular scale attached to the analyzer, we have

$$\frac{J}{J_0} \tan^2 \varphi$$

where J_0 is the intensity of light from the standard; Wien's law gives

$$\log_{10}\left(\frac{J}{J_0}\right) = \frac{C_2}{\lambda} \log_{10} e \left(\frac{1}{T_0} - \frac{1}{T}\right),$$

so that a knowledge of the scale reading when $J = J_0$, *i. e.*, at the normal point, and the black body temperature of the standard source are all the data needed for calibration of the scale if it is correctly graduated.

The constancy of the Hefner as here used is shown by the following observations taken at different times when the current through the electric comparison lamp was kept constant:

Reading Wanner Scale.[1]	Deviations.
39.9	−0.28
39.9	−0.28
40.1	−0.48
39.9	−0.28
39.1	+0.52
39.2	+0.42
39.8	−0.18
39.0	+0.62
39.62	0.38

This shows that the flame is constant enough to reproduce the normal point to 0.5 per cent. in temperature, but this must be increased to about 1 per cent., due to the uncertainties in making a photometric setting. The electric lamp can be set at the normal point to better than 1 per cent. if care is taken that the diffusing glass before the slits S_1 and S_2 is clean and is always inserted in the

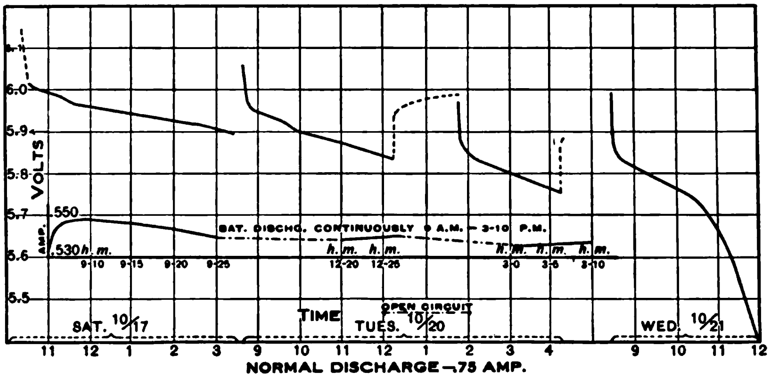

Fig. 3. Discharge Curve of Battery.

of six times the percentage change in current. In order to obtain a precision of 5° at 1,000° C., this current must be constant to 1 per cent., or, as our experiments have shown, to 0.006 amperes for the range 1,000° to 1,500° C. The accompanying table and plot shows the behavior of a three-cell battery of 10 ampere-hours' capacity furnished with the instrument:

Time.	Wanner Scale. Rdg.	Current Through Lamp.	Per Cent. Change in Current Through Lamp.	Apparent Change in Temperature Observed.
15 min.	31.2	0.5645	0.1	1° C.
20 "	31.8	0.5640	1.7	10
27 "	32.7	0.5550	4.3	25
37 "	34.6	0.5400		
38 "		disconnected battery two minutes.		
40 "	32.5	0.5570	1.5	7
42 "	31.7	0.5610	0.6	3
45 "	32.5	0.5560	2.5	15
47 "	33.1	0.5405	4.1	24

A 75 ampere-hour battery gave similar results. It is, therefore, evident that in work of precision it is necessary to control the current through the lamp by means of a rheostat and milliammeter, as otherwise errors of over 25° may be introduced, and greater ones with the battery in poor condition.

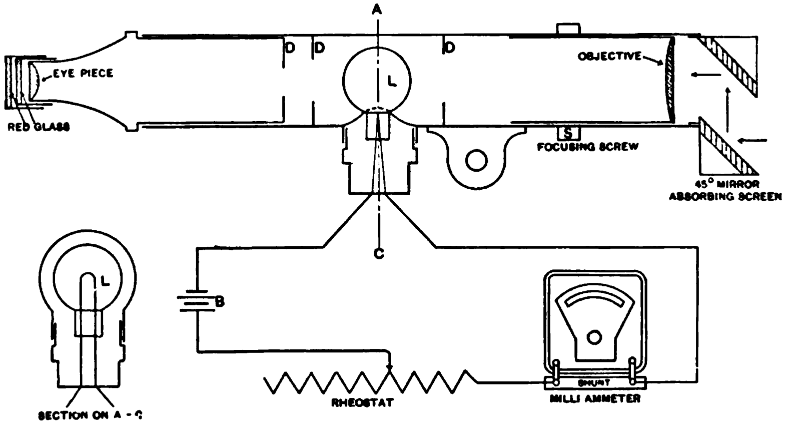

Fig. 4. Holborn-Kurlbaum Pyrometer.

This pyrometer is difficult to sight on small objects, not being a telescope, but on the other hand requires no focussing. Another limitation is due to the fact that the instrument is a polariscope, so that if the light is polarized considerable errors may be introduced in its indications. This effect is eliminated or proved absent by taking readings in four azimuths.

Used with the proper precautions, this pyrometer is one of great precision and convenience.

Holborn and Kurlbaum Pyrometer. — Fig. 4 shows the arrangement of this instrument; the tip of the filament *L* of the four volt incandescent lamp is set to the same brightness as the object whose temperature is sought, by means of a rheostat. When the filament disappears against the bright background, the current through it is a measure of the temperature sought, when the lamp has been calibrated. One or more red glasses is placed before the eyepiece as the temperature rises above 900° C., and for extreme temperatures, above 1,500° C., absorbing glasses or mirrors are inserted in front of the objective.

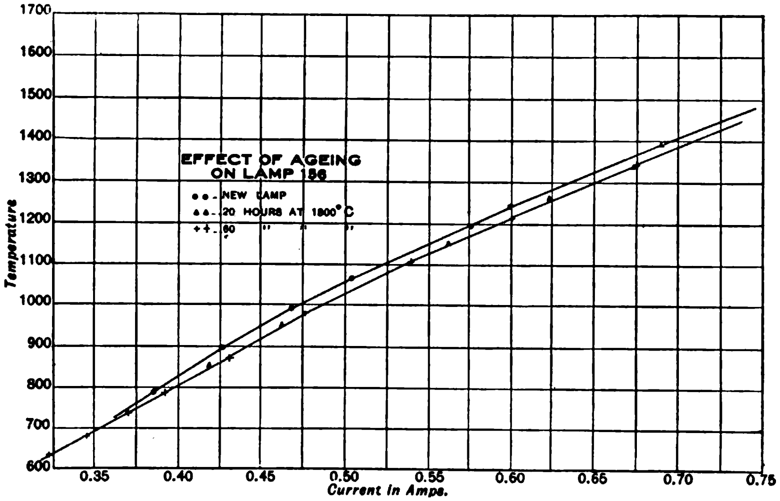

Fig. 5. Ageing of Lamps.

The constancy of indications of this instrument is dependent upon the permanence of the lamp filaments used. Unaged lamps may change by over 25°, as shown by the accompanying plot, but when once aged for 20 hours at 1,800° C., these lamps will not change appreciably in several hundred hours if not heated above 1,500° C. The relation between current and temperature is well given by the quadratic

$$C = a + bt + ct^2$$

where the coefficient c is very small, so that for short ranges of temperature the relation may be taken as linear. A separate calibration is required for each lamp.

The sensibility of this instrument is shown by the following set of observations:

Temperature from H-K Pyrometer.	Temperature from Thermo-couple.	Temperature from H-K Pyrometer.	Temperature from Thermo-couple.
1,347° C.	1,347° C.	632° C.	634° C.
1,351	1,347	634	633
1,343	1,343	633	633
1,338	1,342	633	632
1,342	1,342		

For use at extreme temperatures the absorption factor K of the absorbing glasses used may be calculated from Wien's equation:

$$\log_{10} K = \log_{10}\left(\frac{J_1}{J_2}\right) = \frac{c_2}{\lambda}\log_{10} e\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

where T_1 and T_2 are the apparent black body temperatures (absolute) given by the pyrometer, when sighting on a constant source of radiation first without and then with the absorbing medium. This method of determining K, as well as using Wien's equation in the above form for the estimation of extreme temperatures is applicable also to the other optical pyrometers.

Morse Pyrometer. — This instrument is the same in principle as the preceding, but there is no focussing device provided and the range is limited to that attainable with the unaided eye. A large spiral lamp is used, requiring a more costly battery installation, and it is quite fatiguing to match a particular spot of this spiral with the background. In spite of its crude form, this pyrometer is capable of a precision of better than 5° C. in the limited range for which it is adapted.

Féry Thermo-Electric Telescope. — Radiation from the source whose temperature is sought is focussed by fluorite lens, F (Fig.

6), on one function of a minute iron-constantan couple in series with a potential galvanometer, whose deflections are proportional to the energ yreceived by the couple, and hence to the fourth power of the absolute temperature, according to the Stefan-Boltzmann law, if the radiating source is a black body.

This relation is only true on the condition that fluorite has a constant coefficient of absorption for all wave-lengths. At low temperatures a large proportion of the energy exists in the form of long wave-lengths, and as fluorite has an absorption band in the infra red (near 6μ) the Stefan-Boltzmann law will not hold for this region. Above 900° C., however, Féry has found this instrument to give results to better than one per cent. In making a temperature measurement it is necessary to focus the image of the incandescent body upon the thermo-junction by means of the eyepiece *O* and this image must completely cover the junction.

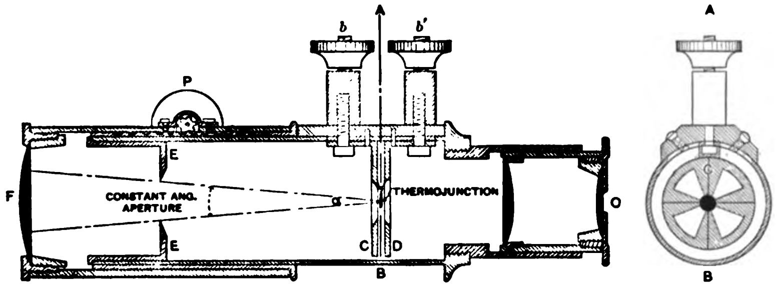

Fig. 6. Féry Thermo-Electric Telescope.

A glass objective may replace the fluorite, when an empirical calibration becomes necessary.

This pyrometer, since its indications are given by a pointer over a scale, is readily made recording, a desideratum in many pyrometric investigations.

COMPARISON OF VARIOUS TYPES OF RADIATION PYROMETER.

An experimental study of the behavior of the various radiation pyrometers shows that no one instrument can claim a decided superiority as to precision, certainty and ease of calibration, permanence of indications, and range. All of the instruments mentioned above are capable of an accuracy of one per cent. in temperature measurement in the ranges for which they are adapted for use, although this accuracy is not attained with the same ease in all cases. With the Wanner pyrometer, for instance, a rheostat and milliammeter have to be inserted in the lamp circuit; and with the Holborn-Kurlbaum and Morse pyrometers, although this accuracy is readily secured, the constancy of the lamp must be assured.

It is possible to control a given temperature or measure temperature differences much more exactly than reproduce absolutely any temperature. For relative temperature measurements the precision attainable is better than one per cent., except possibly for the Féry absorption pyrometer, and may be even as good as 2° at 1,500° C., with the Holborn-Kurlbaum pyrometer.

The lowest temperature measurable optically is about 600° C., which may be reached with the Le Chatelier, Holborn-Kurlbaum and Morse instruments. The Wanner pyrometer and the Féry thermo-electric telescope cannot be used below 900° C.

Regarding the upper limit, there is none except for the Morse, which is difficult to use above 1,100° C.; but for very high temperatures, above 1,500° C., where they can only be determined by extrapolation in terms of some radiation law, the most reliable results will be given by those pyrometers obeying one of these laws the most exactly. Thus, instruments calibrated in terms of Wien's law, but making use of a red glass which is not sufficiently monochromatic, the Le Chatelier, the Féry absorption, and also the Holborn-Kurlbaum pyrometer in the region for which the absorption mirrors or glasses have to be used, will depart from the true temperature scale more than the Wanner pyrometer, which using strictly monochromatic light will obey Wien's law, if its reading circle is correctly graduated and set. The Féry thermo-electric telescope with fluorite lens should agree with the photometric pyrometers, as Wien's and Stefan's laws have been found experimentally in accord to at least 2,000° C., and they probably agree up to the temperature of the arc (3,600° C.).

The only instrument which could readily be made recording is the Féry thermo-electric telescope.

SPECIAL PROBLEMS IN OPTICAL PYROMETRY.

Departure from Black Body Radiation. — Inasmuch as optical pyrometers are calibrated in terms of the radiation emitted by a black body, the temperature as determined by them will depend, as we have seen, on the nature of the incandescent body observed. It is, therefore, of interest to determine the amount by which such indications differ from the true temperatures. With this object in view we have determined the *black-body temperatures* of a number of substances at well-known temperatures, as given by the melting points of pure metals and salts.

Radiation from Platinum. — Among the substances studied was polished platinum, because it deviates farthest from a black body, and thus gives an idea of the maximum difference to be expected. In these experiments for determinations up to 1,500° C. use was made of a Joly meldometer[1] and a Holborn-Kurlbaum optical pyrometer. For experiments at the melting point of platinum a platinum strip about 5 cm. long and 4 mm. wide was cut narrow at the center so that the optical pyrometer could be focussed on the exact spot at which the strip melted. The electric current through the strip was slowly raised until the strip fused, and the reading of the optical pyrometer at this instant gives the black body temperature corresponding to the melting point of platinum (1,780° C.) for the particular color of light used. The results of these experiments are given in the following table: For red ($\lambda = 0.651\,\mu$), green ($\lambda = 0.550\,\mu$), and blue ($\lambda = 0.474\,\mu$) light.

The radiation from platinum at lower temperatures was studied by fixing the temperature of the platinum strip of the Joly meldometer by means of known melting points in the usual way by placing a minute specimen of the metal or salt on the strip and slowly increasing the heating current until the specimen was observed to melt, when the length of the strip was determined by the electric contact micrometer screw, and the current regulated to maintain this length (and therefore the temperature) constant throughout a series of ob-

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[1] Joly, Proc. Roy. Irish Acad., 2, p. 38, 1891.

The Black Body Temperature of Platinum at its Melting Point.

Date, 1904.	No. Lamp Used in H-K Pyr.	Black Body Temp. for Red Light $\lambda = .651\mu$.	Date, 1904.	No. Lamp Used in H-K Pyr.	Black Body Temp. for Green Light $\lambda = .550\mu$.	Date, 1904.	No. Lamp Used in H-K Pyr.	Black Body Temp. for Blue Light $\lambda = .474\mu$.
Apr. 13	No. 156	1,545°C	Apr. 18	No. 156	1,570°C.	Apr. 18	No. 156	1,618°C.
"	"	1,543	"	"	1,585	"	"	1,602
"	"	1,550	"	"	1,578	"	"	1,606
"	"	1,530	"	145	1,577°C.	"	145	1,625°C.
"	"	1,520	"	"	1,577	"	"	1,605
"	"	1,528	"	"	1,561	"	"	1,607
"	"	1,541	"	"	1,577	"	"	1,609
14	"	1,528	"	132	1,591°C.	"	"	1,621
"	"	1,547	"	"	1,587			
Mean	"	1,537°C.	"	"	1,576			
Apr. 14	No. 145	1,550°C.	"	"	1,570			
"	"	1,560						
Mean	"	1,555°C.						
Apr. 14	No. 132	1,540°C.						
"	"	1,550						
"	"	1,533						
"	"	1,542						
"	"	1,548						
Mean	"	1,543°C.						
	Final Mean	**1,541°C.**		Final Mean	**1,577°C.**		Final Mean	**1,612°C.**
		Δ[1] = 239°C.			Δ[1] = 203°C.			Δ[1] = 168°C.

servations with the optical pyrometer. A second determination of the melting point was always made with the meldometer at the end of every series of observations.

The black body temperatures of polished platinum at 1,500° C., 1,064° C., 782° C. and 723° C., as fixed by the melting points of Pd, Au, NaCl, and KBr, and at 1,315° C. and 1,215° C., as fixed by the observed length of the meldometer strip, are given in the following table:

The results of all the experiments on platinum are best shown graphically in Fig. 7, where for comparison we have added the results obtained by Holborn and Kurlbaum[2] for red light by a somewhat different method.

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[1] $\Delta = T - S$, where T = abs. temperature and S = black body temperature.

[2] Holborn and Kurlbaum, Ann. d. Phys., 10, p. 225, 1903.

Radiation from Platinum.

Temp. °C.	Red λ = .651 μ			Green λ = .550 μ			Blue λ = .474 μ		
	Date, 1904.	**Lamp Used in H-K Pyr.**	**Black Body Temp.**	**Date, 1904.**	**Lamp Used in H-K Pyr.**	**Black Body Temp.**	**Date, 1904.**	**Lamp Used in H-K Pyr.**	**Black Body Temp.**
1500°C. (MP of Pd)	Apr. 12	No. 145	1384°C.	Apr. 18	No. 156	1402°C.	Apr. 18	No. 156	1434°C.
	"	145	1368	"	156	1402	"	156	1423
	"	132	1384	"	145	1410	"	145	1430
	"	132	1370	"	145	1398	"	145	1419
	"	156	1368		Mean =	1403°C.		Mean =	1426°C.
	14	145	1378		Δ =	97°C.		Δ =	74°C.
	"	156	1383						
	11	132	1365						
	"	145	1365						
		Mean =	1374°C.						
		Δ =	126°C.						
1315°C.	Apr. 15	No. 132	1208°C.	Apr. 15	No. 132	1233°C.	Apr. 15	No. 132	1269°C.
	"	156	1210	"	156	1233	"	156	1244
		Mean =	1209°C.		Mean =	1233°C.		Mean =	1256°C.
		Δ =	106°C.		Δ =	82°C.		Δ =	59°C.
1215°C.	Apr. 15	No. 132	1122°C.	Apr. 15	No. 132	1135°C.			
	"	156	1117	"	156	1138			
		Mean =	1119°C.		Mean =	1136°C.			
		Δ =	96°C.		Δ =	79°C.			
1064°C. (MP of Au)	Apr. 11	No. 156	969°C.	Apr. 14	No. 132	987°C.			
	"	145	967	"	156	985			
	Mar. 23	156	978	"	156	985			
	Apr. 14	132	978	" 15	156	987			
	"	145	973	"	156	982			
	"	156	980		Mean =	985°C.			
	Mar. 23	with	970		Δ =	79°C.			
	"	Wanner	971						
	"	Pyrom.	974						
			972						
		Mean =	973°C.						
		Δ =	91°C.						
782°C. (MP of NaCl)	Apr. 11	No. 145	714°C.						
	"	132	720						
		Mean =	717°C.						
		Δ =	65°C.						
723°C. (MP of KBr)	Apr. 11	No. 145	660°C.						
	"	132	665						
	"	156	662						
		132	670						
		156	670						
		145	671						
		Mean	666°C.						
		Δ =	57°C.						

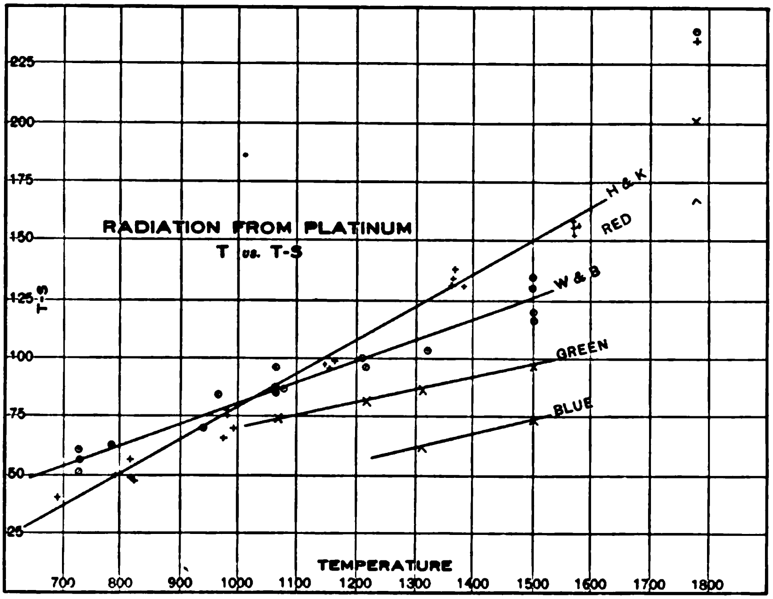

Fig. 7. Radiation from Platinum.

The black-body temperature of the melting point of platinum for red light, as found by these observers, is 1545° C. ($\Delta = 235°$), which is in most satisfactory agreement with the results found by us, which is 1541° ($\Delta = 239°$), obtained by the same method of experiment.

Our results would seem to indicate that the values of Δ increase somewhat less rapidly with rise in temperature than is indicated by the results of Holborn and Kurlbaum. These differences may arise from several causes. The meldometer strip is probably at a slightly higher temperature than indicated by the melting-point calibration, and by an amount which is probably greater at high temperatures. This would tend to make the value of Δ small at high temperatures. In the experiments of Holborn and Kurlbaum the platinum was in the form of a cylinder surrounding a thermo-couple. It is thus possible that the couple was at a slightly higher temperature than the outer radiating surface of the platinum which would tend to increase the values of Δ, especially at high temperatures.

In order to pass through the platinum point ($\Delta = 239°$, red light) a very sudden bend in the curves is necessary. No evidence is yet at hand to explain whether this can in any way be associated with a decreased radiating power of a substance as it approaches a change of state.

It will also be seen, as has been previously shown, that platinum approaches more nearly to black-body radiation for green and blue than for red light.

Radiation from Other Substances. — Experiments were made on a number of other substances, including clay, copper and iron oxides, and unglazed porcelain to determine the departure of these bodies, at various temperatures, from black-body radiation. Another method of experiment was here necessary, as it is not possible to accurately fix the temperature of oxidizable substances by the meldometer method. The method used will be understood from the following diagrammatic sketch, Fig. 8 :

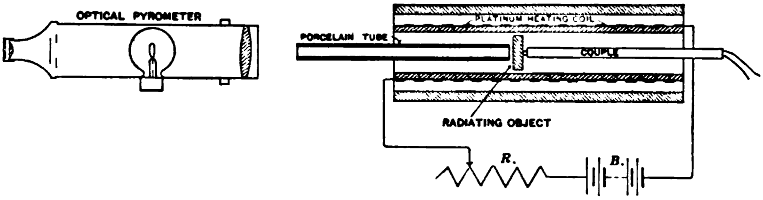

Fig. 8. Radiation Furnace Method.

The radiating substance is placed at the center of an electric furnace. The radiation emitted by the substance under these conditions is a very close approximation to black-body radiation, and its temperature as measured by the optical pyrometer will be the true temperature. As a further check on the optical pyrometer a thermo-couple in contact with the radiating substance was always read. A porcelain tube, open at both ends and blackened on the interior, was then thrust into the furnace and the radiation coming out of this cold tube, which is the radiation of that particular substance, was observed with the optical pyrometer. The temperature measured in this way is the equivalent black-body temperature of the substance for the particular color of light used. The tube was then quickly withdrawn and another measurement taken of the true temperature. There was always a slight cooling due to insertion of the tube, which was allowed for by taking the observations in the order indicated above. The results of these experiments giving the departure from black-body radiation are given in the following table.

The experiments at the melting point of copper were carried out in a manner similar to those at the melting point of platinum. A copper strip cut narrow at the center was observed with an optical pyrometer at the instant it was fused by an electric current. The temperature of melting was assumed to be 1065° C., corresponding to the melting point of copper in an oxidizing atmosphere, and the corresponding black body temperature was given by the optical pyrometer.

In order to compare the *meldometer method* with the *furnace method* we determined the departure of platinum from black body radiation by both methods. The agreement is even better than we have any right to expect, thus at 950° C., the furnace method gave 74° C., and the meldometer method (by interpolation) 75° C., as the difference between the true temperature and the black body temperature of platinum.

The interesting range of iron and steel in which hardening and annealing operations are carried out is from about 600° C., to 950° C., and as will be seen from the table, the indications of an optical pyrometer in this range, if calibrated in the usual way in terms of black body radiation, will be low by about 20° at the lower temperature and about 40° at the upper temperature.

The radiation from the fine-grained crucible seemed to be at least as black as that from iron, which would hardly be expected. When a fresh piece of thin Russian iron was inserted into the furnace and measurements were immediately taken with an optical pyrometer it was observed that the initial readings were some 20° higher at 1000° C., than when the exidation of the surface was complete, which takes two or three minutes.

Polarized Light from Incandescent Surfaces. — Arago was among the first to observe that the light emitted by incandescent surfaces

Departure from Black Body Radiation.

Temperature (Cent).	Platinum.	Iron Oxide.	Fine Grain Battersea Crucible.	Copper Oxide.	Unglazed Porcelain.
730°C.					31° 35 25 30 30°C.
750°				22° 28 25 25°C.	
770°		20° 26 23°C.	11° 16 14°C.		
790°		20° 21 22 26 23 23°C.			
950°	76° 71 77 81 82 60 67 74°C.				
980°		43° 43 46 53 45 46°C.			
1050°			50° 58 57 52 54°C.		
1065°				37° 50 33 37 43 43 30 39°C.	

waves are also polarized in the same plane as the light waves. Arago attributed the polarization to refraction, near the surface, of the light coming from molecules below the surface.

Millikan,[1] who has done the most complete work in this field, has pointed out that this view of Arago fails to explain why the light coming from platinum near grazing emergence is practically completely polarized, for such light must to a large extent be surface light. His experiments indicate that the polarization is a phenomenon of refraction, and that all the light suffers refraction at the surface. Millikan has examined the amount of polarization in the light emitted by a large number of substances. The phenomenon is most marked in platinum, silver and gold, and is very feeble in the readily oxidizable substances, such as iron and in such substances as glass and porcelain. It is quite marked for iron in the molten state.

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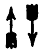
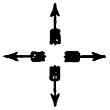
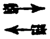
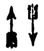
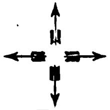
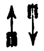
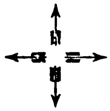
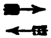
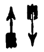
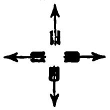
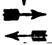
[1] Millikan, PHYS. REV., 3, pp. 81, 177, 1895.

Our experiments have been made mainly with a view to determining the amount by which this phenomenon could influence the indications of optical pyrometers that make use of polarizing devices to adjust to equality the light from the incandescent body observed and from the standard comparison light.

In the experiments on platinum the temperature of the horizontal strip of a Joly meldometer, maintained constant by regulating the electric heating current, was measured with a Wanner optical pyrometer, which employs polarizing devices. To eliminate the effect of the sag of the strip the readings were taken with the pyrometer in four positions at right angles. For each temperature the measurements were made at different angles with the normal to the surface. In rotating the pyrometer the portion of the radiating strip viewed was slightly changed, but an examination of the strip showed that the heating was very uniform, the variations not exceeding 4° over a distance of 2 cm. The results of the measurements on platinum are given in the following table:

From the above table it will be seen that if the temperature of a platinum surface is measured with a polarizing pyrometer large errors may result if the effect of polarized light is neglected. Thus, if the surface be viewed at an angle of 50° with the normal, two readings may be obtained differing from one another by 80° C. at 1450 C., *i. e.*, the reading may differ from the actual temperature by 40° or thereabouts.

As the light emitted in a direction perpendicular to surface is not polarized, this source of error can always be avoided by viewing

Angle at Which Incandescent Surface is Viewed.	Position of Slit of Pyrometer with Reference to Incandescent Strip.	Mean Reading Wanner Pyrometer.	Maximum Δ for Two Positions of Slit at Right Angles.
		PLATINUM AT 1450° C.	
30°		63	
0		64.6	3.3 divs. ∽ 34°
30		66.3	
50		61.9	
0		66.0	7.6 divs. ∽ 80°
50		69.5	
70		60.8	
0		64.9	8.4 divs. ∽ 90°
70		69.2	
		PLATINUM AT 1150° C.	
50°		23.0	
0		24.5	2.8 divs. ∽ 22°
50		25.8	

the surface normally. For most substances on which optical pyrometers are used the amount of polarized light is very feeble compared with platinum, so that its effect is entirely negligible. In some experiments where the incandescent surface of iron was viewed at an angle of 75° the effect was less than 5°. In any case the effect may be eliminated by taking readings in four azimuths, at in-from other sources which is not reflected directly into an optical pyrometer, will not seriously effect its readings, a rise of only 5° being caused by changing from a dark room to bright daylight.

The magnitude of the effect of directly reflected light is very variable, depending upon the reflecting power of the incandescent body observed, the size and location of the disturbing sources, and difference in temperature between the latter and the observed body.

A platinum strip at 800° C. had its temperature apparently raised 120°, due to direct reflection from a large gas flame, and by 300° when placed within a ring of gas flames.

Iron (oxide), which unlike platinum is a poor reflector at high temperatures, underwent an apparent rise in temperature of 35° for the first case and its temperature could not be observed in the second. At 1100° C., however, turning off and on the ring of flames made no appreciable difference in the reading for iron.

This disturbing effect may be very nearly eliminated by viewing the bright object through a tube which cuts off most of the light from surrounding flames, thus the error of 35° above noted was reduced to 5° by this means.

The Measurement of Very High Temperatures. — The temperatures that have been discussed thus far are within the range controlled by thermo-couples calibrated to agree with the gas scale to about 1150° C., which marks at present the upper limit of satisfactory gas thermometry. The thermo-couple scale is then extrapolated for 500° or 600° more.

Already there are many operations, such as those carried out in the Moissan furnace, the Goldschmidt thermite process, the production of carbides and metallurgical products in electric furnaces, and many pyrochemical reactions, that involve temperatures of 2000° or over. It therefore becomes necessary to establish at least some tentative scale that can be used at these extreme temperatures.

Attempts are being made by Nernst and others to estimate these high temperatures by means of chemical phenomena taking place at high temperatures, but this work is still in a preliminary state. For this purpose, therefore, recourse must be had alone to the

Lummer and Pringsheim[1] have recently taken a single set of observations on an electrically heated carbon tube in an atmosphere of nitrogen, using three radiation methods: photometric (Wien's law), spectrophotometric ($\lambda_m T = A$), and total radiation (Stefan-Boltzman law), the results agreeing to 20° at 2300° C. absolute.

From our own work it would seem that the radiation laws are still in agreement at the temperature of the arc.[2] Our measurements have given as the black body temperature of the hottest part of the positive crater 3690°, 3680° and 3720° absolute, as determined with the Holborn-Kurlbaum, Wanner, and Le Chatelier pyrometers, based on the extrapolation of Wien's law. Féry[3] gets for this temperature 3760° by a method based on Stefan's law.

On the basis of these experiments it would seem that the several laws of radiation are in quite satisfactory agreement at the highest attainable temperatures, and thus serve to define the same scale of temperatures.

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[1] Lummer and Pringsheim, Verh. d. Deutsch. Phys. Ges. (5), 1, p. 3, 1903.

[2] Waidner and Burgess, PHYS. REV., 19, p. 241, 1904; Bulletin, Bureau of Standards, 1, p. 109, 1904.

[3] Féry, C. R., 134, p. 977, 1902.

A LIMITATION IN THE USE OF THE WANNER PYROMETER.

By Leon W. Hartman.

THE accurate determination of the temperature of a body heated to a state of incandescence presents many vexatious difficulties to the scientific investigator. As yet the most satisfactory as well as the most accurate method of measuring high temperatures is based on the thermo-electric principle, in which the electromotive force generated by a thermo-element, raised to the temperature of the source in question, is measured. This measurement having been made, the required temperature can at once be read from the calibration curve of the thermo-element. This method, however, has its limitations and objections. It is often inconvenient or impossible to introduce a thermo-element into the heat source whose temperature is sought; suitable sensitive apparatus in portable form may be wanting; and in all cases one is limited to temperatures lower than the melting point of the thermo-element. To avoid these difficulties, therefore, pyrometers of another sort, depending upon various principles, have been designed. Among the latter are those depending upon optical and photometric principles. Two forms which have been elsewhere described may be mentioned as illustrations, *viz.*, the optical pyrometer of Holborn and Kurlbaum [1] and the Wanner pyrometer. [2]

As illustrating the optical principle involved in the Kurlbaum instrument, one may place in front of an incandescent body whose temperature can be arbitrarily increased or decreased, a second incandescent body for example the filament of a glow lamp whose bodies are equally bright, both have the same temperature and the lamp filament will apparently vanish, leaving the first source only as a background. This condition can be most simply brought about by varying the temperature of the filament until the desired point is reached. If the source serving as a background is the interior of a platinum oven whose temperature can be measured by means of a thermo-element, the temperature of the filament of the glow lamp can be determined for varying current density, and this in turn will serve as a sensitive and portable form of pyrometer. Naturally the two sources should emit light of the same color; otherwise, light of a definite wave-length should be used. In the Wanner instrument, which is in reality a photometer with polarizing Nicols, light from a glow lamp mounted beneath the apparatus, and burning under fixed conditions, is compared with the light from the source whose temperature is desired. When the intensity of the field due to the glow lamp is seen to be equal to the intensity of the field produced by the other source, the position of the analyzer is read off from a graduated circular scale, and from a prepared table given out with the instrument the corresponding temperature can be taken. Simplicity of form and construction, and accuracy and rapidity in making temperature readings, are the advantages claimed for this instrument.

Recently in making a series of measurements to determine the temperatures of a number of different-sized platinum wires, when the temperatures varied from 700° C. to 1500° C., it was the writer's privilege to use a Wanner pyrometer. From the values obtained it was suspected that the instrument gave temperature readings much below the true values. To test the matter, a platinum oven containing a piece of magnesium oxide enclosing the end of a thermo-element was mounted on a stand in front of the telescope of the pyrometer. It was found that the temperatures of the heated oven as recorded by the pyrometer and by the thermo-element were in good accord. The platinum wires mentioned above were then the oven, were observed through a telescope and the current flowing in the wires was gradually varied until they apparently vanished from the field of view. The platinum wires and the furnace were then at the same temperature. The temperatures of the oven and wires, and the temperatures of the wires alone, were then measured by the pyrometer, and a marked difference between the two series of readings was found to exist. Then a whole series of measurements was made with the platinum wires placed in front of the oven. In all four different platinum wires were thus tested. In Fig. 1 the curves expressing the relation between temperature and watts per cm. length are drawn. The deviation of the one series of observations from the other is thus graphically shown. Each curve in this figure is the mean of three other curves, each of which was made at a different time. The curves obtained from the observations made with the pyrometer and wires alone, are designated I., II., III. and IV.; the primed curves, obtained by placing the wires in front of the oven, as above described, are designated I′., II′., III′. and IV′. The power consumed was obtained in the usual way from the

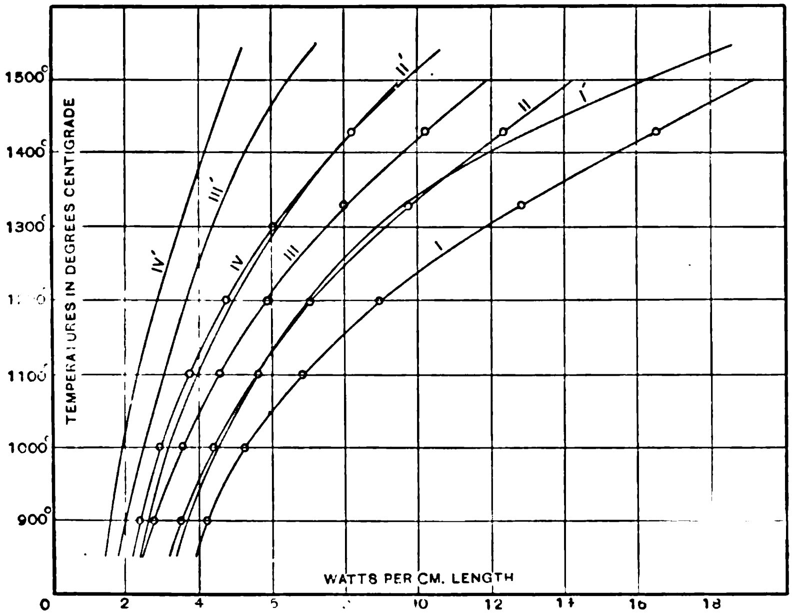

Fig. 1.

measurements of the current strength and potential difference between the terminals of the wires. The instruments used throughout had been carefully calibrated so that no error from this source is present.

Under the two different conditions stated in the preceding paragraphs the values of the Watts per cm. length, required to give temperature readings on the pyrometer equal to those designated below, were determined. These values, together with the diameters of the wires, are recorded in the following table. The primed and unprimed letters have the same signification here as above.

Wire.	Diameter.	Temperatures in Degrees Centigrade.						
		900°	1000°	1100°	1200°	1300°	1400°	1500°
		Watts per cm. Length for Above Temperatures.						
I	0.0690 cm.	4.20	5.60	6.52	8.95	11.85	15.44	19.13
II	0.0440 "	3.50	4.40	5.65	7.07	9.11	12.58	14.24
III	0.0300 "	2.75	3.55	4.55	5.83	7.50	9.60	11.91
IV	0.0194 "	2.40	2.95	3.75	4.75	6.05	7.70	9.50
I′	0.0690 "	3.67	4.57	5.61	7.12	8.85	11.38	16.33
II′	0.0440 "	2.60	3.14	3.98	4.98	6.16	6.90	7.68
III′	0.0300 "	2.18	2.43	3.03	3.70	4.38	5.28	6.47
IV′	0.0194 "	1.55	1.90	2.30	2.87	3.50	4.14	4.88

From a mere inspection of this table it will be seen that the percentage error in the energy measurement was the greatest in the case of the smallest wire; and that this error varies with increasing temperature from 20 per cent. to 100 per cent. The probable cause of this error is a diffraction effect produced when one has a source of illumination with such small dimensions. In any case this instrument is to be used with caution in measuring the temperatures of small narrow sources such as incandescent rods or wires. For incandescent bodies presenting considerable surface such as the interior of furnaces or pottery ovens, the instrument seems to satisfy the claims made for it.

For the apparatus furnished, for the many courtesies shown, and *for the* helpful suggestions given I wish, in conclusion, to acknowledge my indebtedness to Prof. W. Nernst, in whose laboratory the

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SPECTROPHOTOMETRIC STUDY OF SOLUTIONS OF COPPER AND COBALT.[1]

By B. E. Moore.

THE effect of dissociation upon the color of solutions may be studied in two ways — by the photographic or by the spectrophotometric method. The latter has the advantage of quantitative determination of absorption both in strong and in dilute solution. The latter method has been used by a number of observers, who have generally confirmed Ostwald's law, viz: "That the color of dilute solutions having a common ion is identical."

The method consists in the study of the absorption, of such solutions in the concentrated and dilute form. It occurred to the writer that these solutions could be diluted in another uncolored solution containing a common ion of great concentration, and thus prevent a large part of the dissociation, or, what is the same thing, repress the dissociation by the principle of mixed electrolytes.

The present experiments are limited to the study of the sulfate, chloride, nitrate and acetate of copper; and the sulfate, chloride and nitrate of cobalt. They have been given the usual treatment and have besides been diluted in strong acids corresponding to the negative ion of the salts. The method eliminates the absorption of all components except the one which is studied.[2]

All the copper salts, in dilute water solutions have identical absorption per equivalent in the strong absorption region at the red end of the great dilution. They are, however, far from it at the dilutions studied. In the regions nominally transparent, the copper salts show great divergence in the dilute water solutions, and the indications are for larger divergence with greater dilution. The standard solution of copper sulfate and this solution diluted both in water and acid give identical absorption per equivalent in the red band, *i. e.*, in this region the color of the ion and molecule are identical. In this same region the nitrate and chloride become relatively more transparent upon dilution.[1] When diluted in acid they become relatively darker and, if the acid is of adequate strength, even become darker than the original solutions. The acetate shows identical absorption in all solutions, and in the red region the absorption is more than twice that of the sulfate. The three cobalts become relatively more transparent upon dilution. Treating then with acids darkens the dilute solutions of the chlorides and nitrates but makes the sulfate more transparent. Possibly some of the cobalt sulfate was precipitated by the sulfuric acid treatment.

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[1] Müller in Drude Ann., XII., p. 767, states there is an increased absorption of the copper nitrates and acetates upon dilution.

NEW BOOKS.

Thermodynamics and Chemistry; a non-mathematical Treatise for Chemists and Students of Chemistry. By P. DUHEM. Authorized translation by GEORGE K. BURGESS. New York, John Wiley & Sons, 1903. Pp. xxi + 445.

This book presents the main facts of physical chemistry in a very clear way without the use of elaborate mathematical methods which would only bewilder those for whom the book is intended. The various theorems, which stand out prominently in italics, are stated very concisely after a short preliminary discussion in each case, and, nearly every theorem is illustrated by one or more suitable examples chosen from the research literature of physical chemistry, references being given.

These illustrative examples, in consideration of the great number of them, constitute the most valuable feature of the book. It is perhaps the best treatise on physical chemistry at present available for chemists and for students of chemistry. The author and his translator have done a good service and they deserve much credit.

The ideas of work and energy, of heat and internal energy, of chemical calorimetry, of thermal equilibrium and reversible transformations, and the fundamental principles of chemical equilibrium from the point of view of thermodynamics are briefly developed in the first five chapters. Chapters VI. and VII. are devoted to the phase rule in general, introducing the ideas of monovariance and multivariance of systems, and these ideas are developed in detail in Chapters VIII., IX., X., XI., XII., XIII., XIV. and XV. The chemical mechanics of perfect gases is discussed in Chapter XVI, capillary actions in Chapter XVII., false equilibria in Chapters XVIII. and XIX., and explosions in Chapter XX.

The first five chapters are not so satisfactory as the remainder of the book. The old notion of the mechanical equivalent of heat is retained by the author. No quantitative notion of heat can be more satisfactory or more simple than that which is based directly upon the equivalence of heat and work, and it is time that every one should speak of the specific heat of water rather than of the mechanical equivalent of heat. Also we do not believe that the author has reduced to its simplest and most precise terms the thermodynamics which he gives in Chapter V. Thermodynamics presents at the present time a great many theorems which are developed in many of our present day manuals in ways which never could lead to anything but error or to previously known conclusions and the severe limitations which Professor Duhem has chosen to place upon this book do not wholly clear him of this criticism, in our opinion.

Mr. Burgess' work is on the whole well done inasmuch as it seems to be faithfully accurate and it is intelligible, although in some places slightly awkward, partly because of the numerous French idioms which have been retained and partly because of apparent carelessness of expression; although the evidences of carelessness are not many. To justify this criticism, which we hesitate to make in view of Mr. Burgess' service to English-speaking students, we give one of the very few evidences: "According to the law of the displacement of equilibrium by the variation of temperature, the mass of hydrogen sulphide gas formed within a system where hydrogen and sulphur are heated at constant volume should be, at the instant of equilibrium, the feebler as the temperature were the higher."

In conclusion we are pleased to state that the reading of the book has been on the whole very pleasant and very profitable to us.

W. S. FRANKLIN.

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